AN INTRODUCTION TO ASTRONOMY

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AN INTRODUCTION

TO

ASTRONOMY

BY

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PREFACE

An attempt has been made in this volume to give an introductory account of the present state of the science of astronomy. The aim has been to present the subject so that it shall be easily comprehended by the student without mathematical or extensive scientific training, and so that he may obtain from it not only some knowledge of scientific achievements, but also something of the spirit which inspires scientific work. Numerous brief historical references have been introduced to show by what steps the marvelous results of astronomical investigations have been reached.

The value of the "laboratory method" in science is universally recognized. The intimate knowledge which it requires gives a subject a reality that can be obtained in no other way, and it is the method which often develops in the student a deep love for nature. Every one who is familiar with the planets and brighter stars looks with delight up into a clear evening sky. To secure for the student the advantage and pleasure of this first-hand knowledge, a chapter on the constellations has been introduced almost at the beginning. Numerous suggestions and exercises for practical observations, both with and without a telescope, have been given. It is intended that the student shall begin to acquire an acquaintance with the heavens very early in his study of the subject, and that he shall keep up his observations until it is finished; it is even hoped that, whatever his subsequent interests may be, a cloudless sky will never fully lose its charm for him.

One of the chief reasons why the "laboratory method" is valuable is that it requires the active attitude of the mind rather than the passive. Fortunately, this mental attitude is not necessarily limited to those activities in which observational or experimental data are being found. The mind is quite as active in seeking out the relations among facts and in deriving general theories from them. Consequently, in expounding the vast mass of material which the student could not verify if he should devote his whole life to astronomy, yet which is necessary for his understanding of the subject, the deliberate plan has been to explain the facts of observation in connection with the theories which astronomers have built upon them. For example, in the chapter on the motions of the earth, the various facts which prove the heliocentric theory are given sequentially, while in a catalogue of astronomical data, arranged according to the methods of making observations, they would be widely separated. The danger of overestimating the value of theories is avoided by the numerous examples where they have been abandoned or modified as a result of new observational data, or of a more critical analysis of the old.

Finally, the aim has been to give the student a well-balanced conception of the astronomy of the present day. It is desired that it shall become an appreciable part of his mental picture of the general universe, which strongly colors all his opinions even though he is not aware of the fact. When considered in relation to its influence upon the general intellectual horizon, astronomy has claims which are second to those of no other science; and plain duty demands that in outlining courses of study these claims shall not be ignored.

The author desires to express his appreciation of the illustrative material so generously furnished, particularly by the

Lick and Yerkes observatories. Professor G. W. Myers has offered many valuable criticisms and suggestions on a large part of the manuscript, and Messrs. W. D. MacMillan and E. J. Moulton have been of great assistance in reading the proofs. The author desires to express his sincere thanks for this efficient assistance.

F. R. MOULTON.

THE UNIVERSITY OF CHICAGO,
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CHAPTER I

PRELIMINARY OUTLINE

1. Object of a Preliminary Outline. — The greatest benefit is derived from the study of the details of a science when the relations which they have to the subject as a whole are known. It is always found that the chief difficulty in getting a thorough understanding of a scientific subject consists in obtaining a satisfactory appreciation of the bearings of isolated facts upon the general theories to which they are related. After the whole science has been gone over, a review gives a good opportunity for bringing out the essential interrelations, and this is one of the main reasons why a final recapitulation of important data and fundamental principles is oftentimes so valuable. But the same ends may be attained to a considerable extent from a preliminary outline of the scope of the science in question, the methods of investigation employed in it, the results of the observations and experiments which have been made, and the theories which flow from them. For this reason a very brief discussion of the scientific method and an outline of the science of astronomy will be given at once.

2. Astronomy as a Science. — Astronomy is the science which treats of the positions, motions, shapes, sizes, masses, physical conditions, and evolutions of all the heavenly bodies. It is a *science*, for the facts which have been acquired by observations and experiments are classified on the basis of

their essential relations to each other and to the facts and principles of other sciences. It is this which makes the subject valuable.

The fact that astronomy is a science implies that there are astronomical theories, or *doctrines*, giving unity to the great variety of data upon which they depend. It is found upon reflection that, although theories must be based upon the facts obtained by observation, they are not less important and interesting than the facts themselves. Indeed, facts of observation attain their full importance and interest only when their bearings upon the theories are understood. Thus, some things will be discussed which would be trivial and out of place if they were not of assistance in proving some of the most fundamental and far-reaching theories in astronomy. Consequently, since astronomy is a science, it is not sufficient to regard it as a catalogue of facts. The significance of the facts must be given a prominent place. When considered in this way they possess the highest interest and are easily retained.

Notwithstanding the importance and necessity of considering data in connection with the general principles and theories to which they are related, there is a danger which always threatens. It is that a theory may become a hobby, and that it may induce a false interpretation of facts. The scientist must be absolutely honest in his work, and he must use every means to keep himself from being prejudiced. A very effective way of guarding against prejudices is to consider alternative hypotheses simultaneously. This method of always keeping in mind various possibilities undoubtedly has been followed more or less consciously by all the great discoverers.

3. Inductive and Deductive Methods of Investigation. — Methods of scientific investigation are of two kinds, the *inductive* and the *deductive*. The inductive method consists in drawing somewhat general conclusions from a large num-

ber of separate facts established by observation or experiment. For example, it was found by experiment something over a century ago that when two elements unite chemically, they invariably unite in definite proportions. It was inferred from this, as an induction, that the elements are composed of ultimate small particles, called *atoms*, which are not divided by chemical processes; that two kinds of atoms unite with each other in a perfectly definite way; and that, therefore, *any* two elements will *always* unite in a perfectly definite proportion which need be determined but once. This induction has played an important rôle in the splendid achievements in the science of chemistry.

There is an obvious chance for error in the method of induction, because the conclusions reached are broader than the data upon which they are based. Thus, it has been found recently that atoms are highly complicated bodies instead of being ultimate small particles, and that they are divisible under proper conditions. And there is a long array of scientific theories which new facts, furnished by observation, have shown to be partially incorrect or imperfect inductions. This does not mean that the inductive method should be abandoned. The method is, on the contrary, absolutely essential for the development of science. The point to be acquired here is that the method of induction has a possible weakness, and that the results obtained by it should be continually tested by observations and experiments.

The deductive method consists in deriving particular results in special cases from general principles which are known, or are assumed, to be correct. If the logical processes agree with the relations in the physical world, the only chance of error lies in the fundamental assumptions or premises. This is a very real and dangerous source of error, for it is so liable to be overlooked, particularly if the reasoning has been made in mathematical terms. The deductive

mathematical part is seldom open to question, but this in no way insures the correctness of the premises (inductions) upon which it is based. The only safe method is to compare carefully and unceasingly the results of deduction with actual phenomena. Their agreement verifies and increases confidence in the general principles from which they were deduced; their disagreement often effectively and quickly discloses errors.

4. The Steps of Scientific Inquiry. — Astronomy, as well as every other natural science, is partly inductive and partly deductive. In its very beginning it consisted of the noncorrelated facts of common observation. Then followed a period of inductions in which many of the conclusions were incorrect. After a considerable body of inductions had been made, it became partly deductive and mathematical. The most important basis of deduction has been the Newtonian law of gravitation. In the hands of mathematicians it has been an instrument for making investigations of the most far-reaching character; the results obtained from it have very often directed and anticipated observations, and their subsequent verification has established its correctness with a certainty that is not equaled in the case of any other principle. For example, without going beyond the walls of his study, Newton proved from the laws of motion and the law of gravitation that the earth is bulged at the equator. It was sixty years before this fact was verified by the actual observations, which cost journeys of thousands of miles and several years of time (Arts. 95 and 96).

The steps of scientific inquiry are, therefore, observations and experiments, inductions, deductions, and verification of deductions by more observations and experiments. As a science is developed the deductive work relatively increases, but never becomes sufficient alone. Among the natural sciences astronomy is preëminently deductive; this is due partly to its relatively great age and partly to its nature.

5. The Divisions of Astronomy. — Just as the whole field of natural science is divided into a large number of separate sciences, so each separate science gradually becomes divided into a number of parts which are fairly distinct. They are generally distinguished by the methods of investigation which are employed in them, and in an old science like astronomy a single person rarely makes important discoveries in more than one division. It is not because the possibility of making discoveries becomes exhausted, for as the circle of the known grows larger the circumference of the unknown which surrounds it correspondingly increases, but it is because the methods of making them become more and more specialized and distinct.

Astronomy has gradually become separated into the following divisions: Descriptive Astronomy, Practical Astronomy, Theoretical Astronomy, Celestial Mechanics, and Astrophysics. Some of these have several subdivisions, as will be found when they are described more in detail. To these should be added some of the arts which are of great importance to the science, such as instrument making and mounting.

A specialist in any one of the divisions of astronomy should be familiar with the problems and conclusions of all the others in order to appreciate fully the points of contact of his own work. This is especially true at the present time, for astronomers were never before face to face with so many problems demanding solution. On the other hand, there was never a time when so many discoveries were being made as now. When it is said that the present progress is faster than ever before, it is not meant that principles of immense importance, like the law of gravitation, are being established every little while. Such an achievement is the culmination of a long period of progress, just as the sudden view of a lofty plateau follows a long and toilsome ascent.

6. Descriptive Astronomy. — As the name implies, descriptive astronomy consists of a popular statement of the methods, facts, and theories of the whole science. It is that which is given in elementary astronomical text-books, and is generally all that is acquired by those who are not professional astronomers. It is also developed somewhat less systematically by public lectures, magazine articles, and journals of popular astronomy. It is worthy of the warmest support, for it lays open to the cultured public a broad field of ideas in which many minds find the rarest delights. It is characteristic of modern life that its intellectual treasures are scattered as widely as possible, instead of being hoarded by the few as was done in antiquity.

7. Practical Astronomy. — Practical astronomy treats of the adjustment and manipulation of astronomical instruments, the methods of observation, and the processes by which the facts are obtained from them. It is composed of several quite distinct parts, depending upon the purposes for which the observations are made. The principal ones are: (1) the determination of the positions of the stars; (2) the measurement of the dimensions and the study of the markings of the moon, sun, and planets; (3) the search for comets and nebulas; (4) the search for and measurement of double stars; (5) the measurement of the brightness of stars, particularly those which are variable; and (6) photographic work.

(1) The determination of the positions of the heavenly bodies is a work of the highest importance. To show that this is so it is only necessary to point out that it is from observations of the positions of the heavenly bodies that their motions are found, and that it is from their motions that their masses, the law of gravitation, and much of the philosophy of astronomy are derived.

The stars are the basis of reference of position, and it was supposed in ancient times that they were absolutely fixed

with respect to each other. In the year 134 B.C. a bright star, visible even in the daytime, suddenly appeared where none previously had been known, and this stimulated the astronomer Hipparchus (about 180–110 B.C.), to make a catalogue of all the stars he could easily see, 1080 in number. Since the invention of the telescope the possibilities in this line have been enormously increased. The instrument which has been almost universally used is a small telescope delicately mounted so as to turn in the plane of the meridian. The observations are excessively laborious and entirely devoid of anything bordering on the sensational. The most extensive piece of work carried out by a single observer is Argelander's catalogue of 324,198 stars in the northern hemisphere. About fifteen years ago a plan was developed by international coöperation for obtaining a more exhaustive catalogue of the whole sky by photographic processes.

The theory of making and reducing observations was revolutionized by the great German astronomer Bessel (1784–1846), who is known as the father of modern practical astronomy.

(2) The study of the moon, sun, and planets has been carried on with large telescopes, which are mounted so that they may be readily pointed to any part of the sky. By their aid nearly all that is known about the moon and the planets has been discovered. In the case of the sun the spectroscope has been of relatively greater value.

(3) In the search for comets a comparatively small telescope, with low magnifying power, mounted so that it may be pointed at any part of the sky, is used. As only four or five comets are discovered yearly, the amount of labor that may be expended without results is easily imagined.

(4) The search for, and measurement of, double stars is made with a large telescope and high magnifying power. A great amount of time has been spent in this work, and results of much interest have been obtained. For example, the mo-

FIG. 1. — The Great 36-inch Telescope of the Lick Observatory.

tions of double stars make it practically certain that the law of gravitation holds true in every part of the visible universe. The double stars are the only ones, except the sun, whose

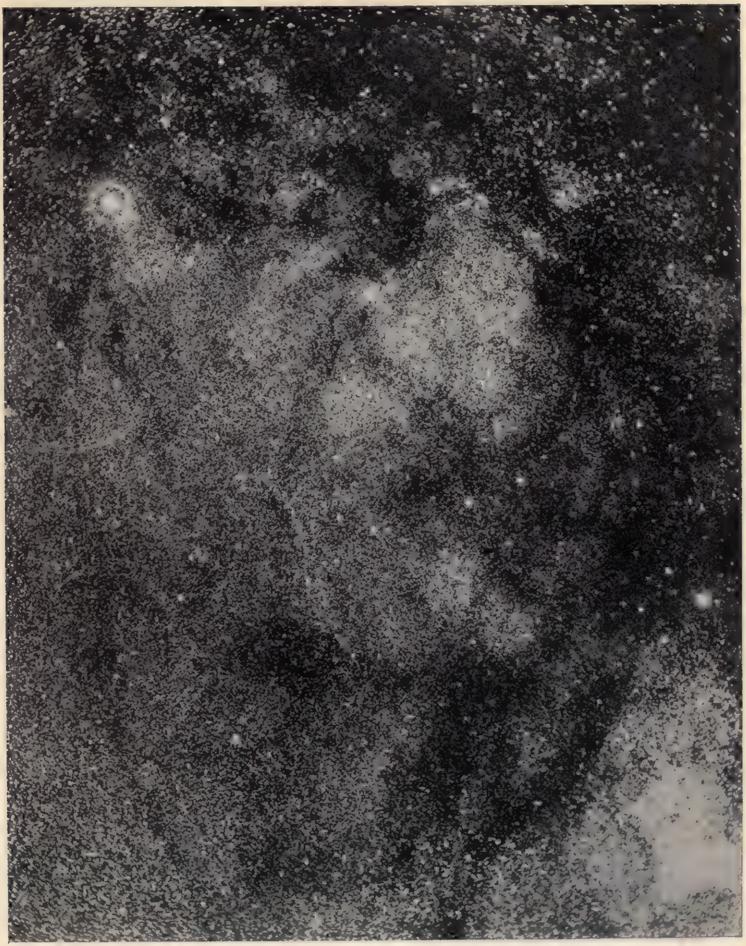


FIG. 2.—Star Cloud in Sobieski's Shield (Milky Way). *Photographed by Barnard.*

masses can be determined, and from the study of them it has been found that the sun is greatly surpassed in size and splendor by some of the other stars.

(5) The measurement of the brightness of stars, or *photometric* work, has received relatively little attention from astronomers, but it is rapidly increasing in importance. Among the problems which lie in this province are the interpretations of the strange phenomena of the various types of variable stars. The results already obtained show, for example, that there are many cases in which a dark sun and a bright sun revolve around their center of gravity.

FIG. 3. — The Moon. Age 21 d. 16 hr. *Photographed at the Lick Observatory.*

(6) Astronomical photographic work on any considerable scale is very recent. Star prints were secured at the Harvard College Observatory in 1850, and Draper obtained a most excellent picture of the moon in 1863; but it is only since the advent of the dry-plate processes, in the seventies, that astronomical photography has made its rapid advances. Its uses are very numerous, and for some purposes it has superseded the eye. It has made possible many of the more delicate spectroscopic investigations. Likewise it is particularly valuable where the object to be observed is of large apparent angular dimensions or faint. By the cumulative effects of long exposures, light which is so feeble that it leaves no sensible

impression on the retina of the eye will write the image of its source on the photographic plate. Objects in these classes are faint comets, faint nebulas, and faint stars. Photography shows, by counting the number of stars obtained in sample regions, that there are probably more than one hundred millions of these bodies.

Photography is also very useful in obtaining permanent records, all on the same scale, of objects which can be equally well viewed directly. Pictures of the moon fall in this class. Up to the present time it fails in showing fine markings on the planets, in the measurement of close double stars, and in general where high magnifying powers are required.

8. Theoretical Astronomy. — Theoretical astronomy treats of the computations of orbits from a sufficient number of observations, the computation of apparent positions, or *ephemerides*, of bodies whose orbits are known, the corrections of approximate orbits, and the computation of the effects of disturbing forces. Considerable mathematics is involved in this division of the science, but more stress is laid upon the methods and results of computation. Much of the work is very tedious, and it is largely carried out under the direction of the governments of the prominent countries, which publish annual ephemerides of the moon, sun, planets, and the phenomena of eclipses, etc.[1]

9. Celestial Mechanics. — When a science reaches such a high state of development that the laws connecting its phenomena are known, it becomes more largely deductive. When these laws and all the phenomena at one epoch are known, a knowledge of the phenomena of any past or future time can be obtained, provided sufficiently powerful deductive processes are available. For example, if the position and velocity of a falling body at a certain instant are given,

[1] Every school should have the current copy of the "American Ephemeris and Nautical Almanac," which can be obtained from Washington for one dollar.

it is possible to determine its position and velocity at any time before it strikes some obstacle, for the laws of its motion are simple and the necessary data are known. But, on the other hand, if the seed of an entirely strange plant were given, it would not be possible to tell the color of the flower it would produce, for the laws of its development would be largely unknown, and the necessary data could not be determined; and even if the laws and the data were known, the necessary deductive processes for deriving the consequences which follow from them would be lacking.

Fig. 4. — Sir Isaac Newton.

More than two centuries ago Newton formulated the laws which describe the way in which all material bodies move. On the basis of these laws he proved that the sun, moon, and planets interact on each other according to the law of gravitation. When the positions, velocities, masses, shapes, and dimensions of these bodies are known at any epoch, the determination of their positions, velocities, and shapes at any other time is simply a question of mathematics. Such problems as these constitute the field of *celestial mechanics*. Its development started with Newton, and it has been cultivated

with increasing zeal up to the present time. Most of the problems of immediate interest in the solar system have been solved, principally by methods of approximation, but there are whole domains as yet entirely unexplored. The unsolved problems are being conquered simultaneously with the development of mathematics.

One of the ultimate aims of all work in astronomy is to discover the general laws in accordance with which phenomena succeed each other, and in this quest the researches in celestial mechanics are of a twofold value. In the first place, they furnish nearly all the predictions that can be made; and in the second place, they constitute essential and very important steps in the verification of general principles obtained by induction.

One of the most remarkable examples of the efficiency of general principles in leading by mathematical processes to important results is the discovery of the planet Neptune. From certain very small, hitherto unexplained, peculiarities in the motion of the planet Uranus, Adams and Leverrier showed almost simultaneously by profound and laborious processes that an unknown planet existed in a certain part of the sky. Under the direction of theory Galle found the new planet within half an hour. This signal triumph of prediction from theory is almost without a parallel in any other science (Art. 285).

The methods of celestial mechanics treat also many questions where the influences at work are so feeble that their results do not become sensible in the time covered by observations. For example, Darwin has shown that most interesting results flow from the tidal interactions of the earth and moon, and of non-rigid bodies in general.

The methods of celestial mechanics are also very important, for they lead to results which never can be reached by direct observation. Thus, it has been shown by tidal, precessional, and rotational phenomena that the interior of the

earth is on the average a very rigid solid, a far-reaching result which is evidently incapable of direct verification.

10. Astrophysics. — The spectroscope, which is the chief instrument of astrophysical research, has been in active use in astronomy less than fifty years, yet its contributions to knowledge have been truly amazing. The results which it has furnished are the more precious, for they cannot be obtained in any other way; and, indeed, until after the application of this instrument to astronomical investigations, no one had the boldness to hope that the regions which it has revealed would ever be accessible to us. It furnishes us with no less marvelous information than the chemical constitution of the sun, and of stars so remote that it takes their light many years to come to us; it distinguishes for us luminous gaseous bodies from those which are solid or liquid; it tells us much of their temperature and physical condition; and it gives us the means of determining the rate at which we are relatively approaching toward, or are receding from, them. Its application to astronomical problems has been only begun, and no one can predict what the future developments will be.

The instruments employed in astrophysical researches are entirely different from those used in other classes of investigations. They are of the most delicate type, and no one but a specialist can manipulate them so as to secure valuable results. The interpretation of their revelations depends very largely upon physical and chemical experiments which they oftentimes suggest. A well-equipped modern astronomical observatory contains a physical and chemical laboratory; and it is here, where three sciences unite, that the essential unity of the visible universe, notwithstanding its great diversities, is demonstrated. These investigations have become such an important part of practical astronomy that this branch of the subject has been given the name *astrophysics*.

11. Astronomical Doctrine. — The labors of astronomers have resulted in the accumulation of a large number of facts and established theories which will be called *astronomical doctrine.* In this preliminary outline only enough of a glimpse can be given of the wealth of the material which is to be unfolded to assist one in properly recognizing the application, and estimating the value, of the separate facts.

12. Motions of Planets. — The earth is nearly round, rotates on its axis once in a day, and revolves around the sun once in a year. The orbit in which it moves is an ellipse with the sun at one of its foci, and the motion is such that the line joining the earth and the sun sweeps over equal areas in equal times. The motions of the seven other planets, except for their different periods, are similar. It was from these facts that Newton derived the law of gravitation.

According to the law of gravitation, every particle attracts every other particle with a force which is proportional to the product of their masses and inversely proportional to the square of their distance apart. No law has had more ample verification than this has in the solar system, and there are excellent reasons for believing that it holds true throughout the visible universe. The law states how gravitation acts, but there is no generally accepted theory respecting its nature. The proof of this law and the deduction of a large number of its consequences make the name of Newton immortal. The law of gravitation is involved in every mass and motion, and every phenomenon depending upon mass and motion, and if it should ever be shown that it is seriously in error, which is very unlikely, the greater part of the edifice of astronomical and physical theory would fall with it.

13. The Sun. — The sun is a vast, intensely hot globe which is gaseous in its exterior parts, and probably fluid, viscous, or possibly solid, in its deep interior. It is surrounded by an enormous envelope of gaseous material. It rotates on

its axis in the same direction that the planets move in their orbits. The spectroscope shows that its lower atmosphere contains the vapor of iron, calcium, sodium, and more than thirty other terrestrial elements, mostly metals. This fact and the appearance of the planets strongly suggest a common origin of all the members of the solar system.

The sun is losing enormous amounts of energy by the radiation of heat and light, which in the course of millions of years will become exhausted, however great may be its actual and potential store. Its temperature is kept up, at least partly, by its shrinking, due to the mutual attraction of its parts, the energy of position being transformed into the energy of molecular motion, that is, into heat. This points to the conclusion that in past times it had vastly greater dimensions than it has at present.

14. The Stars. — The stars are suns which are so remote that they appear to be very small, and that their motions produce only very slight changes in their apparent directions. The nearest star known is 275,000 times as far from us as we are from the sun, and nearly all of the remainder are immensely farther away. In most cases it takes the light centuries to come from them to us, although it travels at the rate of 186,330 miles per second.

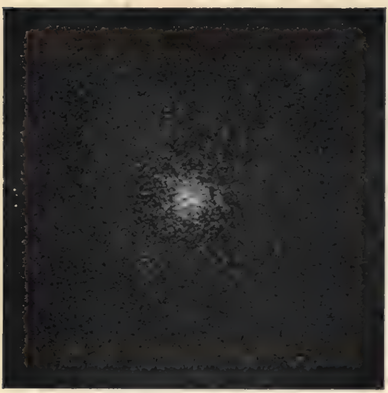


Fig. 5. — Great Star Cluster in Hercules. *Photographed with the Crossley Reflector at the Lick Observatory.*

The millions of stars which can be seen are spread out in a sort of disk, or perhaps in a series of vast streams of stars, in the plane of the Milky Way. They are moving in all directions, some-

times at the rate of more than one hundred miles per second. The spectroscope shows that they generally contain familiar elements. The stars do not always occur singly like our sun, but frequently in pairs and greater numbers, and sometimes in dense clusters made up of thousands of separate bodies. The masses of the few which have been determined are comparable to that of the sun, some being greater and others smaller. There are many stars of unknown masses which radiate hundreds of times as much light as the sun.

FIG. 6. — Spiral Nebula. M. 33. *Photographed by Keeler with the Crossley Reflector at the Lick Observatory.*

15. Nebulas. — There are vast gaseous masses called *nebulas* scattered through the sky, their volumes being millions of times greater than those of the stars. They present the most varied forms, some surrounding stars like exaggerated

FIG. 7.—Nebula in Cygnus. *Photographed by Ritchey with the 2-foot Reflector at the Yerkes Observatory.*

atmospheres, others being wound out in great spirals, others faint and diffuse and widely scattered, and others long, scraggy streaks.

16. Evolution in Astronomy. — It is believed that the solar system has developed out of a nebula, possibly a spiral something like that shown in Fig. 6. This theory is of great interest, and has many points of essential contact in other sciences, particularly with geology. One of the problems confronting astronomers is the induction of the mode and laws of this development so that the theory may become a firm basis for the interpretation of facts. In this ambitious task nothing less is contemplated than a philosophical and historical description, at least in general outlines, of the development of the whole system from a primitive nebulous mass to its present state, and a confident prediction of what its evolution will be during millions of years to come. It is proposed to sweep aside, by inductive and deductive processes, the narrow bounds of space and time, and to furnish a view of the panorama of events in which the development of continents is but an incident. It should be added promptly that this problem is to be solved, not only for the solar system, but also for all the stars and systems of stars which fill the sky. In a way this complicates a difficult problem, but in another it simplifies it immensely, for the diversity of conditions and states of development furnishes opportunities for the comparison and testing of theories. The laboratory, the sun, the solar system as a whole, the stars, and the nebulas must be made to respond to the draft for information and to furnish suggestions and means of verification.

17. The Value of Astronomy. — In this practical age there is a tendency among many to regard astronomy as of little direct importance. But this idea is quite erroneous, for every day in the year astronomy furnishes time to the railroads and makes it safe to run trains, it makes it possible to navigate the sea, it enables surveyors to fix accurately the

boundaries of countries, it has given the very laws upon which mechanics is based, and it has made unrivaled contributions to our general understanding of nature. The ancient Greeks had keen minds, but knowing little beyond the shores of the Ægean Sea, most of them supposed that the earth is flat and the center of the universe, that the sun is a piece of burnished metal, and that the gods live at the summit of cloudy Olympus. How different our present conceptions! They have been obtained by the painstaking observations of Hipparchus, Ptolemy, and Tycho Brahe, by the perilous voyages of Columbus and Magellan, and by the theories of Copernicus, Kepler, and Newton.

In the following chapters an exposition will be given of the elementary geometrical machinery which is necessary for the concise and accurate description of astronomical facts; then will follow a direct study of the constellations, of the solar system and its evolution, and of the sidereal universe in general. A growth into an understanding of these great subjects will be the source of the keenest pleasure and of the greatest intellectual benefit.

QUESTIONS

1. Enumerate as many benefits as possible which may be derived from a review. How does the answer differ in the case of science study from that of language study?

2. Which of these benefits can be largely derived from a preliminary outline?

3. Can the method of alternative hypotheses be employed in subjects which are not sciences?

4. If two hypotheses satisfy all the data equally well, what conclusion is to be drawn? What would be the conclusion if but one hypothesis were under consideration?

5. Give an example of a correct and of a false induction.

6. Give an example of a correct and of a false deduction.

7. What science seems to you to be most largely inductive? Is any science purely deductive? How do you class mathematics?

8. What is an art? What arts are used in astronomy?

9. Is any science independent of all arts? Is any art independent of all sciences?

10. Is it an axiom or an induction that all natural phenomena succeed each other according to unvarying laws? This is ordinarily called *the law of cause and effect.* If it were not true, what would become of science?

11. Is the law of cause and effect true in intellectual matters? For example, will the study of astronomy produce effects on your mind which will permanently modify it?

12. What relation does a superstition bear to the law of cause and effect?

13. Explain, with illustrations, the steps in the development of a science.

14. Enumerate the principal divisions of astronomy, and explain in what respects they differ from one another.

15. Outline the astronomical doctrine respecting the motion of the planets, the character of the sun, and the nature of the stars.

16. In what ways is a theory of evolution in astronomy of interest and value?

17. Discuss the direct and indirect value of astronomy to mankind.

CHAPTER II

REFERENCE POINTS AND LINES

18. Problem of describing Directions. — The motions of the earth and of all the heavenly bodies are obtained directly or indirectly from observations of directions, just as one finds that a distant train is moving by noticing that its direction is different at different times.

Since astronomy is a very exact science, it follows that some scheme of exactly and concisely describing directions must be devised. When one looks at a star he can not tell how far away it is; in fact, all the celestial objects seem to be on a spherical surface commonly called the sky, but known in astronomy as the *celestial sphere*. This appearance is so striking that the ancients supposed that the stars are actually small, bright bodies set in a crystalline sphere. Although it is now well known that the stars are immense suns whose distances differ greatly, yet it is convenient for many purposes to consider that they are on the surface of the celestial sphere. This means simply that as seen from the earth they are projected upon this surface. Consequently, to describe the direction of a star from us it is only necessary to give its position on this fictitious celestial sphere.

The relationship between angles and arcs occurs also in solid geometry. For every proposition relating to trihedral angles there is a corresponding one relating to a triangle drawn on the surface of a sphere. Since the lines on a spherical surface are so much more easily thought of than the corresponding angles at its center, the former are more commonly used. Similarly, the conception of a figure on

the celestial sphere is simpler than that of the corresponding angles whose vertices are at the observer.

This chapter will be devoted to an exposition of the systems of points and lines by means of which positions on the celestial sphere, which are equivalent to directions from us, are described; to showing how to find their defining points and lines; and to exhibiting the relations which exist among the systems.

19. Object of finding Directions. — It has been pointed out that one of the purposes of measuring directions is to determine how the heavenly bodies move. This work is of much value, for it leads to the laws of their motions, and from these laws many important consequences follow. But there is another reason for measuring directions. The distance of an object can not be found from a single observation of its direction; but from two observations, made at different points, it can.

Suppose one desired to know accurately the distance across an impassable river, say the Niagara. It could easily be found as follows:

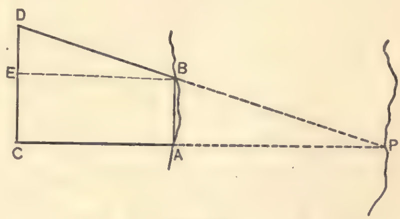


FIG. 8.

Let the distance required be $\overline{AP}$. Suppose $\overline{CAP}$ and $\overline{DBP}$ are straight lines, and that $\overline{BA}$ and $\overline{DC}$, and $\overline{AC}$ and $\overline{BE}$ are respectively parallel. Then the triangles PAB and BED are similar. Hence the following proportion is true:

$$\overline{PA} : \overline{BE} = \overline{AB} : \overline{ED}.$$

Suppose the distances $\overline{BE}$, $\overline{AB}$, and $\overline{ED}$ are measured; then $\overline{PA}$ can be computed from the proportion, and is given by the equation

$$\overline{PA} = \frac{\overline{AB}}{\overline{ED}} \overline{BE}.$$

In practice the work would be done a little differently. Suppose the distance $\overline{AB}$ and the angles PAB and ABP are measured. Then two angles and the included side of the triangle are known, which, according to plane geometry, completely define the triangle. From these data the side $\overline{AP}$ can be computed by plane trigonometry. In fact, trigonometry enables one to compute all the unknown parts of any triangle which is completely defined by having a sufficient number of parts given.

In quite a similar way the distance to the moon is measured.

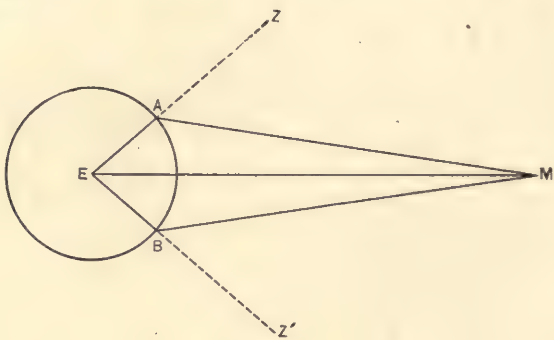


FIG. 9.

Suppose the moon is observed simultaneously by an observer in the northern hemisphere at A and by one in the southern hemisphere at B; and, for simplicity, suppose that A, B, M, and E are in the same plane. The zenith of the observer at A is Z, while that of the observer at B is Z'. The observer at A measures the angular distance of the moon from his zenith, or the angle MAZ, and the observer at B measures the angle $Z'BM$. Suppose their latitudes are known, and that the radius of the earth $\overline{AE} = \overline{BE}$ is known. The sum of the latitudes is the angle BEA. In the quadrilateral $EAMB$ the three angles MBE, BEA, and EAM, and

the two sides $\overline{EB}$ and $\overline{EA}$ are known. Consequently, the quadrilateral is defined and the other parts may be computed. When $\overline{AM}$ has been found, the distance $\overline{EM}$ can be computed from the triangle EAM.

There are many practical difficulties in solving the problem which have not been pointed out here, but the method is essentially that which has been given. It is to be noted that the results are perfectly reliable; in fact, the percentage of error is much less than in most measurements made on the surface of the earth.

It follows that not only the determination of motions, but also of distances, is a problem of measuring and describing directions.

20. Relations between Arc and Linear Measure. — The apparent distances between the stars and the diameters of such objects as the sun will be expressed in degrees, minutes, and seconds of arc. The connection between the arc measure and the actual diameter of an object depends on its distance, and this relation will now be shown. When the angle subtended is small, fairly accurate computations may be made by very simple means, which will be used over and over again throughout this book, and an explanation of this process will be given before the attention becomes absorbed in the mastery of the systems of reference points and lines and their relations.

The circumference of a circle is divided into 360 equal parts called *degrees*. It is shown in plane geometry that the ratio of the radius to the circumference of a circle is $2\pi = 2 \times 3.1416 \cdots$. The equation for this relation is

$$2\pi r = 360 \text{ degrees of arc},$$

where r is the radius of the circle. From this equation it is found that

$$r = 57.3 \ldots \text{ degrees of arc.}$$

Suppose the angular diameter of an object is D degrees, that its linear diameter is l units, and that its distance is r units; it is required to find the relation among these quantities. The angle D has the same ratio to 57.3 that the arc l does to the arc r, whence the proportion

$$D : 57.3 = l : r.$$

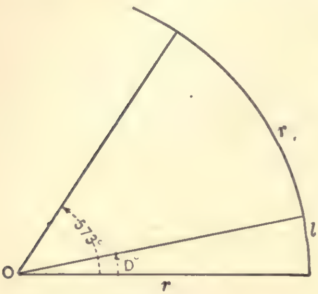


FIG. 10.

When the angle D is small, the subtended chord is nearly equal to the arc and may be used in place of it. As an illustration, suppose the distance from A to B, Fig. 9, is equal to the radius of the earth, or 4000 miles. Suppose it is known from the three measured angles that the angle BMA is one degree, and consider the problem of finding the distance $\overline{MA}$. In the formula D equals one degree, l equals 4000 miles, and r is required. The result is

$$r = \frac{57.3\,l}{D} = 229{,}200 \text{ miles.}$$

The data and result are approximately correct, the true distance to the moon being 240,000 miles in round numbers.

Consider another example. Suppose the distance to the moon is 240,000 miles and that its apparent angular diameter is one-half of a degree; required its linear diameter. In this case $r = 240{,}000$, $D = 0.5$, and l is required, whence

$$l = \frac{D}{57.3}\,r = 2094 \text{ miles.}$$

Likewise, if l and r were given, D could be found.

QUESTIONS AND EXPERIMENTS

1. Do the stars actually seem to you to be all at the same distance?

2. Does the moon seem to be at the same distance when high in the sky as when near the horizon?

3. Does the sky seem to you to be truly spherical?

4. What are the different cases in which a triangle is defined by three of its parts?

5. Actually measure some convenient distance, as across the street, by the method explained by Fig. 8.

6. When you look at a near object, your two eyes see it from slightly different directions. This gives it the appearance of solidity, a fact used in stereoscope pictures. Measure the distance between the pupils of the eyes and compute the amount in angular measure they must turn in to see an object at reading distance, 18 inches.

7. Can you estimate distances accurately with one eye? Catch a ball with one eye closed. See how near you can put the tip of your finger to an object when one eye is closed.

8. How many minutes of arc equal in length the radius of the circle? How many seconds of arc?

9. Let the sun shine through a small circular opening on a screen held perpendicular to the axis of its cone of rays. From the diameter of the image of the sun on the screen and the distance of the screen from the opening, compute its angular diameter. If the distance of the sun is 93,000,000 miles, what is its diameter?

21. The Geographical System. — One of the problems of geography is to describe the positions of places on the surface of the earth. Since this is essentially the same problem as locating points on the celestial sphere and is solved in the same manner, it will be reviewed.

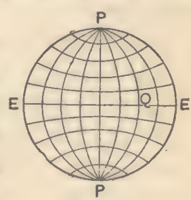


FIG. 11.

Geographical positions are defined by two sets of circles on the surface of the earth. The first set consists of the equator and the small circles parallel to it; the second set consists of great circles perpendicular to the equator.

In the figure, EE' is the equator and PQP' is one of the great circles of the set which is per-

pendicular to it. The circles parallel to the equator are called *latitude circles*, and the circles of the second set are called *meridians*. They cover the whole sphere, one circle of each set passing through every point. Distances are counted from a particular meridian, called the *prime meridian*. In the United States the prime meridian passes through the Washington Observatory, in England through the Greenwich Observatory, etc. The location of a place is defined by its distance north or south of the equator (latitude) and east or west of the prime meridian (longitude), these distances being given in degrees.

It is to be observed that the positions of all the circles are defined when the equator is given, or the poles, which define the position of the equator. The prime meridian of the second set is selected so as to be most convenient. In developing systems of circles the first thing is to define what shall correspond to the equator, or to its poles; then, to define what circle shall correspond to the prime meridian; then, to give the circles appropriate names; and finally, to give names to the distances corresponding to latitude and longitude. These distances are called *coördinates*.

22. The Horizon System. — The horizon, which separates the visible portion of the sky from the invisible, is a curve which can not escape attention. If it were a great circle, it might be taken as a great circle of the first set for a system of circles on the celestial sphere. For an observer on the land it is very irregular, and for one on the sea more than half of the sky is visible unless he is precisely at the surface of the water. This horizon which we see is called the *sensible horizon*.

An astronomical horizon will now be defined and made the great circle of the first set. The direction of the plumb line at any place is perfectly definite. The point where the line having this direction pierces the celestial sphere overhead is called the *zenith*, and the opposite point, the *nadir*. These

may be taken as the poles of the first set of circles. The great circle 90° from them, or midway between them, is the *astronomical horizon.* The small circles parallel to the horizon are called *parallels of altitude*, or sometimes *almucantars.* The great circles of the second set pass through the zenith and nadir, and intersect the horizon perpendicularly. They are called *vertical circles.*

The standard vertical circle is the *meridian*, or the one passing through the north and south points of the horizon. The cardinal points will be defined in the next article. The vertical circle passing through the east and west points, called the *prime vertical*, is used sometimes as a standard.

The angular distance above or below the horizon is called the *altitude*, being considered positive when above and negative when below the horizon. The angular distance from the zenith, called the *zenith distance*, is also used frequently. The angular distance from the meridian, starting from the south point and counting westward along the horizon around up to 360°, is called the *azimuth.*

Consider an example. Suppose a star is directly in the northeast, and midway between the horizon and the zenith; required its altitude and azimuth. Think of the vertical circle passing through it, and count the distance from the horizon along this circle up to the star. From the conditions given it is 45°, and the altitude is therefore + 45°. The vertical circle through the star cuts the horizon in the northeast point. Keeping this point in mind, start at the south point and count along the horizon westward around to it. The distance is 225°; that is, the azimuth is 225°.

23. The Equator System. — The sun and moon pass across the sky from east to west in diurnal circles, and an hour's observation will show that the stars do the same thing. The circles become smaller and smaller the nearer the star is to a certain point in the northern sky. These circles all have the same center, and if there were a star at this point it would

have no diurnal motion. This point, which can not escape attention, is called the *north pole of the sky*, and will be taken as a pole in a second system of reference circles. The star nearest to this point which can be seen with the unaided eye is the Pole Star, called *Polaris*. It describes a small circle with a radius of about a degree and a quarter, or a little more than twice the diameter of the moon. There is, of course, a corresponding point in the southern sky, called the *south pole*, which is invisible to observers in the northern hemisphere of the earth. Unless otherwise stated, these points will be meant when the north and south poles are mentioned.

FIG. 12.—Circumpolar Star Trails. *Photographed at the Goodsell Observatory, Northfield, Minn.*

Since the apparent motion of the stars is due to the rotation of the earth, it follows that the poles of the sky are the points where the line of the earth's axis pierces the celestial sphere.

The great circle on the celestial sphere 90° from the north and south poles is called the *celestial equator*, and is the defining circle of the first set for this system of coördinates. The small circles parallel to it are called *declination circles*. The great circles perpendicular to it are called *hour circles*. The fundamental hour circle is the one cutting the equator in the vernal and autumnal equinoxes, and is called the *equinoctial colure*. The equinoxes will be defined in the next article.

The distance north or south of the celestial equator is called the *declination*, being considered positive if north and negative if south. The distance from the equinoctial colure, starting at the vernal equinox and counting *eastward* along the equator up to 360°, is called the *right ascension*.

The meridian of the horizon system can now be defined. One of the vertical circles passes through the north pole of the sky. The one of the two points where it cuts the horizon which is nearest the north pole is the north point, and the other, the south point. The other cardinal points follow directly from these two.

It is easy to see why the great circles perpendicular to the equator are called hour circles. The whole sky appears to turn around from east to west once in a day. Suppose the hour circle whose right ascension is 0° coincides with the meridian at a given instant. An hour later it will have moved westward 15°, and the hour circle whose right ascension is 15° will coincide with the meridian. If the whole circumference is divided into 24 hours (of arc, not time) of 15° each, the circles through these points will pass the meridian at intervals of 1 hour. The hours mentioned here are about 10 seconds shorter than ordinary hours, as will be shown when this subject is discussed in Art. 185.

24. The Ecliptic System. — If we could see the stars near the sun, we should find that it apparently moves eastward among them, completing a revolution in a year. A telescope would enable us actually to make these observations, but the same conclusions can be reached as readily in an indirect way from observations with the unaided eye.

Suppose we notice the stars which are on the meridian toward the south at 9 o'clock in the evening. These stars are 9 hours, or 135°, east of the sun. On the next evening it will be observed that the same stars cross the meridian nearly 4 minutes earlier, and so on for every succeeding evening. Consider the conditions after 15 days have elapsed.

The stars under observation will then cross the meridian at 8 o'clock in the evening. Now these stars are only 8 hours, or 120°, east of the sun. That is, the sun has gone eastward 15° among the stars. This relative apparent motion keeps up indefinitely. It was discovered in very ancient times by such observations as those just described, and every student should repeat them.

The next question is whether the sun travels in this motion along the equator, or a declination circle, or in some other curve. The pole is a fixed point in the sky, and the equator, which is 90° from it, always cuts the meridian at the same altitude for an observer at a given place. So, also, do all the declination circles. Consequently, if the annual motion of the sun is along the equator, or a declination circle, it will in its diurnal motion always cross the meridian at the same altitude. Observations soon show that this is not so. If its eastward motion, determined by the change in the time at which the observed stars cross the meridian, and its northward and southward motion, determined by the altitude at which it crosses the meridian, are plotted on the surface of a sphere, it will be found that its apparent annual path is a great circle making an angle of about 23.5° with the celestial equator. This great circle is called the *ecliptic*.

The ecliptic will be made the defining great circle of the first set for the ecliptic system of coördinates. The small circles parallel to it are called *parallels of latitude*. The great circles perpendicular to it are called *longitude circles*. The fundamental longitude circle is the one passing through the points where the equator and ecliptic intersect. These points are the equinoxes, the *vernal equinox* being the one at which the sun crosses the equator from south to north, and the *autumnal equinox* the other one.

The north side of the ecliptic is the one toward the north pole of the sky. The distance north or south of the ecliptic is called the *latitude*, being considered positive if north and

negative if south. The distance from the fundamental longitude circle, starting from the vernal equinox and counting eastward along the ecliptic up to 360°, is called the *longitude*.

25. Comparison of the Systems of Coördinates. — All three of the systems are geometrically just like the one used in geography, but there are important differences in the way in which they arise. The horizon system has its origin in the fact that the earth separates the sky into two parts, the visible and the invisible. It is defined by the position of the observer on the earth, or, more exactly, by the direction of gravity at his point of observation. The equator system is defined by the apparent rotation of the sky, which is due, of course, to the actual rotation of the earth. The ecliptic system is defined by the apparent annual path of the sun among the stars, which is caused by the actual motion of the earth around the sun.

The horizon system depends upon the position of the observer. Consequently, to have the position of a celestial object completely defined when its altitude and azimuth are given, the observer's position must be given also. The sky has a diurnal motion with respect to the horizon system. Therefore it is necessary to give the time of day at which the altitude and azimuth have the given values. It has been seen that given stars cross the meridian at different times on succeeding days; that is, their relations to the horizon system vary with the day as well as with the time of day. Consequently it is necessary to give also the day of the year on which the altitude and azimuth had the given values. On the other hand, the equator and ecliptic systems are fixed (except for *very slow* changes) on the sky, and when they are used the coördinates are independent of the place of the observer, the time of day, and the time of year.

If a catalogue of stars is to be made, it is clear that the horizon system is much less convenient than either of the other two. It will be shown in connection with the dis-

cussion of astronomical instruments that for such purposes the equator system is the most convenient of the three. In fact, it is the one which is always used in catalogues. The ecliptic system is comparatively little used, while the horizon system is of more service, because it is employed when we wish to say whether an object is at any time in a position to be visible or not.

QUESTIONS AND EXPERIMENTS

1. If the earth's surface were a plane, as it was supposed to be in antiquity, what sort of a system would be used in describing geographical positions?

2. Is it absolutely necessary that the two sets of circles of a system intersect each other perpendicularly?

3. What is the longitude of the earth's north pole?

4. What is a great circle? How many points on a given sphere are necessary to define a great circle? How many to define a small circle?

5. Is there any natural reason for counting azimuth along the horizon westward?

6. What are the horizon coördinates of the north pole at your latitude?

7. Estimate the horizon coördinates of the sun at 10 o'clock this morning; at 10 o'clock this evening.

8. Verify the diurnal motions of the stars by observing the increase in altitude of eastern stars during an evening.

9. In what direction do the stars under the pole move?

10. At what points on the surface of the earth must an observer be in order that the first set of circles in the horizon and equator systems shall coincide? What is the relation of their second sets of circles for these points?

11. How are the horizon and equator systems of coördinates related for an observer at the earth's equator?

12. Why is it simpler to count right ascension eastward than it would be to count it westward? Why not count it both eastward and westward, as longitude is counted on the surface of the earth?

13. How long does it take the sky to turn 1 degree?

14. How far, in angular measure, does the sky turn in 1 minute?

15. Make a smooth pinhole through a large piece of stiff paper. Hold it perpendicularly to the rays of the sun and another paper at a convenient distance back of it. An image of the sun will appear on the second piece of paper. Keep both of them stationary, and observe how

long it takes the sun to move through its diameter. From this compute its angular diameter. From the angular diameter find the relation between the linear diameter of the sun and its distance from us.

16. Invent a simple apparatus for showing that the stars cross the meridian nearly 4 minutes earlier every night. Use stars near the equator. What results do you get?

17. Would it not have been more appropriate, from comparison with the system used in geography, to have interchanged the names of the coördinates in the equator and ecliptic systems?

18. Fill in the following table:

	Geographical System	Horizon System	Equator System	Ecliptic System
Defined by	Rotation of the earth			
Fundamental Circle	Equator			
Associated Circles of First Set	Parallels of latitude			
Second Set of Circles	Meridians			
First Coördinates	Latitude: N., if north; S., if south			
Second Coördinates	Longitude, counted both eastward and westward 180°			
Circles Fixed or Movable	Fixed on surface of the earth			

26. Method of locating the Defining Elements in the Horizon System. — It is absolutely essential for practical astronomy to be able to locate the defining points and lines of the various systems used. An exposition of the principles of these methods will greatly assist in obtaining the necessary mastery of this part of the subject.

In the horizon system it is necessary to know the zenith and the north and south points. The angular distance from the zenith to an object can be measured, and, therefore, its altitude found by subtracting this distance from 90°. As a matter of fact, the horizon is not directly used except in determining the position of a ship at sea. The north and south points can be found, when the zenith point is known, after the position of the celestial pole has been determined.

The position of the zenith is found from the direction which a plumb line takes, but astronomical instruments can not be put in adjustment with any considerable degree of precision by this method. The surface of a stationary fluid is perpendicular to the direction of a plumb line, and the determination of the horizontal can be quite accurately obtained from this principle by means of a level. Then, by means of a graduated circle, the zenith point can be found.

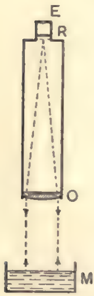


FIG. 13.

But the most accurate method, and the one actually in use in work of much precision, is to find when the telescope is pointed at the nadir by means of reflection of the reticle (Art. 79) from a basin of mercury.

In the figure, E is the eyepiece, R the reticle, which consists of a number of parallel spider lines (it is sufficient to consider one here), O is the objective of the telescope, and M the basin of mercury. When parallel rays come into the telescope through the objective, they converge at R. Consequently, when the telescope is pointed exactly at the nadir, the light from R will pass down to O and emerge in parallel

lines. From *M* it will be reflected and pass back over the same paths. When the telescope is not exactly in the vertical, the reticle and its reflected image will not coincide, and they are to be brought into coincidence by a slow-motion screw. When the telescope is mounted on an axis carrying graduated circles, after the nadir reading is taken, the horizon and zenith can be found (Fig. 39).

27. Method of locating the Defining Elements in the Equator System. — The elements to be located are the equator, or pole, and the vernal equinox. By the definition of the latter it can be found only in connection with the ecliptic system, and its consideration will therefore be deferred until the next article.

To find the celestial pole consider one of the circumpolar stars, — Polaris, for example. The center of the circle which it describes (see Fig. 12), when corrected for refraction and other errors which enter, is the pole. The celestial equator is the great circle 90° from the pole.

Another definition of the equator is that it is the circle of intersection of the plane of the earth's equator with the celestial sphere; and from this it follows that, as seen from a given place on the earth's surface, it always crosses the meridian at the same altitude.

The cardinal points, and therefore the meridian of the horizon system, can now be located from their definition as given in Art. 23.

28. Method of locating the Defining Elements in the Ecliptic System. — The elements to be found are the ecliptic and the equinoxes. The method of finding the ecliptic was explained in Art. 24. If a map of the stars has been made previously, the apparent path of the sun among them will be given by the observations.

The vernal equinox is the place where the ecliptic cuts the equator from south to north. It is found by comparing the altitude of the sun as it daily crosses the meridian with

that of the equator. Suppose the sun is at 18, ⋯, 23 (Fig. 14) on March 18, ⋯, 23, respectively, as it crosses the meridian of the observer. The vernal equinox, which is denoted by the symbol ♈, can be found by drawing a smooth curve through these points. A graphical process is subject to errors in the drawings. Actually, numerical processes are employed. The declinations and distances of the sun in right ascension from some fixed star (see Art. 24) are measured day by day. Thus, suppose the declinations on the dates above are respectively $-1° 1' 30''$, $-0° 37' 47''$, $-0° 14' 4''$, $+0° 9' 38''$, $+0° 33' 18''$, $+0° 56' 58''$, and a corresponding set of distances in right ascension. From this set of numbers, by a systematic process called *interpolation*, the time at which the declination was zero can be found with an accuracy corresponding to that of the observations. The date now being known, the corresponding distance in right ascension from the fixed star can be found from the set of distances in right ascension by a little different application of the process of interpolation. Suppose the vernal equinox is $120° 8' 30''$ (*i.e.* 8 hr. 0 m. 34 sec.) west of the star which has been used as the basis of measurement. Then the right ascension of this star is 8 hr. 0 m. 34. sec. The distances in right ascension of any other stars may be measured from this one, from which their true right ascensions may be found.

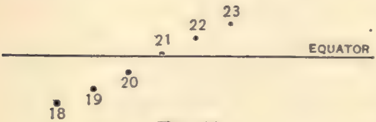


Fig. 14.

This completes the determination of the defining points and circles upon which all astronomical observations of position depend. It is to be understood that only the essence of some of the methods has been given, and that there are many details which must be considered in practical work, and which increase the accuracy of the results to a remarkable degree.

29. Relation of the Position of the Observer to the Altitude of the Pole and of the Equator on the Meridian. — Figure 15 represents a meridian section of the earth through the place of the observer, O. The plane passes through the polar axis of the earth and cuts the plane of the horizon in the line marked "horizon," and the plane of the equator in the line marked "to equator." The polar axis really points to the pole of the heavens, but the stars are so remote that a parallel line from O may be used in its stead. That is, as seen from the distance of the stars, the radius of the earth subtends an angle so small that it can not be measured by any known means. When a nearer object, as the moon, is observed, corrections must be applied for the position of the observer. The corresponding statements are true for the equator.

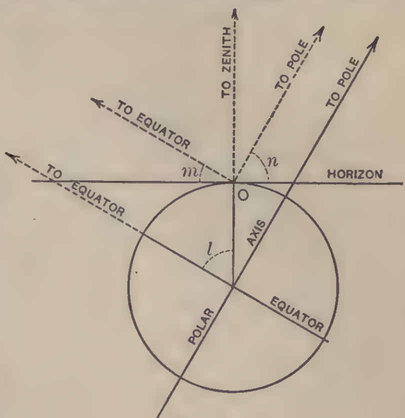


FIG. 15.

The angle l is the latitude of the observer at O, n is the altitude of the pole as seen from this place, and m is the altitude of the equator. It follows from the principles of plane geometry that $n = l$ and $m = 90° - l$. For example, suppose the latitude of the place of observation is 40°; then the altitude of the pole from the north point is 40°, and the altitude of the equator where it crosses the meridian is 50°. The equator and horizon are great circles and consequently bisect each other, and since the equator cuts the meridian perpendicularly, they intersect in the east and west points. The relations among the reference points and lines are illustrated in Fig. 16.

In the figure $NESW$ is the horizon circle with its cardinal points, Z is the zenith, P and P' are the poles, EQW is the equator, NV is the portion of sky always visible, and SI is the portion never visible.

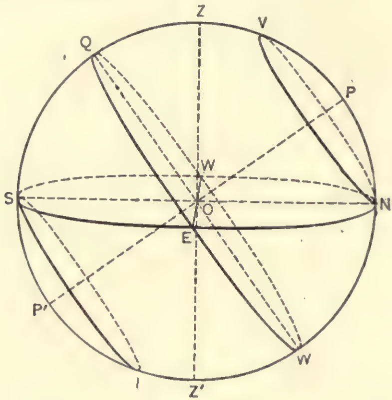


FIG. 16.

30. Finding the Horizon Coördinates when the Equator Coördinates are Given. — As was stated in Art. 25, the equator system of coördinates is much more convenient than the horizon system for catalogues and general descriptions of position, and it is almost invariably used. Suppose one should read in a daily paper that a new comet visible to the unaided eye had been discovered. Its position would be described by its right ascension and declination. He would want to see it, but it is probable that he would be unable to tell when it could be seen and in what part of the visible sky. The method of solving such problems as this will be treated in this article. There are also direct applications which will be given in connection with the chapter on telescopes. The exact solutions in general depend upon spherical trigonometry, but this is not needed in handling a telescope or in ordinary work with unaided vision.

Suppose the right ascension and declination of a celestial object are given, together with the latitude of the observer, the time of year, and the time of day; the altitude and azimuth are required. This problem will be solved in four steps.

(1) *The right ascension of the sun on the date in question.* It is found from observation that the sun passes the vernal equinox on March 21. (This date may vary a day because of leap year, but it will be sufficiently accurate for present purposes to use March 21 in all cases.) In a year the sun moves (apparently, of course) around the sky 24 hours in right ascension, or at the average rate of 2 hours a month. Consequently, to find the right ascension of the sun at any date, count the number of months from March 21 and multiply the result by 2. Thus, October 6 is $6\frac{1}{2}$ months from March 21, and the right ascension of the sun on this date is about 13 hours. The sun's motion in right ascension is not perfectly uniform, so the results obtained are slightly inexact.

(2) *The right ascension of the meridian at the given time of day.* It will be supposed that the right ascension of the sun has been found by step (1). In the next steps one must imagine the lines on the celestial sphere, and nothing makes it so clear as actually to point them out. Suppose the time in question is in the evening, say 8 o'clock. The right ascension of the hour circle through the sun is known; the thing required is the right ascension of the hour circle which coincides with the meridian. It will be remembered that right ascension is counted eastward. Therefore, starting with the sun, which is now below the western horizon, and coming eastward 8 hours to the meridian, its right ascension will be found to be 8 hours greater than that of the sun. For example, suppose the sun's right ascension is 5 hours; therefore the right ascension of the meridian is 13 hours. If the sun's right ascension were 19 hours, the right ascension

of the meridian would be 27 hours, which is the same as 3 hours.

If the time of day is before noon, the right ascension of the meridian is less than that of the sun.

(3) *The hour angle of the object.* Suppose the object is east or west of the meridian. A certain hour circle passes through it, and the angle between this hour circle and the meridian is called the *hour angle* of the object. It must be specified whether the hour angle is east or west, and it is generally simplest to think of its magnitude as being defined by the arc of the equator intercepted between the hour circle and the meridian.

The right ascension of the meridian was found in step (2) and the right ascension of the object is given. Since one is at liberty to go either way along the equator, the hour angle can not exceed 12 hours. The object is east if its right ascension is greater than that of the meridian, and its hour angle is their difference. If its right ascension is less, it is west, and its hour angle is their difference. For example, if the right ascension of the meridian is 7 hours and that of the object 10 hours, the hour angle is 3 hours east. Or, if the right ascension of the meridian is 2 hours and that of the object 20 hours, the hour angle is 18 hours east, which is the same as 6 hours west. But the right ascension of the meridian may be called 26 hours instead of 2 in order to prevent the difference exceeding 12 hours. Then the hour angle comes out directly 6 hours west.

(4) *Application of declination and estimation of altitude and azimuth.* Let us take special examples and consider the case first where the declination of the star is zero. Suppose that the latitude of the observer is 40°, and that the hour angle of the star is 4 hours east. The altitude of the equator on the meridian is 50°. Estimate the position of this point and keep one hand pointing toward it. With the eyes sweep over the arc of the equator reaching from this point to the

east point of the horizon. The distance is one-quarter of a circumference, or 6 hours. Since the hour angle is 4 hours east, it is two-thirds of this distance, which is to be estimated. This is the point occupied by the object, and in the present example its altitude and azimuth are about 22° and 280° respectively.

Suppose now that the declination is + 30° and that the other data are the same. Keep one hand pointing at the place on the equator previously found, and locate the pole, which is directly in the north with an altitude of 40°. The distance from the equator to the pole is 90°. Start from the point on the equator at which the hand is pointing and follow along an hour circle toward the pole a distance equal to the declination of the object. This is the point occupied by the object, and its altitude and azimuth are to be estimated. In the present example they are about 40° and 265° respectively.

An astronomical globe is of great assistance in this work, and every school should possess one, but it should not be allowed to supplant the actual pointing at the sky which has just been described. There is nothing else that so thoroughly fixes the ideas in one's mind and develops the imagination which is so essential in much of the work in astronomy.

31. Illustrative Example. — An illustrative example of as complicated a type as will arise will now be solved. Suppose the data are:

Right ascension $= \alpha$ (Greek letter alpha) $= 3$ hr.
Declination $= \delta$ (Greek letter delta) $= +75°$.
Time of year $= T =$ Feb. 5.
Time of day $= t = 7$ o'clock A.M.
Latitude of observer $= l =$ N. 40°.

The following are the successive steps of the solution:

(1) The time from March 21 to February 5 is 10.5 months; therefore the right ascension of the sun is 21 hr.

(2) The time of day is 5 hours before noon; therefore the right ascension of the meridian is 21 hr. – 5 hr. = 16 hr.

(3) The right ascension of the object is 3 (or 27) hours; therefore it is 11 hours east of the meridian.

(4) It is simpler in this case to count from the *antimeridian*, for the object is only 1 hour east[1] from it. It is also simpler to use the polar distance than it is to use the declination, for the object is only 15° from the pole. To find the object, start from the pole and go down along the hour circle which is 1 hour east of the antimeridian. The altitude and azimuth are approximately 25° and 185° respectively.

32. Finding the Equator Coördinates when the Horizon Coördinates are Given. — There are many variations of the problems which have just been treated, and one more case will be explained. More variations are given in the appended examples, which should be carefully worked out.

This problem of finding the equator coördinates when the horizon coördinates are given, will also be solved in four steps. The first two are the same as before. They give the right ascension of the meridian at the given time.

(3) *Determination of the declination and hour angle.* From the altitude and azimuth, locate the position of the object and keep one hand pointing at it. Starting from the pole, sweep the eyes along the hour circle which passes through the object, and note where it strikes the equator. This point must be kept in mind. The distance from the star to the equator is the declination, being positive or negative according as it is on the north or south side. Count from the

[1] That is, east as seen by the observer, but really west in the sky.

point where the hour circle strikes the equator along the equator to the meridian. The distance in hours is the hour angle, which may be either east or west.

(4) *Finding the right ascension.* The right ascension of the meridian and the hour angle of the object are now known. If the hour angle is east, the required right ascension is their sum; and if the hour angle is west, it is their difference. If the right ascension of the meridian were small, as 2 hours, and if the hour angle were west and considerable, as 5 hours, 24 hours would have to be added to the former in order to keep the result positive.

QUESTIONS AND EXPERIMENTS

1. Hold a pencil at arm's length between your eye and a mirror. When is the line from your eye through the point of the pencil perpendicular to the mirror? (Compare this experiment with the problem of locating the zenith, Fig. 13.)

2. Can you devise a simpler set of steps than that given for solving the problem considered in Art. 30? If so, you should use them.

3. Suppose $\alpha = 16$ hr., $\delta = 0°$, $T =$ July 21, $t = 8$ P.M., $l =$ N. 43°; find the altitude and azimuth. (See Art. 31 for meaning of α, δ, T, t, and l.)

4. Suppose $\alpha = 4$ hr., $\delta = 0°$, $T =$ Nov. 21, $t = 8$ P.M., $l =$ N. 40°; find the altitude and azimuth. What would be the answer for $l =$ N. 30°?

5. Suppose $\alpha = 16$ hr., $\delta = +60°$, $T =$ Aug. 21, $t = 6$ A.M., $l =$ N. 40°; find the altitude and azimuth.

6. Suppose α of sun = 6 hr., δ of sun = 23.5°, $T =$ June 21, $t =$ midnight, $l =$ N. 45°; find the altitude and azimuth of the sun. Point in the direction.

7. Suppose $\alpha = 9$ hr., $\delta = 0°$, $T =$ May 21, $t = 8$ P.M., $l =$ N. 40°; find the altitude and azimuth.

8. Suppose $\alpha = 9$ hr., $\delta = +45°$, $T =$ Feb. 21, $t = 9$ P.M., $l. =$ N. 45°; find the altitude and azimuth.

9. Suppose $\alpha = 17$ hr., $\delta = 0°$, $T =$ Oct. 6, $l =$ N. 40°; at what time of the day does the object rise?

10. Suppose $\alpha = 1$ hr., $\delta = 0°$, $t = 9$ P.M., the object is setting; what is the date?

11. Suppose the zenith is in question, $T =$ July 21, $t =$ noon, $l =$ N. 40°; find α and δ. Find the same for the nadir.

12. Do any stars as seen from any place on the earth move in vertical circles in their diurnal motions?

13. Are there any places on the earth from which the apparent diurnal motions of the stars are along the parallels of altitude?

14. If the earth revolved around the sun in the opposite direction, what changes in counting the coördinates would be necessary to preserve simplicity?

15. What changes in counting the coördinates would be required if the earth rotated in the opposite direction?

CHAPTER III

THE CONSTELLATIONS

33. Origin of Constellations. — A moment's observation of the sky on a clear and moonless night will show one that the stars are not uniformly scattered over its surface. Every one is acquainted with such groups as the Big Dipper and the Pleiades. This natural grouping was noticed in prehistoric times and was of more interest to primitive peoples, spending their lives in the open air under skies which were nearly always clear, than it is to the ordinary person in this age of houses and artificial lights. Besides, the ancients were highly imaginative and, like children, they secured a sort of companionship for themselves by seeing all kinds of living creatures in inanimate objects, and they often wove about them the most fantastic romances. They saw in the natural aggregations of stars the forms of all sorts of animals, whose names were given to the groups, or *constellations*. Only a few constellations have been added in modern times, except in that part of the southern sky which was always invisible to ancient observers.

34. Naming the Stars. — The ancients gave proper names to many of the stars and described the others by their relation to the anatomy of the fictitious creature in which they were supposed to be situated. Thus, there were Sirius, Altair, Vega, etc., with proper names, and "the star at the end of the tail of the Little Bear" (Polaris), "the star in the eye of the Bull" (Aldebaran), etc., designated by their positions.

In modern times the names of forty or fifty of the most conspicuous stars are used very frequently; the remainder

are designated by letters and numbers. A system in very common use, which was introduced by Bayer in 1603, is to retain the ancient constellations, and to give to the stars in each constellation, in the order of decreasing brightness, the names of the letters of the Greek alphabet in their natural order. In connection with the Greek letter the genitive of the name of the constellation is used. Thus, the brightest star in the whole sky is Sirius, in Canis Major. Its Greek letter name is Alpha Canis Majoris. The remarkable variable star, Algol, is the second brightest star in Perseus, and in this system is called Beta Persei. When the Greek letters are exhausted the Roman letters are used, and after them numbers are given to the stars in the order of decreasing brightness. There are some exceptions to this arrangement, such as the lettering of the stars in the Big Dipper. (See Fig. 18.)

Another method, adopted by Flamsteed about 1700, is to number all the stars catalogued in each constellation according to their right ascensions, and independently of their magnitudes.

Still another method is to pay no attention to the constellations, but simply to give the number of the star in some of the large star catalogues. This is used especially in the hundreds of thousands of small stars which have been catalogued. Bayer's system for the brighter stars and the star catalogue number system for the fainter ones are now very generally used.

35. Star Catalogues. — Star catalogues are lists of stars, usually all above a certain brightness in certain parts of the sky, together with their right ascensions and declinations at a given epoch. It is necessary to give the epoch, for the stars slowly "drift" with respect to each other, and the reference points and lines are not absolutely fixed in position.

The earliest-known catalogue is one of 1080 stars by Hipparchus (180–110 B.C.) for the epoch 125 B.C. Ptolemy

(100–170 A.D.) revised it and reduced the star places to the epoch 150 A.D. Tycho Brahe (1546–1601) made a catalogue of 1005 stars in 1580, and since that time they have become quite numerous. Among these may be mentioned that of Lalande (1732–1807), 47,390 stars; of Argelander (1799–1875), whose work has been already referred to, 324,198 stars; of Schönfeld (1828–1891), who extended Argelander's work to a portion of the southern sky, 133,659 stars; and of Gould (1824–1896), in Argentine, 32,468 stars. These catalogues are very extensive, but they do not give the positions of the stars with the high degree of precision demanded in some of the modern work.

There are several catalogues containing from a few hundred to a few thousand stars whose positions have been determined with the highest possible degree of accuracy.

The project of photographing the whole heavens by international coöperation has been mentioned. Each plate will cover four square degrees of the sky, and since they overlap so that the whole sky is photographed twice, nearly 22,000 plates will be required. When the position of one star on a plate is known, the positions of all the others can be found by measuring their distances and directions from it. On these photographs, a large part of which have been taken, something like 15,000,000 stars will be shown, although it is planned now to measure and catalogue only about a million and a quarter of the brightest of them.

The photographic catalogue is an indirect outgrowth of the photographs of the great comet of 1882 taken by Gill at the Cape of Good Hope. The number of star images obtained at once showed the possibilities of the method. Plates covering all the sky from declination $-19°$ to the south pole were obtained by Gill in 1889, and the enormous labor of measuring the positions of 350,000 star images has been carried out by Kapteyn of Groningen.

36. Magnitudes of Stars. — The amounts of light we re-

ceive from the different stars differ greatly, probably no two sending us precisely the same quantity. The *magnitude* refers to the amount of light we receive from a star and has nothing directly to do with its actual dimensions.

The stars which are visible to the unaided eye are divided arbitrarily into six groups, or magnitudes, depending upon their apparent brightness. The twenty brightest stars constitute the first magnitude group, and the faintest which can be seen without optical aid are of the sixth magnitude, the other four magnitudes being distributed at equal intervals between these two. It has been found from experiments that the light received from a first-magnitude star is, on the average, about 100 times as much as that received from a sixth-magnitude star.

Let r be the ratio of the light given by a star of one magnitude to that given by one of the next fainter set. Then stars of the fifth magnitude are r times brighter than those of the sixth; those of the fourth are r times brighter than those of the fifth, or r^2 times brighter than those of the sixth, etc., to the stars of the first-magnitude, which are r^5 times brighter than those of the sixth. But observations show that they are 100 times brighter; whence $r^5 = 100$, from which it follows that $r = 2.512 \cdots$.

Since there are stars sending us all amounts of light, from the brightest to the faintest, it has been necessary to introduce fractional magnitudes. They are now estimated to nearest tenths. A star which is brighter than the average first-magnitude star has a magnitude smaller than unity, as 0.4, or it may be so bright as to have a negative magnitude as -1.2.

37. The First-magnitude Stars. — As the first-magnitude stars are conspicuous and relatively rare, they serve as sort of guide-posts in the study of the constellations. They are given in the following table together with their magnitudes, positions, and colors. From their positions and the principles of the last chapter they can easily be found.

Star	Magnitude	Right Ascension		Declination		Color
Sirius; *α Canis Majoris* .	−1.4	6 hr.	40 m.	−16°	34′	Bluish white
Arcturus; *α Boötis*	0.0	14	10	+19	48	Orange
Vega; *α Lyræ*	0.2	18	33	+38	40	Pale blue
Capella; *α Aurigæ* . . .	0.2	5	8	+45	52	Yellowish
Rigel; *α Orionis* . . .	0.3	5	9	− 8	20	White
Canopus; *α Argûs*	0.4	6	21	−52	38	Bluish
Procyon; *α Canis Minoris* .	0.5	7	33	+ 5	32	White
Betelgeuse; *β Orionis* . . .	0.9	5	49	+ 7	23	Ruddy
α Centauri . . .	1.0	14	31	−60	20	White
Achernar; *α Eridani* . . .	1.0	1	33	−57	51	White
Altair; *α Aquilæ* . . .	1.0	19	45	+ 8	33	Yellowish
Aldebaran; *α Tauri*	1.0	4	30	+16	16	Red
Antares; *α Scorpionis* . . .	1.1	16	22	−26	10	Deep red
Pollux; *β Geminorum* . .	1.1	7	38	+28	19	Orange
Spica; *α Virginis* . . .	1.2	13	19	−10	32	White
β Centauri . . .	1.2	13	55	−59	48	White
α Crucis	1.3	12	20	−62	26	Bluish white
Fomalhaut; *α Piscis Australis* .	1.3	22	52	−30	16	Ruddy
Regulus; *α Leonis*	1.4	10	2	+12	33	White
Deneb; *α Cygni*	1.4	20	38	+44	53	White

38. The Number of Stars. — The number of stars in the first six magnitudes are: —

1st magnitude	20	4th magnitude	425
2d magnitude	65	5th magnitude	1100
3d magnitude	190	6th magnitude	3200

Each fainter magnitude has approximately three times as many stars as the preceding one, and there are all together about 200,000 stars in the first nine magnitudes. They all can be seen with a good telescope a little more than one inch in aperture, while only about 5000 are visible with the unaided eye. Only a rough guess can be made respecting the number of stars which are still fainter, but there are probably more than 100,000,000 within the range of present visual and photographic instruments.

39. Proper Motions of Stars. — The stars have motions with respect to each other which in the course of immense ages change the outlines of the constellations, but which during historic times have not been important. Yet they are large enough so that they must be known and the corresponding corrections applied in catalogues. The motion with respect to a fixed system of reference lines is called *proper motion*. The greatest proper motion known is that of an eighth-magnitude star in the southern heavens which drifts in the sky about $8.7''$ yearly. The ineffectiveness of even this largest known motion in changing the general appearance of the constellations can be seen from the fact that it would take this star nearly 220 years to travel over an arc equal to the apparent diameter of the moon.

The stars also have motions toward and from us, to be discussed later. These motions in the course of time change the magnitudes of the stars appreciably, but their effects are not measurable for thousands of years.

40. The Milky Way, or Galaxy. — The *Milky Way* is a hazy band of light, averaging about 20° wide, stretching in nearly a great circle entirely around the sky. The telescope shows that it is made up of an enormous number of small stars which can be separately distinguished only with optical aid. It intersects the equator at the points whose right ascensions are 6 hr. 47 m. and 18 hr. 47 m., and its inclination to the equator is about 63°. Its north pole is at right ascension 12 hr. 47 m. and declination + 27°. Its borders are very irregular and it is divided for a long distance into two parts. It is even cut entirely across by a dark streak near the south pole. The "star gauges" of the Herschels show that the stars are much more numerous in the Milky Way than they are in other parts of the sky.

FIG. 17. — The Milky Way.

41. The Constellations and their Positions. — The preceding work is sufficient to prepare one to study the constellations with interest and profit, and he should not stop short of an actual acquaintance with all the first-magnitude stars and the principal constellations which are visible in his latitude. The following table of constellations and their positions is taken from Young's "Elements of Astronomy." The numbers at the top show the degrees of declination between which the constellations lie, the Roman numerals at the left show their right ascensions, the numbers after the names give the number of stars in the constellations easily visible to the unaided eye, the names of the ecliptic constellations are italicized, and the modern constellations are marked with asterisks.

R. A. \ Dec	+90° to +50°	+50° to +25°	+25° to 0°	0° to −25°	−25° to −50°	−50° to −90°
I–II	Cassiopeia, 46.	Andromeda, 18; Triangulum, 5.	*Pisces*, 18; *Aries*, 17.	Cetus, 37.	Phœnix, 32; *Apparatus Sculptoris, 13.	(Phœnix); Hydrus, 18.
III–IV . . .	—	Perseus, 46.	*Taurus*, 58.	Eridanus, 64.	(Eridanus.)	*Horologium, 11; *Reticulum, 9.
V–VI . . .	Camelopardus, 36.	Auriga, 35.	Orion, 58; *Gemini*, 33.	Lepus, 18.	*Columba, 15.	*Dorado, 16; *Pictor, 14; *Mons Mensa, 12.
VII–VIII . .	—	* Lynx, 28.	Canis Minor, 8; *Cancer*, 15.	Canis Major, 27; *Monoceros, 12.	Argo-Navis, 149.	*(Argo-Navis, Puppis); *Piscis Volans, 9.
IX–X . . .	—	* Leo Minor, 15.	*Leo*, 47.	Hydra, 49; *Sextans, 5.	—	(Argo-Navis, Vela.)
XI–XII . .	Ursa Major, 53.	—	* Coma Berenices, 20.	Crater, 15; Corvus, 8.	Centaurus, 56.	(Argo-Navis, Carina); *Chameleon, 13.
XIII–XIV .	—	* Canes Venatici, 15; Boötes, 36.	—	*Virgo*, 39.	Lupus, 34.	(Centaurus); *Crux, 13; Musca, 15.
XV–XVI . .	Ursa Minor, 23.	Corona Borealis, 19; Hercules, 65.	Serpens, 25.	*Libra*, 23.	Norma, 14.	*Circinus, 10.
XVII–XVIII.	Draco, 80.	Lyra, 18.	Aquila, 37; Sagitta, 5.	*Scorpio*, 34; Ophiuchus, 46.	Ara, 15.	*Triangulum Australis, 11; *Apus, 8.
XIX–XX . .	—	Cygnus, 67.	*Vulpecula, 23; Delphinus, 10.	*Sagittarius*, 48.	Corona Australis, 8.	*Telescopium, 16; Pavo, 37; *Octans, 22.
XXI–XXII .	Cepheus, 44.	* Lacerta, 16.	Equuleus, 5.	*Capricornus*, 22.	Piscis Australis, 16.	Indus, 15; (Octans).
XXIII–XXIV	—	—	Pegasus, 43.	*Aquarius*, 36.	*Grus, 30.	(Octans); *Toucan, 22.

The following maps show the constellations from the north pole to declination $-50°$. When Map I is held up toward the sky, facing north, it shows the circumpolar constellations in their true relations. The other maps are to be held up toward the sky, facing the south.

QUESTIONS

NOTE. — Let s be the ratio of brightness of a star to one one-tenth of a magnitude fainter. Then the brightness of a star of one magnitude is s^{10} times brighter than a star one whole magnitude fainter. But in Art. 36 it was found that the ratio of brightness of two consecutive magnitudes is $2.512 \cdots$. Therefore $s^{10} = 2.512 \cdots$, whence $s = 1.1 \cdots$.

1. What is the ratio of the light received from Sirius to that received from Aldebaran?

2. Find how many first-magnitude stars would be equivalent in giving us light to all the stars of the first six magnitudes.

3. Make a list of the constellations which are on the meridian between eight and nine o'clock in the evening for every month in the year.

4. What part of the sky is richest in the stars which are visible to the unaided eye?

5. At what time of the year do these constellations cross the meridian at eight o'clock in the evening?

6. Why are there vacant places, that is, fewer constellations, in the first column of the table?

7. Mark the constellations which contain first-magnitude stars.

8. What constellations pass through your zenith in the diurnal motions of the stars?

9. It will be seen later that the sun is traveling nearly toward the star Vega in Lyra. Point in that direction at your class hour. What will be the direction with respect to your horizon twelve hours later?

10. At what time of the year is the earth on the opposite side of the sun from Vega?

11. Make an observing programme for to-night, consisting of the constellations to be seen on the meridian and those immediately to the east and west, and the apparent positions of the circumpolar constellations, together with all the first-magnitude stars which are in all of them.

12. How many of them can you identify on the sky? (The next articles will assist in this work.)

42. How to find the Pole Star. — It is not easy for an observer to pick out the constellations, unless he is quite familiar with them, when he does not know the cardinal points. The first thing to be done in any case is to find the pole star. It is a second-magnitude star in Ursa Minor and stands quite apart from all other bright stars. Its altitude is equal to the latitude of the observer.

The Big Dipper is one of the most conspicuous and well-known of the groups of stars. It is a part of the constellation Ursa Major and is always above the horizon in the latitude of the United States and Europe. It is composed of seven stars of about the second magnitude which form the outline of a great dipper, and it can always be found without difficulty.

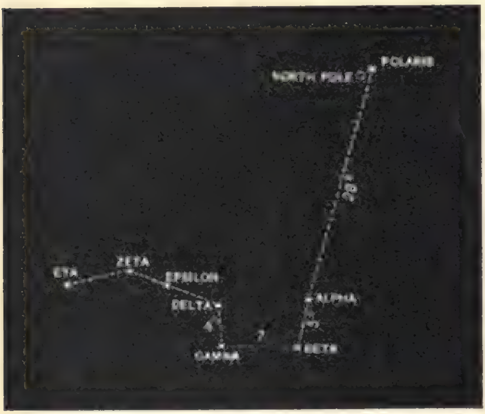
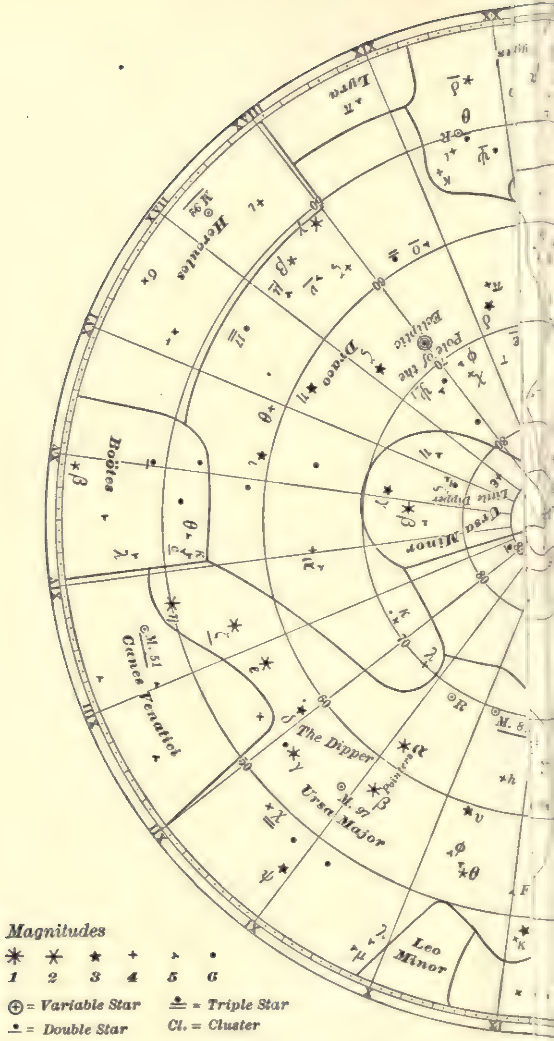


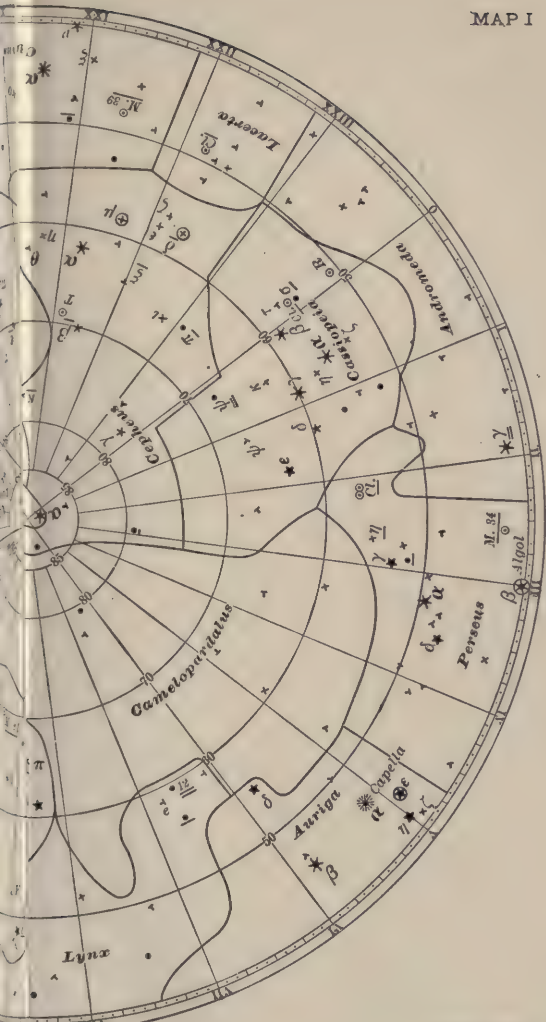
FIG. 18.

The stars Alpha and Beta are called the "pointers," for they are almost directly in a line with the pole star, Polaris, and about five times their distance apart from it. Consequently, when the dipper has been found, it is a simple matter to find the pole star.

Besides being a sort of guide in the study of the sky, the pole star is of much interest in other respects. It is a noted double star, the brighter component being a little fainter than the second magnitude, and the companion fainter than the ninth. Their distance apart is about 18.5″. It is impossible for the unaided eye to separate two objects as close


Lyra
Hercules
M. 92
Draco
Pole of the Ecliptic
Boötes
Little Dipper
Ursa Minor
Canes Venatici
M. 51
The Dipper
Pointers
Ursa Major
M. 97
Leo Minor
Magnitudes
1 2 3 4 5 6
= Variable Star
= Triple Star
= Double Star
Cl. = Cluster

MAP I



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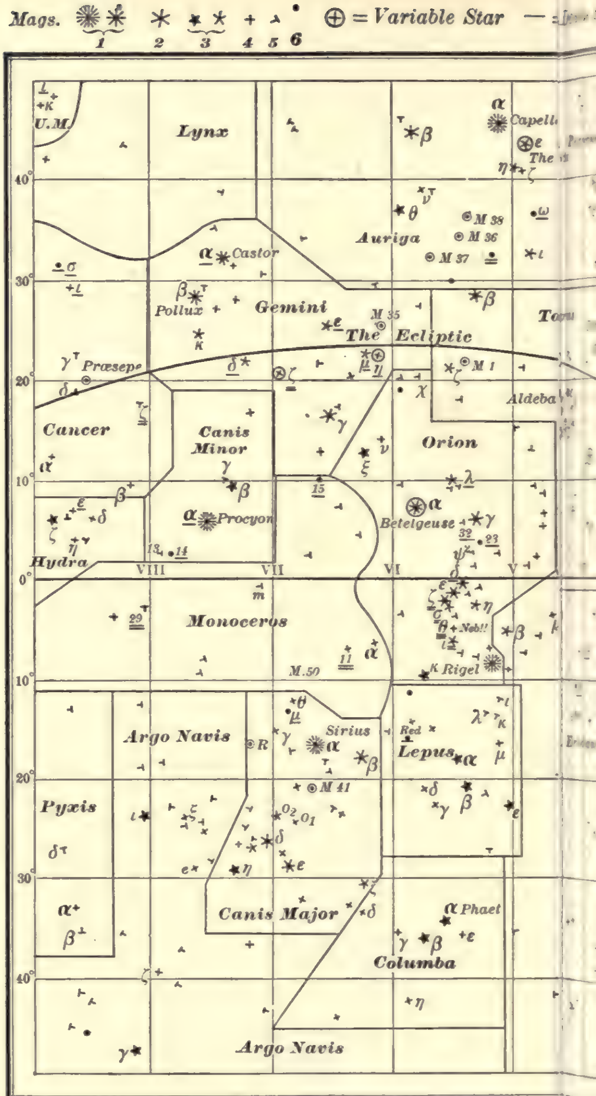
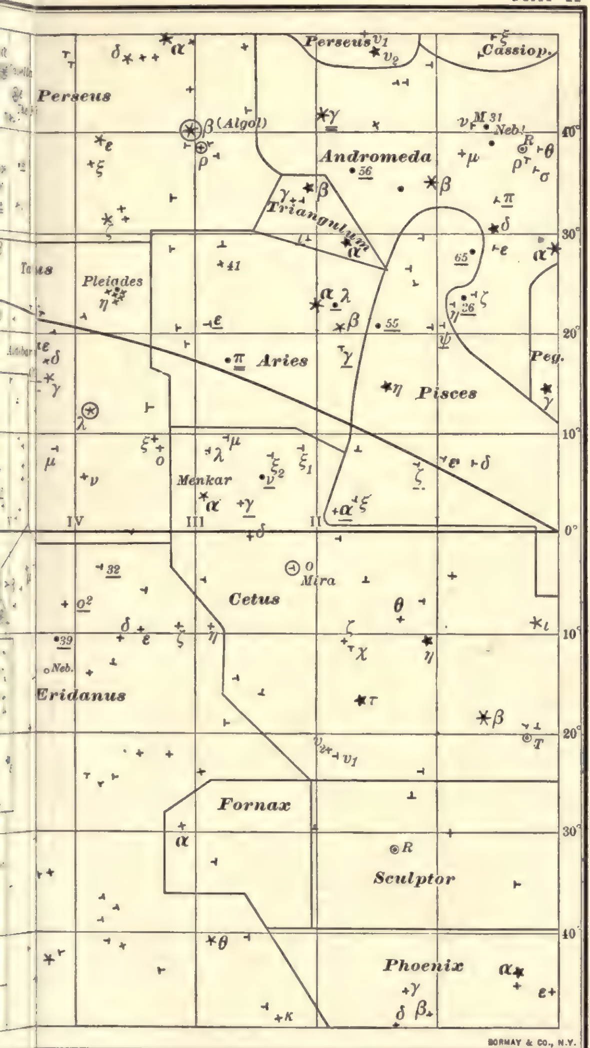


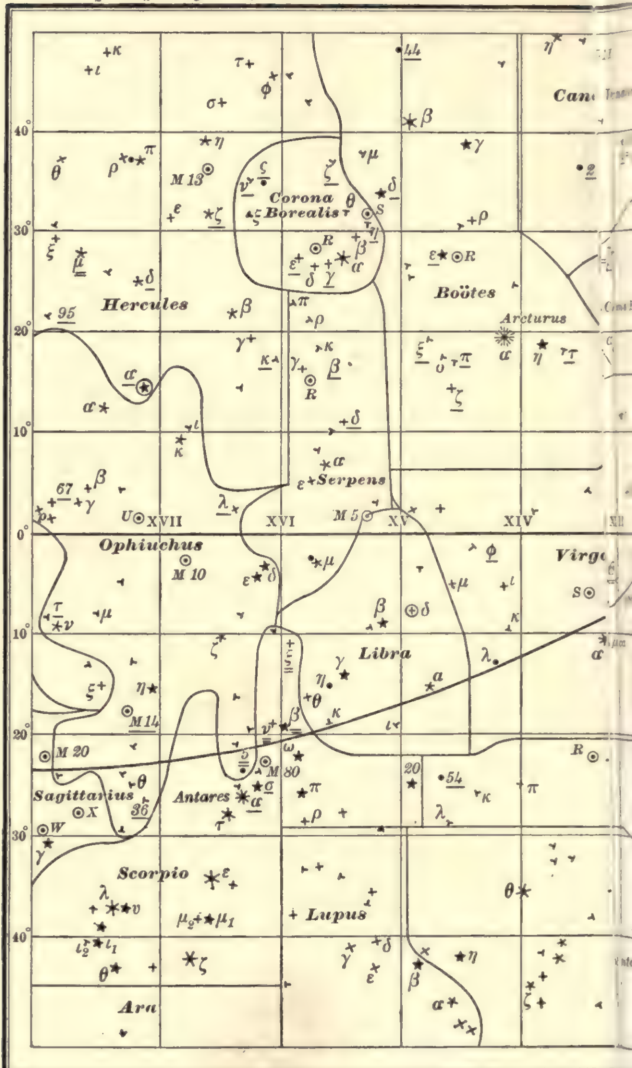
Mags. 1 2 3 4 5 6 = Variable Star
U.M.
Lynx
Capella
The
Auriga
M 38
M 36
M 37
Castor
Gemini
Pollux
M 35
The Ecliptic
Taurus
Præsepe
M 1
Aldebaran
Cancer
Canis Minor
Orion
Procyon
Betelgeuse
Hydra
VIII
VII
VI
V
Monoceros
Neb!!
Rigel
M.50
Argo Navis
Sirius
Red
Lepus
Pyxis
M 41
Canis Major
Phaet
Columba
Argo Navis
40°
30°
20°
10°
0°
10°
20°
30°
40°

--ouble Star ∸ = Triple Star Cl. = Cluster

MAP II

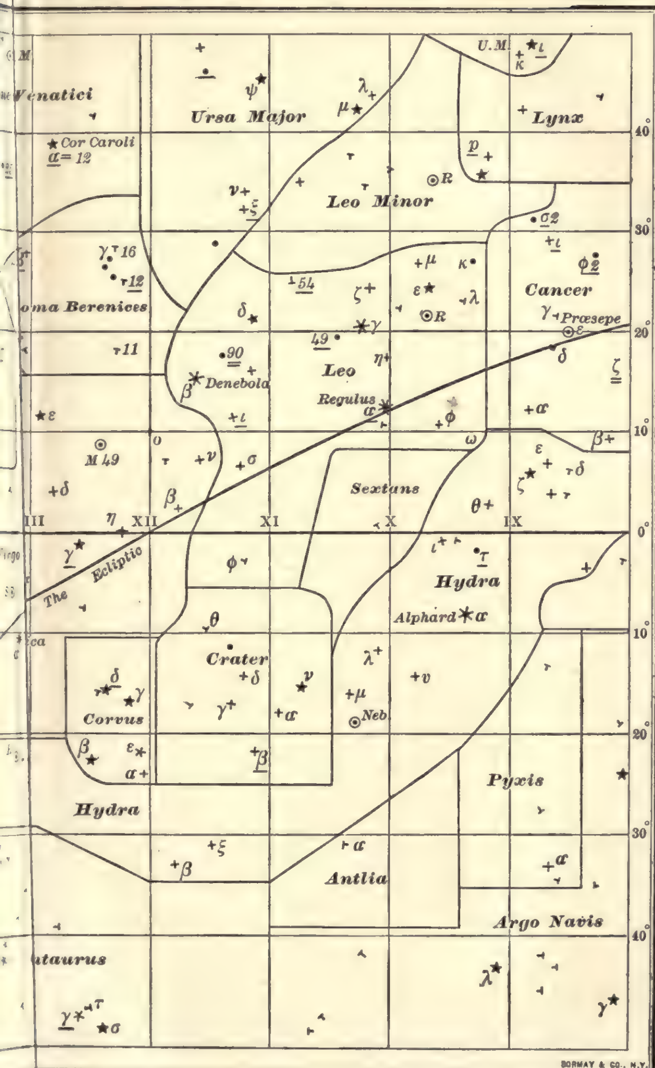


Mags. 1 2 3 4 5 6 ⊕ = Variable Star

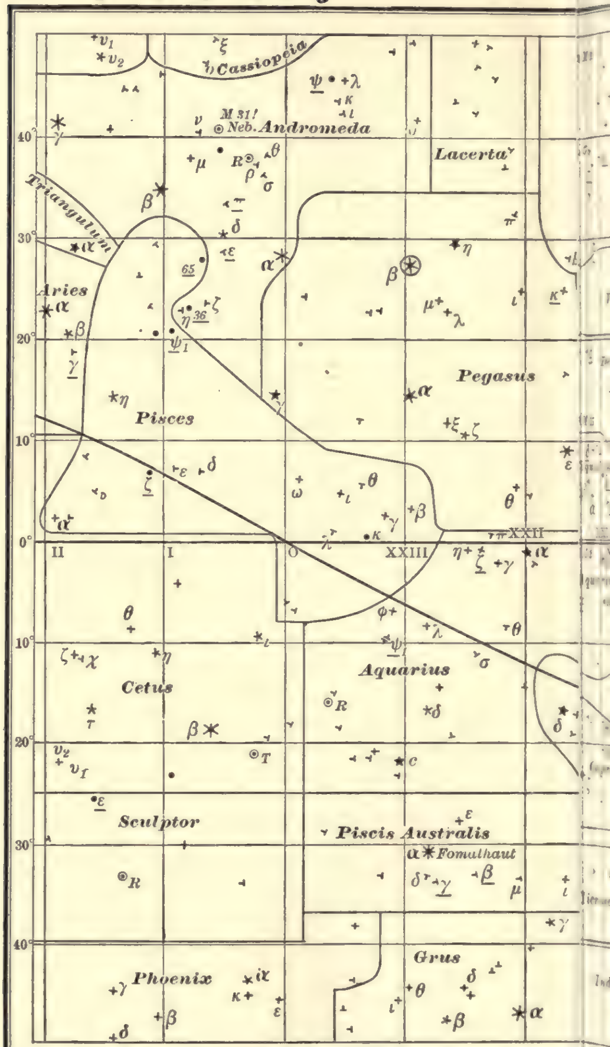


Double Star ≐ = Triple Star Cl. = Cluster

MAP III

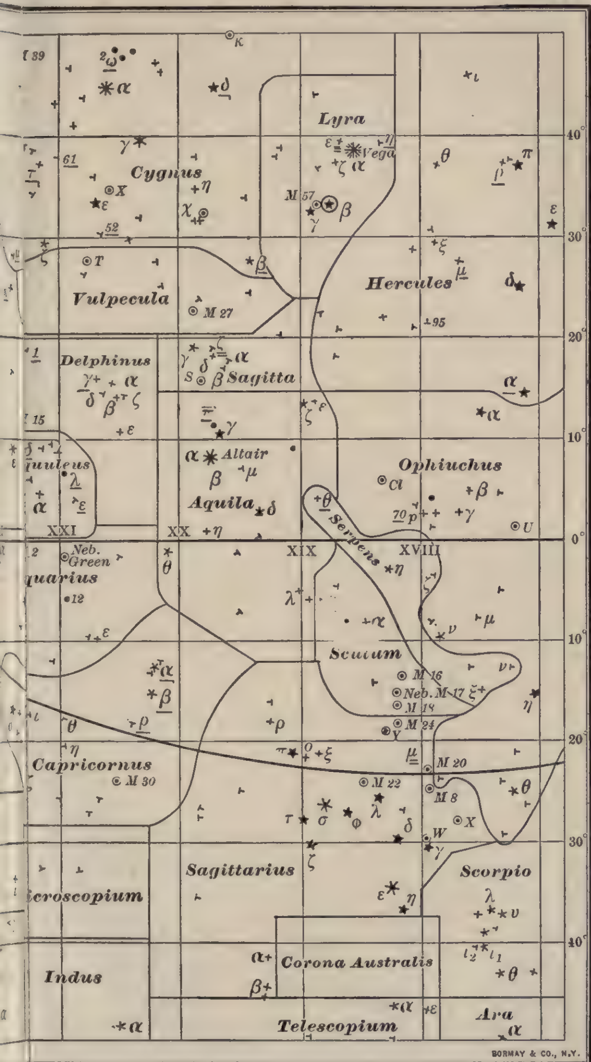


Mags. 1 2 3 4 5 6 ⊕ = Variable Star



∴ Double Star ∴ = Triple Star Cl. = Cluster

MAP IV



together as these stars are, for we cannot see separately stars which are much less than 3′ apart, and then they must be not very bright. The two components of Polaris can be seen with a telescope of three inches aperture and a magnifying power of from 75 to 100. The larger one is yellowish, and the smaller one white. In 1899 Campbell found by the spectroscope that the brighter component is a triple. It is so distant that it takes the light from it more than 40 years to come to us. If it were in some way extinguished, it would be "seen," in the ordinary sense, for more than 40 years, as though nothing had happened.

43. Units for estimating Angular Distances. — There is no scientific value in a statement that two stars seem to be a yard apart, for no two people would agree in such a description of estimates. The scientific and simplest way is to give their apparent angular separation, but to do this with any considerable accuracy some instruction and practice are required. Such work is essential in the study of the constellations.

It is 90° from the horizon to the zenith. This furnishes a unit for estimating large angles. One has to be somewhat careful in estimating the position of the zenith. For example, with the face to the south look up and fix the eyes on a star which seems to be at the zenith. Keeping the eyes directed toward this object, turn around so that the face is toward the north. The first trial is apt to furnish a surprise.

The altitude of the pole is equal to the latitude of the observer. This furnishes a unit which for the United States is in the neighborhood of 40°, and for England, 50°. It is not so satisfactory as a unit depending on the position of two stars, for it varies with the position of the observer, and it is more difficult to estimate distances from the horizon than from a star.

The distance from Alpha Ursæ Majoris to Polaris (see

Fig. 18) is about 28°. This unit, which can always be seen in our latitude, is very convenient for comparison in estimating large distances.

The distance between the pointers of the Big Dipper is about 5° 20′. This is a very convenient unit for comparison in estimating distances of this order of magnitude.

The diameter of the moon is a little more than half a degree. This gives a unit for measuring small distances, though it is not very satisfactory, because the moon looks larger than it really is. Other units will be given in connection with the description of some of the principal constellations which follow.

A very convenient and simple aid in estimating distances may be secured by holding a pencil at arm's length up toward the sky and observing how much of it is required to reach from one of the pointers to the other, or over any other known angular distance. With this as a basis, the whole pencil may be divided by marks into lengths which will cover known arcs when held at arm's length. To find the distance between any two objects, hold the pencil at arm's length and find how much of it is covered by the arc joining them, and reduce by the known scale on which the pencil is divided. It will be found that it takes about 2.75 inches to cover an arc of 5°.

In the search for the first-magnitude stars one must take care not to be misled by the planets, some of which nearly always can be seen. Since they move in the sky, their positions can not be described in connection with the constellations. They can be recognized from the fact that they are always near the ecliptic, and that a few days' observations will show their motions.

44. Ursa Major (the Greater Bear). — The most conspicuous part of Ursa Major is the Big Dipper, which occupies its eastern part.[1] It extends north, south, and west of the

[1] *East* and *west* are to be understood here as being measured *along* the declination circles around the pole. Thus, below the pole east in the sky is

bowl of the Dipper for more than 10°, but all the stars in these parts are of the third magnitude or fainter.

Near the star Zeta (Mizar), which is at the bend in the handle, there is a little star of the fifth magnitude called Alcor. Its distance is 11.5′, and it can be seen easily by people with good eyes. Mizar is itself a fine double, composed of a white star and one of an emerald color. The distance of the two components from each other is about 14.6″, and a 3-inch telescope will easily show them. It is not known just how far the system is from us, but it is certainly so far away that it takes the light more than 100 years to come to us, and perhaps very much longer. The two components are actually so far apart that if an observer were on one, the other would probably look like a distant, though bright, star. The larger of the two components has been found to be approaching the solar system at the rate of nearly 20 miles per second.

The first of a series of very important discoveries was made in 1889 by E. C. Pickering in spectroscopic observations of the brighter component of Mizar. It was found by methods which will be discussed at the proper place (Art. 401) that this star is itself a double in which the components are so close together that they can not be distinguished separately by any telescope. It is composed of two great suns whose combined mass is about 20 times that of our sun, and which revolve around their common center of gravity in a period of 20.5 days, and at a distance of 36,000,000 miles from each other.

45. Cassiopeia. — To find this constellation, go from the middle of the handle of the Big Dipper through Polaris and about 30° beyond. That is, the pole is about midway between the two groups, as can be seen from the table in Art. 41.

west with respect to the horizon. The following statements all refer to directions in the sky except when otherwise indicated, and care must be taken not to understand them in any other sense.

The constellation is distinguished by a zigzag, or letter W, composed of seven stars from the second to the fourth magnitudes. The brightest one is at the bottom of the second part of the W, and is a fine double, colors rose and blue, which can be seen separately with a 2-inch telescope.

One of the most interesting objects in the constellation is the star Eta Cassiopeiæ, which is near the middle of the third stroke of the W, and about 2° from Alpha. It is a fine double, which can be separated with a 3-inch telescope. These two stars form a binary system, revolving around their common center of gravity in a period of about 200 years. It takes the light from this pair a little over 9 years to come to us. If there are planets in such systems, the phenomena of night and day and the seasons must be very complicated.

In 1572 a temporary star suddenly blazed forth in this constellation, and became so bright that it could be seen in full daylight. It did much to stimulate the interest and zeal of Tycho Brahe (Art. 135), who was then a young man of twenty-six.

46. The Equinoxes. — To find the vernal equinox, draw a line from Polaris through the most westerly star in the W of Cassiopeia, and prolong it 90°. This point, which is on the equator, is very near the vernal equinox. There are, unfortunately, no conspicuous stars near this point, which is in the constellation Pisces.

The autumnal equinox is found by drawing a line from Polaris through Delta Ursæ Majoris, and prolonging it 90° to the equator. This point is in Virgo, which contains the first-magnitude star Spica. The autumnal equinox is about 10° north and 20° west of Spica.

47. Lyra. — Lyra is a small, but very interesting, constellation. Its mean right ascension is about 18.7 hours, and it is about 50° from the pole. No other description of its position is needed, for it is made conspicuous by the brilliant first-magnitude star Vega.

As we shall see later (Art. 146), the pole of the heavens is not absolutely fixed, and in 12,000 years it will be very near Vega. What a splendid pole star it will make! This star is approaching us at the rate of about 10 miles per second, but it is so far away that 30 years are required for its light to come to us. The sun, at the same distance, would be only $\frac{1}{100}$ as bright as Vega is.

Lyra is a constellation of interest, for it is nearly in its direction that the sun, with its retinue of planets, is moving.

There are two stars of the fourth magnitude, Epsilon and Zeta Lyræ, each about 2° from Vega. One is northeast and the other southeast, and the three stars form a nearly equilateral triangle. The star Epsilon is a double, composed of two nearly equal stars separated by a distance of $207''$. A person with good eyesight, under favorable conditions, can distinguish the two components without optical aid. A century ago astronomers gave their ability to separate this pair as proof of their having exceptionally keen sight. Perhaps with more exacting use the eyesight of our race is improving, for a majority of students now can separate the pair without any practice.

The star Epsilon Lyræ should be carefully observed. The distance between the two components seems small, but astronomers regularly measure $\frac{1}{2000}$ of this angle. The discovery of Neptune (Arts. 9 and 285) was based on the fact that it had pulled Uranus from its predicted place only a little more than half of this angular distance as seen from the earth. When Epsilon Lyræ is viewed with a telescope, it presents a great surprise. The two components appear to be far apart, and it is seen that each one of them is a double, thus forming a fine system of four suns.

About 5.5° south of Vega and 3° east is the third-magnitude star Beta Lyræ. It is a very remarkable variable, its brightness changing in a strange manner by more than a magnitude in a period of 12 da. and 22 hr. The subject

of variable stars is to be discussed later (Arts. 405 to 409), but it may be remarked in passing that, according to the work of Myers, this is really a double whose components are more than 10,000,000 miles in diameter, whose masses are 10 and 21 times that of our sun, and whose mean densities are about $\frac{1}{1000}$ that of water. This is a very difficult case, and the results are perhaps still open to some question. When you look at the star, though, you see one of great interest, which is apt to throw much light on the early stages of double-star evolution.

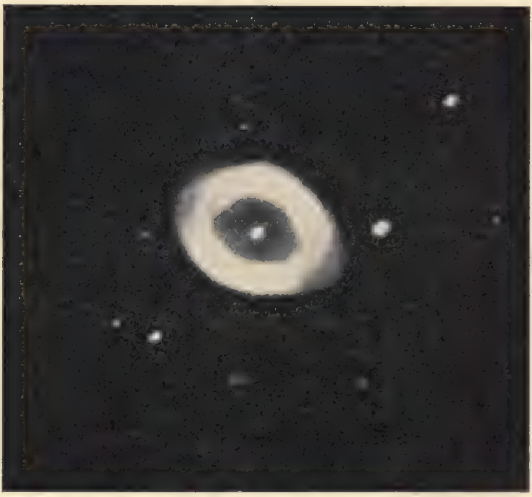


FIG. 19. — The Ring Nebula in Lyra. *Photographed at the Lick Observatory.*

About 2.5° southeast of Beta Lyræ is the third-magnitude star Gamma Lyræ. On the line joining these two stars and about one-third of the distance from Beta is a ring, or annular, nebula, the only one of the few that are known which can be seen with a small telescope. It takes a large instrument, however, to show much of its detail.

48. Hercules. — This is a very large constellation lying west and southwest of Vega, but it contains no stars brighter than the third magnitude.

About 25° west of Vega is a trapezoidal figure of five stars. The base is turned to the north and slightly to the east, and is about 6° long. There are two stars in the northeast corner, one of the third magnitude and one of the fourth. The star in the southeast corner is of the fourth magnitude, and those at the other corners are of the third. On the western side, about one-third the distance from the northern end, is one of the finest star clusters in the whole sky, Messier 13. It is visible to the unaided eye on a clear, dark night, and with the telescope is a wonderful object. It was discovered by Halley (1656–1742), but derives its present name from the catalogue of the great French comet hunter, Messier (1730–1817), who did all of his work with an instrument of 2.5 inches aperture.

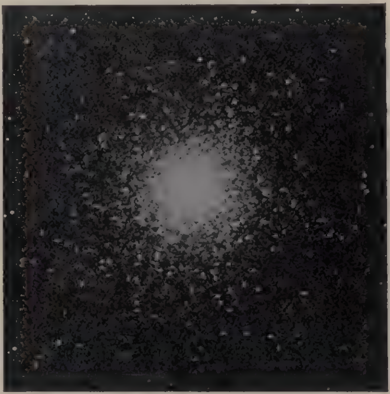


FIG. 20. — Great Star Cluster in Hercules (Messier 13). *Photographed by Ritchey with the 40-inch telescope of the Yerkes Observatory.*

49. Scorpio (The Scorpion). — Twelve constellations lie along the ecliptic and constitute the Zodiac (see italicized names in Table of Art. 41). Scorpio is the ninth of these and the most brilliant one of all; in fact, the finest southern constellation that can be seen in our latitude. It is about 60° south of Hercules, and is always easily recognized by the fiery red first-magnitude star, Antares, which in light-giving power is equal to 900 suns such as ours. This star has a

small green companion of the seventh magnitude 7″ west of it, but a 5 or 6 inch telescope and a good atmosphere are necessary in order that the companion may be seen. About 5° northwest of Antares is a very compact and fine cluster, Messier 80. Scorpio lies in one of the richest and most varied parts of the Milky Way.

50. Corona Borealis (The Northern Crown). — Just west of the great constellation Hercules is the little constellation Corona Borealis. It is easily recognized by a semicircle of six small stars opening to the northeast.

51. Boötes (The Hunter). — Boötes is a large constellation lying west of Corona Borealis and reaching from near the equator to within 35° of the pole. It always can be recognized easily by its brilliant first-magnitude star Arcturus, which is about 20° southwest of Corona Borealis. One hundred years are required for its light to come to us, and it is approaching us at the rate of about 5 miles per second. In light-giving power it is equivalent to about 1300 suns such as ours.

52. Leo (The Lion). — This is one of the zodiacal constellations, and the ecliptic passes very near to its brightest star Regulus. It is about 60° west of Arcturus and is recognized easily by a sickle of seven stars opening to the southwest, with Regulus at the end of its handle. One of the many things of interest in connection with this constellation is that the November 14 meteors seem to radiate from a point within the blade of the sickle.

53. Andromeda. — This is a large constellation just south of Cassiopeia, but it contains no first-magnitude stars. Its most interesting object is the Great Andromeda Nebula, the brightest in all the sky, which is about 15° directly south of Alpha Cassiopeiæ. It can be seen without difficulty on a clear, moonless night as a hazy patch of light. When seen through a telescope, it fills a part of the sky nearly 2° long and 1° wide. In its center there is a star which is probably

variable. Its spectrum presents peculiarities which seem to indicate that perhaps it is composed of solid or liquid material surrounded by cooler gases.

54. Perseus. — Perseus is a large constellation situated in the Milky Way directly east of Andromeda. The second brightest star in it is Algol (The Demon), of the second magnitude, and the earliest-known variable star. Its period is 2 da. 20 hr. 49 m. and at its minimum, which lasts about 18 m., it is of the fourth magnitude. That is, it loses more than five-sixths of its light at intervals of its period. The explanation is that it has a relatively dark companion revolving around it which partially eclipses it at regular intervals. It is not near any bright stars, and it can be recognized from its coördinates (right ascension 3 hr. 1 m., declination 40° 32′), and from the fact that it is about 10° south of a line of small stars running northwest and southeast.

There is also a remarkable double cluster in this constellation about 10° east of Alpha Cassiopeiæ.

55. Auriga (The Charioteer). — This large constellation lies east of Perseus. It is rich in telescopic objects, particularly star clusters. Its principal object of interest for work without a telescope is the great first-magnitude star Capella, which is about 40° from the Big Dipper and nearly in a line from Delta through Alpha Ursæ Majoris. It is receding from us at the rate of 15 miles per second, and is already so far away that its light is 35 years coming to us. The spectroscope shows that it is much like the sun, but the sun would not be such a splendid object at the distance of this star. The computation of Maunder shows that it radiates 220 times as much light as is given out by the sun.

56. Taurus (The Bull). — This constellation is southwest of Auriga and contains two conspicuous groups, the Pleiades and the Hyades, and the brilliant red star Aldebaran. The Pleiades are seven stars about 30° southwest of Capella and nearly 20° south and a little west of Algol. The stars form

the outline of a small dipper, with two stars instead of one at the point opposite the handle. Six of the stars are visible without optical aid, but the seventh, which is near the one at the end of the handle, is more difficult. In ancient times people seemed to have had considerable difficulty in seeing it, for it was frequently spoken of as having been "lost," but any one with fairly good eyesight can see it now. People with exceptionally good eyesight can see eleven stars in the immediate vicinity of the group.

Fig. 21. — Pleiades. *Photographed by Wallace at the Yerkes Observatory.*

In trying these tests of faint stars, one will be surprised to catch glimpses of stars a little to one side of the point where he is looking, and to have them disappear when he looks where he thought he saw them. The explanation is simple. The lens in the front of the eye forms an image on the retina at the back part. This retina is sensitive except at the point where the optic nerve leads out of the eye to the brain. When an image of an object falls on this spot it can not be seen. In addition to this, the retina may be more sensitive to faint light near its margin.

Attempts have been made by direct and indirect processes to find the distance of the Pleiades, but they have not given positive results. The best that can be said is that probably

several hundred years are required for the light to come from them to us. From the proper motion of Alcyone, discussed by Newcomb, and on the hypothesis that it is due indirectly to the actual motion of the sun, it follows that it takes 267 years for the light to come from this star to us. The sun seen at this vast distance would appear as a star of the ninth magnitude, or about $\frac{1}{250}$ as bright as Alcyone. About the middle of the last century Mädler thought Alcyone was the center of the universe and that all the other stars revolved around it, but there is not a particle of evidence to support such an idea.

The ecliptic passes about 4° south of the Pleiades, which are sometimes eclipsed by the moon. About 8° southeast of the Pleiades is the Hyades group, containing the red first-magnitude star Aldebaran.

57. Orion. — Southeast of Taurus and directly south of Auriga is the constellation Orion lying across the equator between the fifth and sixth hours of right ascension. This is the finest region of the whole sky for work without a telescope. About 7° north of the equator and 15° southeast of Aldebaran is the ruddy Betelgeuse. About 20° southwest of Betelgeuse is the first-magnitude star Rigel. About midway between them and almost on the equator is a row of second-magnitude stars running northwest and southeast. This is the Belt of Orion. From its southern end another row of fainter stars runs off to the southwest almost toward Rigel. It is the Sword of Orion. The central star of this row appears a little fuzzy. It is not a star at all, but the Great Orion Nebula, which, through a telescope, strikes many observers as being the most impressive object in the whole sky. There are, indeed, many stars in the nebula, particularly the trapezium of four stars near its center, but the greater part of the light comes from the nebula.

Some idea of the meaning of the photograph (Fig. 22), which shows more than a square degree of sky, can be ob-

FIG. 22.—Great Orion Nebula. *Photographed by Ritchey with the 2-foot reflector of the Yerkes Observatory.*

tained when one realizes that the nebula is at the distance of the fixed stars, which are so remote that their apparent diameters are less than $\frac{1}{50}$ of a second of arc, although they are a million miles or more in diameter. The nebula is a

great mass of glowing gas; at least, that part of it which gives us light is gaseous.

The star Betelgeuse is slightly variable, and Rigel has a faint and close companion, while nearly every star in the Belt and Sword is either double or multiple.

58. Canis Major (The Greater Dog). — This constellation is southeast of Orion and is marked by Sirius, the brightest star in all the sky. Sirius is almost in a line with the Belt of Orion and a little over 20° from it. This is one of the stars which is comparatively near to us, the light coming from it in 8.4 years. Expressed in miles, the distance is 47,000,000,000,000 of miles, and the star is approaching us at the rate of about 10 miles per second. Sirius is really overtaking the sun, for the solar system is moving in the opposite direction.

The history of Sirius for the last two centuries is very interesting, and furnishes a good illustration of the value of the deductive method in making discoveries. First, it was found by Halley in 1718 that it has a motion with respect to fixed reference lines; then, a little more than a century later, Bessel found that this motion is somewhat irregular. He interpreted this as meaning that Sirius and an unseen companion are revolving around their common center of gravity which describes the arc of a great circle as the stars ordinarily do. This companion actually was discovered by Alvan G. Clark in 1862, while testing the 18-inch telescope now of the Dearborn Observatory at Evanston. The two stars revolve around their center of gravity in a period of about 50 years. The distance of the two components from each other is about 1,800,000,000 of miles; Sirius is about 10,000 times as bright as its companion (it is difficult to determine the magnitude of the companion); its mass is a little more than 2 times that of the companion; their combined mass is more than 3.5 times that of the sun, and they radiate about 30 times as much light as the sun does.

59. Canis Minor (The Lesser Dog). — This constellation is directly east of Orion and is of interest here because of its first-magnitude star Procyon, which is about 25° east and just a little south of Betelgeuse. The history of this star is much like that of Sirius, the faint companion having been discovered in 1896, by Schaeberle, at the Lick Observatory.

The period of revolution of the two components of Procyon is 40 years, and their combined mass is 0.6 times that of the sun. If the orbits of such systems as Sirius and Procyon and their companions were edgewise to the earth, the brighter components would be regularly eclipsed, and the stars would be variables of the Algol type (Art. 406), though with such long periods and short time of eclipse that their variability would probably not be readily discovered.

60. Gemini (The Twins). — This is the fourth zodiacal constellation, and lies directly north of Canis Minor. Its two principal stars, Castor and Pollux, are about 25° north of Procyon and about 4.5° apart, Castor being the farther north. Castor is Alpha, and Pollux is Beta, Geminorum, although Pollux is now the brighter of the two, both being between the second and first magnitudes. That Pollux is now the brighter of the two may be due to the fact that Castor is receding at the rate of 4.5 miles per second, while Pollux is approaching at the rate of 33 miles per second.

Castor is a very fine double star, and the two components can be separated with a small telescope. Their period of revolution is nearly 1000 years. In his study of this system, Bélopolsky found with the spectroscope that the fainter component is also a double, one of the pair being dark. Their period of revolution is about 3 days. The mass of the close pair is apparently small, being something like $\frac{1}{10}$ that of the sun.

About 10° southeast of Pollux is the large star cluster Præsepe (The Beehive), which can be seen on clear, moonless nights without a telescope.

61. Observations. — The discussion of the constellations will be closed here, not because all have been described, or, indeed, any one of them adequately, but because enough has been said to show one that the sky is full of objects of interest which can be found and enjoyed with very little optical aid. The reader is expected to observe all the objects which have been described, so far as the time of year and the instrumental help at his command will permit. If he does this, the whole subject will have a deeper and more lively interest, and it will be a pleasure to make constant appeals to the sky to verify statements and descriptions.

The general features of the constellations are very simple, but the whole subject can not be mastered in an evening. One should go over it several times with no more powerful instruments than opera glasses. After that will come work with the telescope; this instrument will be described in the next chapter, where it will be seen how intimately its forms of mounting and its manipulation are related to the reference points and lines.

QUESTIONS AND EXPERIMENTS

1. Which side of the line from Beta Ursæ Majoris to Polaris is Alpha Ursæ Majoris?

2. How far apart would two objects have to be to subtend an angle of 3′ at a distance of 20, 30, and 40 feet? Make two artificial stars on the blackboard at one of these distances at the proper distance from each other, and see how many can distinguish the two points. (See that those who are to observe do not know in advance the direction of the stars from each other.)

3. What is really meant when one says he "sees" an object?

4. Perform the experiment of Art. 43 in locating the zenith.

5. What are the angular distances from Alpha to Delta Ursæ Majoris; from Alpha to Gamma Ursæ Majoris, and from Alpha to Eta Ursæ Majoris?

6. Which way is east on the sky at a point whose altitude is 40° and azimuth 160°?

7. How many stars can you see in the bowl of the Big Dipper? (Very keen eyes under very favorable conditions can see eight.)

8. The word *Alcor* means "the test." Can you see this test object without optical aid?

9. What two stars of Ursa Major are nearly in a line with Vega?

10. What proportion of the class can see that Epsilon Lyræ is a double? What direction do the two components lie from each other? Can you see the dark sky between them?

11. Can you find the "trapezoid" in Hercules and the star cluster Messier 13?

12. Does Antares appear red to you? Do you always agree with the colors as given in the text? (Experienced observers often disagree, especially on faint stars.)

13. Which of the stars of the sickle of Leo has a faint star near it?

14. How is Leo situated with respect to the Big Dipper?

15. Can you find the Great Nebula of Andromeda without opera glasses?

16. From the journal, *Popular Astronomy*, get the phase of Algol and observe a minimum.

17. Does Capella seem to you to be yellowish?

18. Can you see seven stars in the Pleiades?

19. Can you see more than seven?

20. See if you can observe the moon passing near the Pleiades.

21. Compare the colors of Aldebaran, Betelgeuse, and Rigel.

22. Look at the Orion Nebula with opera glasses.

23. At what time of the year does the sun have the same right ascension as Sirius?

24. Can you see any difference in the brightness of Castor and Pollux? How do they compare with Polaris?

25. What constellations north of the equator, which have not been described, contain first-magnitude stars? Describe their positions with respect to the other constellations.

26. What constellations north of the equator are in the Milky Way?

27. What portion of the Milky Way is divided into two parts?

28. From the data of Art. 47 find how long it will take Vega and the sun to pass each other.

NOTE

In addition to the work on constellations which has been given, there are always numerous phenomena to be observed, such as the planets, occultations by the moon, eclipses, meteoric showers, and comets. These phenomena are temporary or changing, and one can not follow them suc-

cessfully without the help of a periodical publication. Then, too, one might desire more extended star charts than those given, or a fuller treatment of some of the subjects. Nearly every one has access to some library where scientific books may be obtained, though they are frequently old, and scientific treatises rapidly lose their value with age in this epoch of many discoveries.

The following is a list of a few of the journals, maps, and books, most of which one either should have in his possession or be able to consult.

1. *The American, English, or German Ephemeris and Nautical Almanac.* — Part III of *The American Ephemeris* is on phenomena, and gives the eclipses of the sun and moon yearly, with maps for the solar eclipses; the moon's phases and librations; the data for predicting occultations; the disks and satellites of the planets (the daily coördinates of the planets are given in Part I), with maps showing the positions of the orbits of the satellites and the epochs of their greatest elongation; and a table giving the positions of all the principal observatories of the world.
2. *Popular Astronomy.* — A monthly journal published at Northfield, Minn. It contains a monthly account of the phenomena of *The American Ephemeris;* announcements of discoveries such as comets, temporary stars, and variable stars; the orbits of comets and binary stars; popular articles on nearly every conceivable phase of astronomical thought.
3. *The Astrophysical Journal.* — A monthly journal published at Chicago. It is much more technical than *Popular Astronomy*, and is largely limited to spectroscopic, photographic, and related original astronomical discoveries and discussions.
4. *The Astronomical Journal.* — A journal published at Boston, Mass. It is technical and is mostly devoted to researches and discoveries in practical astronomy and mathematical astronomy.
5. *The Observatory.* — A semipopular journal published at Greenwich, England. It appeals to a great variety of readers.
6. *Monthly Notices of the Royal Astronomical Society.* — A journal of proceedings, discussions, and investigations in all branches of astronomy, published by the society at London.
7. *Bulletin Astronomique.* — A journal published by the French Astronomical Society at Paris. It is much like *The Astronomical Journal* in scope.
8. *Astronomische Nachrichten.* — A journal published at Kiel. It is the oldest periodical devoted to the interests of astronomy, having been started over eighty years ago. It has served as a sort of model for *The Astronomical Journal*.

STAR MAPS

1. Upton's *Star Atlas.* Boston.
2. Klein's *Star Atlas.* London.
3. Proctor's *New Star Atlas.* London.
4. Peck's *The Observer's Atlas of the Heavens.* London.

BOOKS

1. Proctor's *Half Hours with the Stars.* London.
2. Young's *Elements of Astronomy.* Boston.
3. Todd's *Stars and Telescopes.* Boston.
4. Clerke, Fowler, and Gore's *Astronomy.* New York.
5. Gibson's *The Amateur Telescopist's Handbook.* New York.
6. Smyth's *A Cycle of Celestial Objects.* Oxford.
7. Clerke's *System of the Stars.* London.
8. Newcomb's *The Stars.* New York.

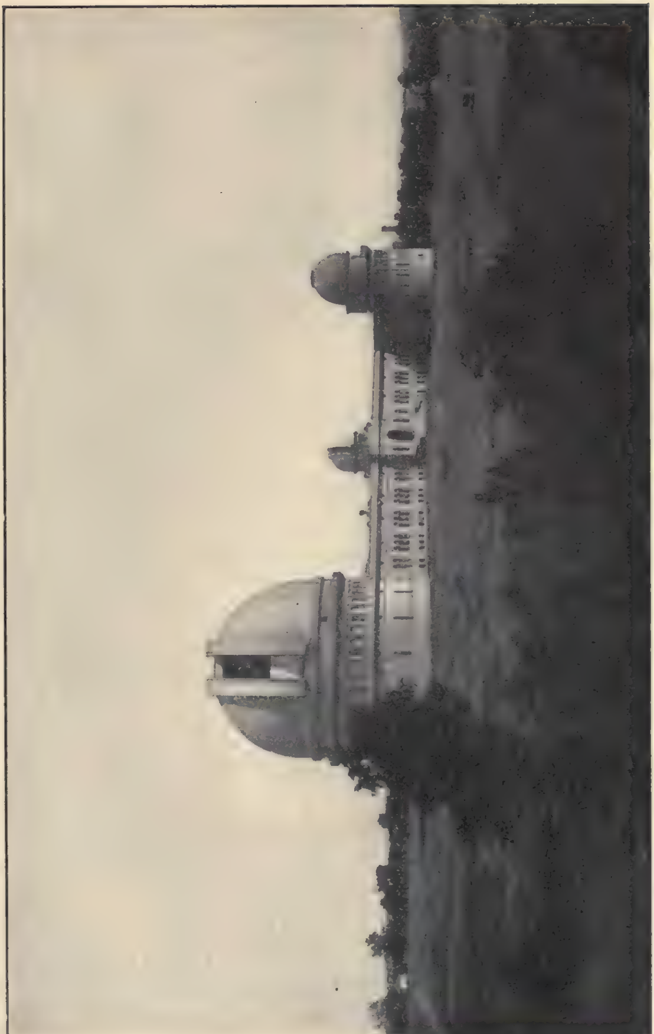


Fig. 23. — Yerkes Observatory of the University of Chicago.

CHAPTER IV

TELESCOPES

62. Kinds of Telescopes. — When classified on the basis of their method of gathering light, telescopes are of two kinds, the *refracting* and the *reflecting*. In the refracting telescope the light from the object is gathered by its passing through a collecting lens. In the reflecting telescope it is gathered by its reflection from a concave mirror.

When classified on the basis of their uses, telescopes are of two kinds, differing principally in their mounting. An important use of a telescope is to enable us to find with great accuracy the direction of an object. This requirement determines the character of one mounting. But if the telescope is to be used for observing the surfaces of such objects as the sun, moon, and plants, and for measuring short angular distances, such as the apparent diameters of the planets, a different type of mounting is required. Both refracting and reflecting telescopes may be adapted to either purpose, though the refracting is now of universal use in determining directions.

The telescope may be used for photographing many celestial objects, and this adaptation of it is becoming more and more important. When employed in this way, a visual refracting telescope must be corrected by means to be explained in Art. 77, but the reflecting telescope requires no changes.

63. The Prism. — A prism, in optics, is a triangular piece of transparent material, usually glass, whose faces are made accurately plane and polished. Let ABC (Fig. 24) be a cross-section of a prism, and suppose the screen S admits a

beam of parallel rays through the narrow slit O which is placed parallel to an edge of the prism. At P the beam strikes the surface AB obliquely. If the prism is denser than the air, the rays are bent *toward the perpendicular to the surface*, and strike the second surface at Q. At this place they emerge from a denser to a rarer medium, and they are *bent from the perpendicular to the surface*. After the second refraction they go on and strike the screen T at R.

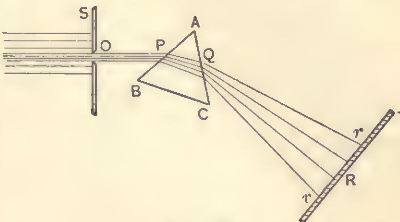


FIG. 24.

Suppose the beam of light is white, that is, a mixture of all colors. Then it will be found that the light at R is not white, but a series of all the colors spread out in a band whose width depends upon the prism. The violet light will be bent the most of that which is visible, and the red the least. The visible colors in their order, from violet to red, are *violet*, *indigo*, *blue*, *green*, *yellow*, *orange*, *red*, and they blend into each other by insensible gradations. Beyond the violet there are rays, the *ultra violet*, to which the human eye is not sensitive; similarly, on the other side of the red there are invisible rays, the *infra red*. The whole series of rays after their separation constitutes the *spectrum*. Until quite recently, none but the visible part of the spectrum was of interest to astronomers, but, as will be seen later, the other parts are becoming of great value.

Prisms of the same shape, but of different kinds of glass, refract light quite differently. As a rule, the denser the glass, or medium, the more the light is refracted. There is also another very important difference. For a given prism the widths of the various colors have a definite relation to one another, but for a prism of another kind this relation may be quite different. Thus, one kind of glass may bend the

red rays to r and spread out the visible light into the spectrum $\overline{rv}$. Another kind of glass may refract the red rays the same amount, but spread out the visible light into a much wider spectrum. This spreading out of the colors is called *dispersion*. It plays an important part in the theory of the refractive telescope, and is the basis of spectrum analysis. (See Arts. 320–329.)

64. The Eye. — The eye is a natural optical instrument whose general structure and uses must be understood in order to comprehend the theory of telescopes.

Figure 25 represents an axial section of the eye. S is the opaque *sclerotic coat*, which is a protective covering of the whole eye except C, the *cornea*, which is transparent. Back of C there is a lens L composed of successive layers which refract the light more and more as the center is approached. At the back of the eye the optic nerve r is spread out into the *retina*, and it is sensitive to light except at the point where it leads out to the brain.

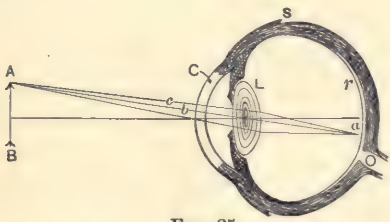


FIG. 25.

Consider an object $\overline{AB}$ situated in front of the eye. (For the sake of clearness in the illustration, it is placed relatively too near the eye.) The light radiates in every direction from every point of $\overline{AB}$. Consider the rays from A. One of them, as b, will strike the lens perpendicularly, and passing through it without change of direction, will fall on the retina at a. Another will strike higher up, but in this case obliquely, and, according to the principles of a prism, it will be bent as indicated in the figure. It will be bent still more on emerging from L, and if the eye is of the proper shape, it will also fall on the retina at a. Likewise every ray from A will fall on a. Every ray from every other

point P will fall on the retina at some other point p. The result is that there is a separate image of every point of $\overline{AB}$ on the retina. In some way, which is not fully understood, the stimulus of this image is conveyed to the brain over the optic nerve, and we are conscious of "seeing" something.

The main point here is that the lens collects the slightly diverging rays from the object and forms an image on the retina. The closer the object is to the eye, the more divergent are the rays from a given point which fall on the lens. If the object is very near, the lens of the eye can not refract them enough to bring them together on the retina. In this condition, instead of all the rays from A being brought together at a, they will tend toward a point back of a and will be spread over a little spot around a, and so with every point. The images of the different points of $\overline{AB}$ will overlap and form an indistinct image on r. Consequently the object can not be seen distinctly.

The conclusion is that the rays may diverge a little, but must not diverge very much if they are to form a distinct image on the retina. The amount of the divergence admissible can be seen from the fact that the pupil of the eye is from one-fifth to one-quarter of an inch in diameter, while the distance of most distinct vision, that is, sharpest images on the retina, is from 15 to 20 inches. From the principles of Art. 20 and these data, it is found that the eye can bring rays to a sharp focus only if the extreme ones which enter the pupil diverge less than about $45'$.

THE REFRACTING TELESCOPE

65. The Simple Lens and its Image. — There are certain serious difficulties to be overcome as far as possible in constructing a telescope, but the subject will be best understood by first considering the general principles which are involved, without reference to these refinements. A refracting tele-

scope consists of two essential parts. One is a lens, the *objective*, which gathers the rays and forms an image of the object, and the other is the *eyepiece*, which enables us to see the image and in a sense magnifies it. The objective is treated in this article.

Let L be a double convex lens whose two surfaces may have quite different curvatures.

Let $\overline{AB}$ be a very distant object. (In the figure it is necessary to represent it as being near.) All the rays from A which strike L will be very nearly parallel. Suppose all the rays from A are refracted to the point a and that those from B are refracted to b, etc.; that is, there is in the air at $\overline{ab}$ an inverted image of the object $\overline{AB}$. Only three rays from each of the two points are given in the figure.

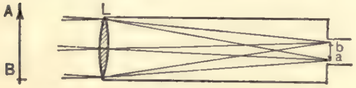


FIG. 26.

66. Magnitude and Brightness of the Image. — It is seen from Fig. 26 that from every point of $\overline{AB}$ there is one ray which passes through the center of L and whose direction is not changed. Consider the rays from the extremities of $\overline{AB}$; then it follows from the figure that the angular diameter of the object and of its image as seen from the center of L are the same. Therefore *the diameter of the image is directly proportional to its distance from the objective.* It is to be observed that this statement is true, whatever the diameter of the lens L may be.

Consider a lens of given size. It admits a definite amount of light from the object. This light is distributed over the image, and it follows that the brightness of the image decreases directly as its size increases. Suppose the object is a surface; then the size of the image increases, and its brightness decreases, as the square of its distance from the objective increases. But the brightness of the image depends also upon the amount of light admitted by the objective.

The light admitted varies directly as the surface of the objective, or as the square of its diameter. Hence the final result is that *the brightness of the image formed by an objective varies as the square of the aperture* (diameter of the objective) *and inversely as the square of the focal length* (the distance from the objective to the image).

67. Photographing the Image. — If a sensitive plate is exposed at the focal plane, the image of the object will be secured. Suppose the object is bright, like the sun or moon; then it is advantageous to use a telescope with a long focal length, for there will be no lack of light when the image is large. Suppose, on the other hand, that the object is very faint, like some of the nebulas; then it is advantageous to use a telescope with a short focal length, for the problem is to secure an image bright enough so that a picture may be secured in the time which can be given to an exposure. In this case the scale of the picture will be small, but when comets, nebulas, and other faint objects of large extent are photographed, this is not an objectionable feature.

68. Seeing the Image. — In treating the problem of seeing the image, a single point of it may be considered; for, if the individual points can be seen separately, there will be no trouble with any number of them together.

Suppose the object is very remote and that the sensibly parallel rays from one of its points strike the lens L and pass through the focus F. The light would make a point on a photographic plate exposed at F. Consider the effect

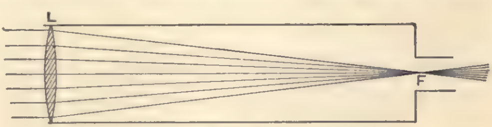


FIG. 27.

on the eye placed at F. The rays come to a point on the lens of the eye from the whole objective. Since they are converging, they will *not* be brought to a focus on the retina, and the whole objective will appear to be filled with light.

If the eye is held just a little back of F, the rays will not all strike it at the same point, but they will be so divergent that the eye lens can not bring them to a point on the retina, and the image will appear blurred. The result is the same as when one tries to see an object held very close to the eye.

Suppose now that the eye is held back a few inches, and let it be remembered that the pupil through which the light must pass is small. Only a small part of the whole cone of rays will enter the eye, and they will be those which are not strongly divergent. When a normal eye is back eight or ten inches, it can focus the rays which enter the pupil, and the image appears as a point.

Let us find whether fainter objects can be seen with the single lens L than can be seen without it, and whether more of the details of such an object as the moon are made visible. The first problem is to find whether more rays from a point of the object enter the pupil of the eye with the lens than without it. The cross-section of the cone of rays back of F is illuminated by the object more than the outside of the telescope until the diameter of the cone is as great as that of the objective. But the cone of rays does not become as large as the objective until the distance back of F is equal to the distance from the lens to F. Consequently, if the eye is held nearer the focus than the focal distance of the objective, it will receive more light than it will on the outside, and within this limit fainter objects can be seen through the objective than with the unaided eye.

Suppose the light comes from two points very near together; then another system of sensibly parallel rays will strike the objective at an angle differing slightly from the angle of those already considered, and they will be brought to a focus at a point very near F. Suppose this point is just below F in Fig. 27, and call it F' (Fig. 28). The central rays of these two cones make the same angle with each other that the two systems of rays do before they strike the objective.

Thus, in Fig. 28 let c and c' be the central rays which meet in L. There are rays from F which intersect c' extended, and from F' which intersect c extended, which make a greater angle with each other than the angle between c and c' at L. Without the use of the objective, all the rays of the first set are parallel to c and all those of the second set are parallel to c', and the distance apart of their two images on the retina depends upon this angle. Suppose the eye is between c and c' and far enough from F and F' so that their rays can be brought to a focus, and on the other hand near enough so that the angle between the two sets is greater than that at L; then the images on the retina will be farther apart than they would be without the use of the objective.

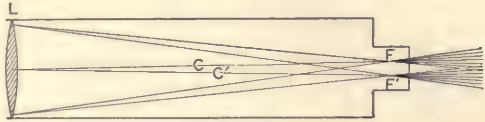


FIG. 28.

The terminal nerve fibers of the retina are not indefinitely fine, and two images must be at some little distance apart (about $\frac{1}{1200}$ of an inch) in order that they may be separately distinguished. Therefore it follows that two objects sometimes may be seen separately with the aid of an objective alone when they can not be seen separately without it.

69. Use of the Eyepiece. — The preceding discussion has been given more in detail than the results merit except for the illumination they give the question now treated. As before, the discussion can be made simplest by treating first the case where the source of light is one point, and then where it is two points close together.

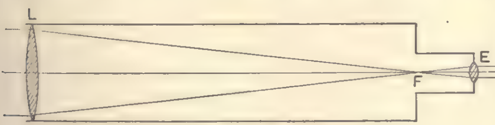


FIG. 29.

Suppose parallel rays from a star strike the objective L and that they are brought to a focus at F (Fig. 29). Place a

small lens E, more convex than L, at such a distance from F that the divergent rays which fall upon it emerge parallel. Then the whole cylinder of rays which entered L parallel emerge from E parallel in a much smaller cylinder. If the eye is placed anywhere back of E, parallel rays will enter it and will be brought to a focus on the retina. The object will appear to be a point just as it does without the telescope, but it will be much brighter because the cylinder of rays has been greatly condensed.

For illustration, compare the eye with a 2-inch telescope. The diameter of the pupil is about one-fifth of an inch. The light-gathering power of the eye and of the telescope are to each other as the squares of their apertures; that is, as $(\frac{1}{5})^2$ is to $(2)^2$, or as 1 is to 100. Therefore the telescope enables the eye to get 100 times as much light from the star as it would get without it. On this basis, since the ratio of the light of a first-magnitude star to that of a sixth-magnitude star is as 100 is to 1, if a sixth-magnitude star is just visible without the telescope, a star five magnitudes fainter, or the eleventh, would be just visible with the telescope. There is, however, quite a little loss of light by absorption in the glass and reflection from the surfaces of the lenses, so that the actual difference is less than the theoretical. But Argelander made his great catalogue of stars down to magnitude 9.5 with a 2.5-inch telescope, which amply illustrates the value of even a small instrument.

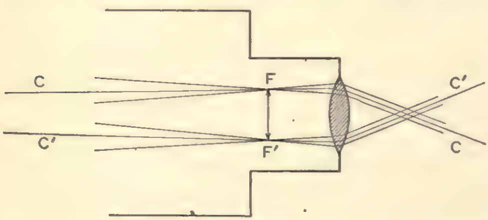


FIG. 30.

Now consider the case of two luminous points, as stars, near together in the sky. After passing through the objective the rays converge at two points, F and F' in Fig. 30. Now place an eyepiece

lens at such a distance from F and F' that the two cones of rays emerge from it in two sets of parallel rays. When the eye is placed back of the eyepiece, the two sets of parallel rays are brought together at two points on the retina, and the object appears to consist of two points, as it does without the telescope. There are two differences. In the first place each point appears brighter, for each cylinder of rays is greatly condensed; and in the second place the two points appear farther apart, for the rays c and c' make a greater angle with each other after passing through the eyepiece than they do before.

70. Magnifying Power. — The ratio between the natural apparent angular diameter of an object and its apparent angular diameter as seen through the telescope is the magnifying power of the telescope. In Fig. 30 the natural angular separation of the two objects is the angle between c and c', or the angle subtended by $\overline{FF'}$ as seen from the center of L. The angular separation as seen through the eyepiece is the angle subtended by $\overline{FF'}$ as seen from the center of E. These two angles, which are small and subtended by the same distance $\overline{FF'}$, are to each other nearly inversely as the distance of $\overline{FF'}$ from the two lenses. For a given objective the shorter the focus of the eyepiece the higher the magnifying power; and for a given eyepiece the longer the focus of the objective the higher the magnifying power. The converse statements are also true. That is, the magnifying power of any combination of objective and eyepiece is the focal length of the objective divided by the focal length of the eyepiece.

For example, if the focal length of the objective is 36 inches, and if the focal length of the eyepiece is three-fourths of an inch, the magnifying power is $36 \div 0.75 = 48$.

A very simple and convenient way of measuring the magnifying power of a given combination of objective and eyepiece is to focus on a star and turn the telescope on the sky

in the daytime. A cylinder of rays will emerge from the eyepiece, and the ratio of the diameter of the objective to that of this cylinder is the magnifying power of the combination. With a given objective, which is the expensive part of a telescope, several eyepieces generally are used, giving various magnifying powers.

71. Seeing Large, Faint Objects. — The more an object is magnified, the farther apart are the images of its parts on the retina. This means that the light which enters the eye is spread over a larger surface and the intensity is correspondingly less. Consequently, it diminishes the apparent brightness of an object to magnify it, and in viewing faint objects of finite apparent dimensions, a low-power eyepiece should be used. The principal objects of this class are comets and nebulas which generally are of considerable angular dimensions.

It is advantageous to see as much of an object as possible at once, and here again the low-power eyepiece has the advantage. A high-power eyepiece must be very convex, and this limits its diameter so that it takes in only a small part of the rays which come to a focus in the focal plane of the objective. Comet hunters always use low-power eyepieces so as to secure high illumination and a wide field.

72. Seeing Small, Bright Objects. — Suppose the object is a very close double star. The image of each star in the eye is a point, for the two systems of rays emerge in parallel lines from the eyepiece. Consequently, a high power does not diminish the brightness, but makes the stars seem farther apart. This is precisely what is desired in order that they may be seen separately. Therefore in such work a high power should be used.

A word of warning needs to be given at this point. The irregular density of the air and the waves in it cause serious troubles in the image. This is the source of the twinkling, or *scintillation*, of the stars as seen with the unaided eye.

The image formed by the objective is subject to many imperfections, some of which will be discussed immediately. The eyepiece also has various faults which increase rapidly as its power increases. The errors due to the atmosphere and the objective are magnified by the eyepiece just the same as the correct parts of the image. When the magnification is small, the eye smooths out most of the irregularities, and a nearly correct impression is obtained; but when the magnification is large, the eyepiece is more imperfect, and the individual oscillations traverse so many of the nerve terminals of the retina that the image appears blurred. For these reasons one must be on his guard against using too high-power eyepieces; the danger lies entirely in this direction.

73. Diffraction. — Light is a wave motion in the luminiferous (light-bearing) ether, which is a fluid filling the interstellar spaces and permeating matter. The waves are something like those on the surface of water, though the illustration is far from perfect. They are more like the waves through an elastic solid, in which the vibrations may be in any direction in the plane perpendicular to the line of propagation.

It follows from the wave theory of light that the image of a point source of light through a perfectly constructed object glass is not exactly a point, as has been supposed above, but a small, luminous circle surrounded by a series of rings of light which very rapidly become faint as the distance from the center increases. The reasons for this can not be gone into here more than to state that the trouble has its source at the circumference of the objective, every point of which acts like a source of light. It is a phenomenon which appears in all classes of optical instruments and is called *diffraction*. The bright circle and series of rings constitute the diffraction pattern. It is desirable to have the bright circle as small as possible and the rings relatively inconspicuous.

The appropriate mathematical discussion shows that the size of the circle is inversely proportional to the diameter of the objective, and that its relative brightness, as compared to the rings, increases in the same ratio. This is the chief reason for the superiority of large instruments in showing fine detail, or in separating very close objects.

74. Spherical Aberration. — Suppose the object glass is a perfectly accurate double-convex lens, and suppose the light is all of the same color. It can be shown by a mathematical discussion that, even though the diffraction be disregarded, the rays from a point source of light do not pass through a single point. Those which pass through the marginal portions of the object glass are brought to a focus nearer the objective than those which pass through the central portions. The image on a screen held at any distance is a small circle instead of a point. This error, which is due to the use of spherical surfaces, is called *spherical aberration.*

It is easy enough to calculate what form the surfaces should have in order to give no errors of this type, but it is very difficult, if not impossible, to grind and polish to these forms. The error can be corrected almost perfectly by using two lenses in the objective, and at the same time the trouble to be discussed in the next article can be largely overcome.

75. Chromatic Aberration. — It was seen in Art. 63 that when light passes through a prism, it is spread out into a spectrum of colors, the violet being refracted most, and the red least, of that which is visible. In a precisely similar way, when light passes through a lens, each beam is spread out into a spectrum. If there were no diffraction or spherical aberration, a single lens would still give a very imperfect image, for every color would be brought to a focus at a different distance from the objective, the violet being the nearest and the red the farthest of that which is visible. This imperfection in the image is called *chromatic* (color) *aberration.*

76. Compound Objectives. — The diffraction is a trouble which can not be remedied, but the spherical and chromatic aberrations can be overcome almost entirely by using compound objectives of two kinds of glass having different properties of refraction and of dispersion. A common type will be explained, though there are several others which are about as good.

The objective consists of two lenses, a double-convex in front and a concavo-convex behind. The first lens is of crown glass with the front surface a little flatter than the second. If it were not for the second lens, the visible rays would be brought to a focus along the line F', the focus of the red

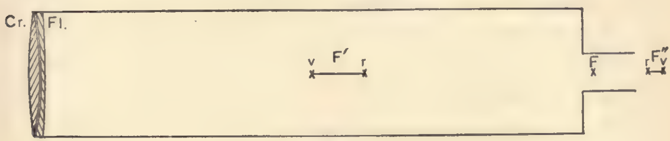


FIG. 31.

being farthest from the objective. These colors are considerably mixed up by the spherical aberration, for the marginal rays are brought to a focus sooner than the central. Thus the marginal red might fall on the central yellow.

The second lens is of flint glass. Its first surface is concave and a little more curved than the second surface of the crown lens. Convex and concave surfaces act oppositely on rays of light, the latter spreading them out. Flint glass refracts light more than crown, and its dispersion is not only greater, but it is greater in proportion to its refraction. Now the first surface of the flint glass not only tends to spread out the light, that is, to bring the rays back parallel, but it acts most on the violet end of the spectrum. The result is, neglecting the action of the second surface of the flint lens, that the focus is the line F'', the focus of the violet being a little farther from the objective than that of the red. The

last surface is very slightly convex and shortens the focus of every color, but the violet the most. The result is that, when these four surfaces are properly chosen, all colors come to a focus very near to the required point F.

The correction just discussed is not perfect, for the two kinds of glass disperse the different parts of the spectrum in relatively different amounts. Thus, if the surfaces are such as to bring the yellow and green accurately together at F, it is found that the other colors are a little inside or outside of F. In the best objectives the yellow and green are brought to a focus at very nearly the same point, and the other colors at slightly greater distances.

Our eyes are most sensitive to light in the yellow near the green, while ordinary photographic plates are most sensitive to the blue. If the telescope is used for photographic work, the objective must have a color correction different from that which is required for visual purposes.

It is easy to show in a general way how the spherical aberration is corrected. The spherical aberration depends not only upon the focal distance of the objective, but also upon the way the work of refraction is distributed among the surfaces. As a general rule, the more evenly the work of refraction is divided among the surfaces, the less the spherical aberration; and also, the greater the number of surfaces which are used, the less the spherical aberration. Surfaces which collect the rays give a positive spherical aberration, and those which spread them out a negative spherical aberration. In the objective represented in the diagram (Fig. 31), the work of collecting the rays is distributed among the first, second, and fourth surfaces, while that of spreading them out is all done by the third surface. It follows that, when the surfaces are properly chosen, even though the whole result is a gathering of the rays, the positive and negative spherical aberrations just balance each other.

This discussion has been made for a point source of light.

When the source is a surface, the light from it must all be brought to a focus in the same plane and without distortion. It is a somewhat complicated mathematical problem to satisfy simultaneously all these conditions, and it must be solved separately for every pair of glasses.

The curvature of the surfaces is much less than one would imagine. For a 5-inch telescope the focal length is generally about 80 inches, and the most curved surface is the same as that of a sphere 30 or 40 inches in diameter.

77. Photographic Refractors. — When an objective is constructed for visual purposes, it is not suitable for photographic work, because photographic plates are ordinarily most sensitive to the indigo-blue light instead of the yellow-green. If the objective is designed for photographic work alone, it can be corrected for the appropriate part of the spectrum by the method explained in the last article. A visual objective may be corrected so that it will focus the photographic rays by placing in front of it another lens having the proper surfaces. Pickering has devised an objective which brings the visual rays to a focus when one side is in front, and the photographic when it is turned around.

Another method of using a visual telescope for photographic purposes is to cut out the poorly focussed indigo-blue rays by a suitable absorbing medium placed in their path near the principal focus, and to use plates specially sensitized to the visual rays.

78. Eyepieces. — The eyepiece has been spoken of heretofore (Arts. 69 and 70) as though it consisted of a single double-convex lens. Such an eyepiece is satisfactory for a small object in the middle of the field; but to secure a large field and good definition compound eyepieces must be used. They are of two kinds: the *positive*, in which the focus of the objective is outside of the eyepiece, and the *negative*, in which it is within the eyepiece. The former was invented by Ramsden and the latter by Huyghens.

The Ramsden eyepiece consists of two plano-convex lenses of equal focal length placed with their convex sides toward each other. The distance between them is about two-thirds of the focal length of either, and the one toward the object is distant one-fourth of its own focal length from the focus of the objective. Thus, in Figure 32, *F* is the focus of the objective, *A* the front lens, called the *field glass*, and *B* the one next to the eye, called the *eye glass*. The figure shows how the converging rays from the objective pass through the eyepiece and emerge parallel. When the eye is placed behind the eyepiece, the parallel rays fall on the retina at a point whose position depends only on their direction.

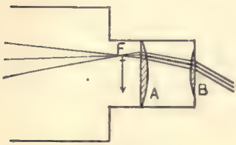


FIG. 32.

The Huyghenian eyepiece consists of two plano-convex lenses with their convex surfaces turned toward the objective. The field glass has a focal length equal to three times that of the eye glass. The distance between the two lenses is equal to twice the focal length of the eye glass, and the field glass is placed between the objective and its focus at a distance equal to one-half its focal length from the latter. Thus, in Fig. 33, *F* is the focus of the objective, but the rays are intercepted by the field glass before they reach this point. The figure shows the path of the rays through the eyepiece. *D* is a diaphragm which cuts off all rays which would not fall on the eye glass.

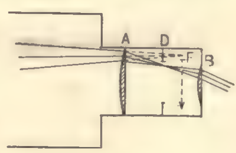


FIG. 33.

In both cases the image is inverted by the objective, and is not again brought upright by the eyepiece.

79. The Reticle. — It is often desired to measure short distances, such as the angular separation of the two components of a double star. This is accomplished by means of the reticle, which consists of a frame holding spider lines, or

something else equivalent, in the plane of the focus of the objective. Thus, Fig. 34 represents a cross-section of the telescope at the focal plane of the objective. A double star is in the field, and the objective makes images of the two components in this plane. The reticle frame is attached in such a way that it may be rotated so as to bring the spider line *a* in line with the two stars. The lines *b* and *c* are perpendicular to *a*. The telescope is moved so that one of them, as *b*, falls on one of the stars. The other is movable, and is shifted by a screw with a graduated head until it coincides with the other star. The number of rotations of the screw and the known angular value of one rotation give the angular distance of the two stars from each other.

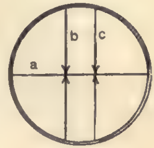


FIG. 34.

The spider lines are illuminated from the side, except when very bright objects are measured. The lines can be seen through the eyepiece, for they are at the plane which is in focus for the eyepiece. They are always used with a Ramsden, or positive, eyepiece; with the Huyghenian it would be necessary to place them in the eyepiece itself, which would involve many difficulties. It is immaterial what power eyepiece is used, for the star distances and reticle lines are always magnified the same.

It is apparent from the preceding discussion that eyepieces are simple and inexpensive as compared with objectives, which must be made with the very greatest care. Eyepieces seldom cost more than twenty-five dollars, while objectives cost all the way from a few dollars up to many thousands.

QUESTIONS AND EXPERIMENTS

NOTE. — Every school should be equipped with at least a small telescope. When work with it is guided by a knowledge of its theory and uses, and when observations are made to see *something*, it becomes a most

fascinating instrument. If a telescope is available, Chapter III should be gone over again and its statements verified as far as is possible.

1. Suppose light passes from water to a prism; will it be refracted as much as when it passes from air into the same prism?

2. Would a fish require a more convex or a flatter eye than a land animal if the lens in his eye were of the same material?

3. Would a fish require a denser or rarer lens in his eye than a land animal if it were of the same shape?

4. If you open your eyes under water, you can see distinctly only a small surface directly in front of them. What is the reason the field of view is so restricted?

5. Are the lenses too convex or too flat in the eyes of a nearsighted person?

6. Does a nearsighted person need convex or concave glasses?

7. Are diffraction phenomena present in the eye?

8. Find your distance of most distinct vision by observing something very fine, as the striæ on a crystal.

9. Is the image on the retina inverted?

10. Could you see the image $\overline{ab}$, Fig. 26, by looking in at the side of the telescope?

11. The angular diameter of the moon is about one-half a degree. What is the diameter of its image in a telescope whose focal length is 32 inches? What is the diameter of its image in the Yerkes telescope whose focal length is 62 feet?

12. How does the brightness of an image in a telescope of 2 inches aperture and 32 inches focal length compare with that in one of 4 inches aperture and 64 inches focal length?

13. Verify by experiments the statements made in Art. 68. For faint objects use the moons of Jupiter, the stars within the bowl of the Big Dipper, or the stars next fainter than the first seven in the Pleiades. For the test of separation look at the markings on the moon, or at Epsilon Lyræ, or at Mizar and Alcor.

14. How large would an objective have to be in order to be as much more powerful than a 2-inch in gathering light as a 2-inch is more powerful than the eye?

15. If an objective of a given diameter shows stars of a given magnitude, how much larger an objective will be required to show stars one magnitude fainter?

16. Try low and high powers on the Orion or Andromeda nebula to verify the statements of Art. 71.

17. Look at Algol to verify the statements of Art. 72.

18. Hold a lighted match a little below the objective and a foot or

so in front of it while observing a bright star to see the effect of air currents.

19. If a pencil were held close in front of the objective, could it be seen through the telescope?

THE REFLECTING TELESCOPE

80. The Mirror. — When a ray of light of any color strikes a smooth reflecting surface at any angle, it is reflected from the surface at the same angle. This law of reflection is at the basis of the theory of reflecting telescopes.

In Fig. 35, *M* is a concave mirror with a system of parallel rays falling on it. When it has the proper shape, they are all reflected to the focus *F*, at which there is an image of the point source of light. The required shape of the mirror is a surface called a *paraboloid*. For the small section used it is nearly the same as a spherical surface, the difference being that it is a little flatter toward the margins.

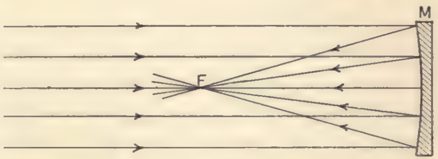


FIG. 35.

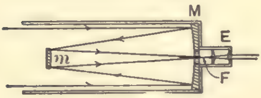
If one could place an eyepiece at the proper distance to the left of *F*, he would see the star just the same as with a refractor. But in this position he would shut off the incident rays of light. The three principal artifices which have been used to enable one to see the image without cutting off the light from the object are illustrated in the diagrams of Fig. 36.

In all the diagrams *M* is the large mirror and *E* the eyepiece, which is the same as that which is used in a refracting telescope. In the Newtonian telescope, the rays are reflected back to a small right-angled prism, which is placed so that its hypothenuse face makes an angle of 45° with the side of the telescope. The light enters the prism, is reflected from

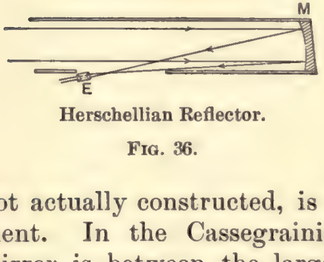
its hypothenuse face, and leaves it nearly at right angles and enters the eyepiece.

In the Cassegrainian telescope, the rays are reflected back to a small, nearly flat, convex mirror, which in turn reflects them through a small opening in *M* into the eyepiece.

Newtonian Reflector.



Cassegrainian Reflector.



Herschellian Reflector.

Fig. 36.

In the Herschellian reflector, the mirror is inclined a little to the axis of the telescope, so that the rays are reflected to a focus at the side. This is the simplest type and has the least loss of light, but there is a slight distortion of the image on account of the obliquity of the axis of the mirror to the incident rays.

The oldest type of reflecting telescope, which was invented by James Gregory and described in 1663, but not actually constructed, is much like Cassegrain's instrument. In the Cassegrainian reflector, the small convex mirror is between the large mirror and its focus; in the Gregorian, a small concave mirror is used beyond the focus of *M*. The Cassegrainian has two advantages over the Gregorian; it is shorter, and it has less spherical aberration.

81. Errors in Reflectors. — There are diffraction difficulties in reflectors precisely as in refractors. The spherical aberration is also present, and even more difficult to eliminate; but there is no chromatic aberration, except the little which may be introduced by the eyepiece. If the telescope is designed for both visual and photographic purposes, this is a very important advantage. The irregularities in the air

affect both kinds of instruments in the same way. One of the chief sources of difficulty in the use of large mirrors is their change of form with varying temperature and position.

82. Comparison of Refractors and Reflectors. — From the preceding discussions of the errors to which the two kinds of telescopes are subject, one can get an idea of the way they compare in performance; but a short recapitulation of these points and the enumeration of some others will help to bring out the similarities and contrasts.

As far as the construction is concerned, the reflector has important advantages. In the first place, there is but one surface to work instead of four; in the second place, it is easier to get material suitable for a mirror than for a lens, for its transparency and homogeneity are not essential. On the other hand, the silvering of a mirror deteriorates with age, while the polish of a lens is very permanent, and an error in the surface of a mirror causes three times the trouble in the image that the same error does in a lens.

As far as the light-gathering power is concerned, the advantage is with the refractor, except, possibly, in the best examples of silvering while it is fresh. The loss from the absorption of light by the lenses and by reflection at their surfaces does not now amount to more than 15 per cent of the incident rays in small objectives a few inches in diameter, while the loss in reflection at the surfaces of the mirrors required in reflectors generally amounts to from 30 to 50 per cent. But in large refractors the loss of light due to the greater thickness of the lenses may equal, or even exceed, that of reflectors of the same size.

As far as the aberrations are concerned the advantage lies with the reflector, for it is entirely free from chromatic errors. This is very important in photographic and spectroscopic work.

As far as the convenience is concerned, the advantage probably lies with the refractor.

The general consensus of opinion seems to be that for ordinary visual work the refractor is preferable, and that for special photographic and spectroscopic work the reflector is preferable. The great observatories are very generally equipped with both kinds of instruments.

83. Size of Telescopes. — Instruments of larger and still larger dimensions have been made until one might be led to suppose that there is almost no limit to the size and power which they will eventually attain. Less than a century ago there was no refractor more than 6 inches in diameter, while now there are seven 30 inches or more in diameter, and more than fifteen others with apertures greater than 20 inches.

Reflecting telescopes have attained still greater dimensions, the largest being 6 feet in diameter, while there are quite a number 4 feet or more in diameter. There has been no increase in the size of reflectors in recent times, for the one 6 feet in diameter was erected in 1845 by Lord Rosse at Parsonstown, Ireland.

There are reasons for believing that it will not be found advantageous to increase much, or even any, the size of either refractors or reflectors. In the first place, the cost of a large instrument is very great, and the cost of keeping it running correspondingly large. For example, the disks from which the 40-inch objective of the great Yerkes telescope was made weighed in the rough over a ton and cost $20,000. The whole cost of constructing and maintaining so large an instrument is on a corresponding scale. The tube is over 60 feet long, and this causes so much motion in the eye end, as the telescope is turned from the zenith to near the horizon, that an observer can not get up to it on any ordinary ladder. To overcome this difficulty the whole floor, 70 feet in diameter, is made so that it may be raised and lowered by electric power. This requires a power house, and an engineer must be on duty every time the telescope is used. An enormous amount of time is consumed in the manipulation of such an

unwieldy instrument. Probably few astronomers would consider it wise to expend the money to construct still larger refractors and reflectors when it could be put to good uses in so many other ways.

There are, however, other reasons more important than the financial for believing that the limit in size has been reached. The lenses of refractors must be supported at their perimeters. As their size and weight are increased, they are subject more to flexure; and if they are made thick enough to withstand distortion, they absorb considerable light and are not easily kept at the same temperature throughout. But probably the most formidable difficulty which confronts astronomers is of atmospheric origin. A large objective or mirror gathers light passing through all phases of air waves and gives an image made up of all of them. When the air is unsteady, the image is very indistinct and even worse than in a smaller instrument, as every observer has found by experience. The air always produces some irregularities in refraction, and the problem is to find what aperture is the greatest that can be used to advantage under the best conditions. Experience seems to show that the limit has already been reached.

The length of the tube of a refractor is generally about fifteen to eighteen times the diameter of the objective, and in large instruments it is difficult to keep the temperature of the air the same throughout its whole length. Wadsworth has shown that even a slight variation in the temperature has a very serious effect on the image, and he concluded from this alone that a limit in size has already been reached, beyond which it is not wise to go.

There are serious mechanical difficulties, but they are probably not insuperable.

84. Historical. — There are indefinite statements that Roger Bacon (1214–1294) and Leonard Digges (?–1571) both invented instruments enabling them to see distant ob-

jects more clearly than they could with the unaided eye, but if so their inventions passed unnoticed and were lost. The actual invention and construction of the telescope dates from Hans Lippersheim (?–1619), a Dutch spectacle maker, who made a telescope in 1608. He applied for a patent on it, but his application was refused. This may, perhaps, be taken as indicating that the idea was somewhat well known at the time.

In 1609 a report of Lippersheim's invention reached Galileo, who discovered the method of construction for himself, and soon made a telescope magnifying three diameters, and then one magnifying thirty diameters. He undoubtedly first introduced the telescope into astronomy, and he very soon discovered the mountains on the moon, spots on the sun, the satellites of Jupiter, star clusters, etc. Compared to the poorest instruments of the present time, his telescope was very imperfect. It was not corrected for either of the aberrations, and its field of view was very small.

Newton was the great successor of Galileo. He recognized chromatic aberration and, supposing it could not be overcome, abandoned refractors and invented a reflector. But an Englishman, named Hall, constructed an objective in 1733 of two lenses in which the chromatic aberration was largely overcome. The invention attracted little attention until 1760, when the method was rediscovered and patented by Dolland, a London optician.

The next important problem was to cast disks of optical glass of large size and the required purity and homogeneity. The first real advance in this problem was made early in the nineteenth century by the Swiss glass maker Guinand and the German physicist Fraunhofer, who together constructed a refractor of the unprecedented size of 10 inches in diameter. Great improvements were made about 1840 by Feil at Paris, and they have been steady and continuous ever since. The studies and experiments of Abbé and Schott at Jena, resulting

in the discovery of many new kinds of glass, and some of them of much importance, are particularly worthy of mention. At the present time most of the optical glass is cast either at Jena or at Paris.

The work of grinding and polishing lenses has correspondingly advanced. The most remarkable progress was made about the middle of the last century by Alvan Clark of Cambridge, Massachusetts. He took up the work as a pastime, but he became so successful that his instruments soon found favor, particularly in England. His first large objective was the 18.5-inch of the Dearborn Observatory at Evanston, which was finished in 1862. Since that time, with the assistance of his sons, he has constructed a majority of the great instruments, including the two largest equatorially mounted refractors in the world, the 36-inch at the Lick Observatory and the 40-inch at the Yerkes Observatory. Although nothing can detract from the splendid successes of the Clarks, they have had worthy rivals in Sir Howard Grubb of Dublin, Steinheil of Munich, and the Henrys of Paris. The success of the Clarks in the mere number of large instruments undoubtedly comes partly from the fact that there are nearly as many in the United States as in all the rest of the world. Their largest instrument sold abroad is the 30-inch refractor of the Russian Imperial Observatory, at Pulkowa.

Until very recent times reflecting telescopes have been constructed largely by the English. The principle of reflectors seems to have been first explained by James Gregory (1638–1675) in 1663, but he did not actually construct an instrument. Newton made one in 1668, for he despaired of overcoming the chromatic aberration to which lenses are subject. Nevertheless, owing to mechanical difficulties in their construction, they did not equal refractors in efficiency until the work of Herschel, about a century later.

No observer ever had a more remarkable career than William Herschel (1738–1822). He was born in Hanover,

and as a young man he was a musician in the German army. His health was delicate and, in 1757, in the midst of the Seven Years' War, he emigrated to England. In 1766 he settled in Bath as a musician and music teacher, but devoted his spare moments to the study of mathematics, optics, and

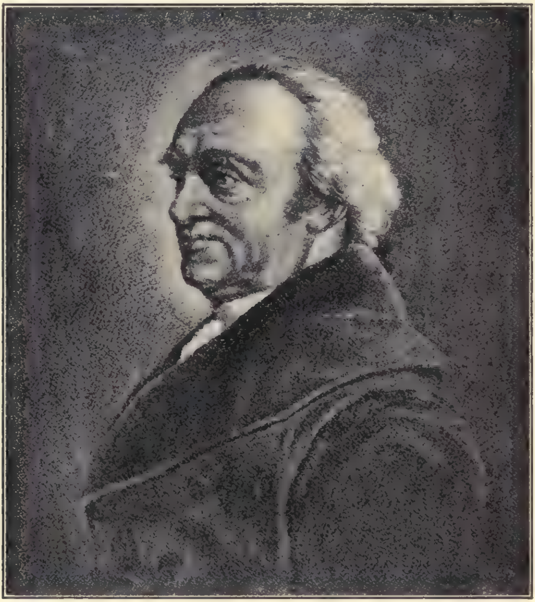


Fig. 37. — Sir William Herschel (1738–1822).

astronomy. Having his interest aroused by a borrowed telescope, he resolved to make as large an instrument as could be constructed, and to study carefully every object within its reach in the whole heavens. In spite of the tedious work of grinding and polishing a mirror, and his many failures, he finally succeeded in getting an instrument giving fair definition, and with it he made his first recorded observation, which was of the Orion nebula, in March, 1774.

Then followed larger and still larger instruments, with which he swept the whole northern heavens four times. In the second of these surveys, which was made with a reflector of the Newtonian type seven feet in length, he discovered on March 13, 1781, a new planet (Uranus), the first discovery of the kind in historic times. He was called to the court of George III, appointed Royal Astronomer with a salary of £200 a year, and given the chance to spend his whole life in astronomical work. His largest reflector, which was finished in 1789, was four feet in diameter and forty feet long. On the first evening of its use a sixth satellite of Saturn was discovered and in less than three weeks a seventh. It was last used to observe the Orion nebula in January, 1811.

In former times reflectors were made of speculum metal, an alloy of tin and copper. They are now made of glass, highly polished, and coated with a very thin film of silver. They reflect much more light than the old mirrors, and though easily injured, it is not very difficult to renew the silvering.

As has been said, Lord Rosse has the largest reflector; but probably the most efficient one actually in use is the 5-foot silvered mirror which Common erected at Ealing, England, in 1889. In the United States the largest reflector in actual use is a 3-foot instrument at the Lick Observatory, although there is one 5 feet in diameter at the Solar Observatory of the Carnegie Institution which is practically finished. The Lick Observatory reflector was made in England by Calver, and the 5-foot by Ritchey of the Solar Observatory staff.

THE MOUNTING OF TELESCOPES

85. The Meridian Circle. — The meridian circle is a telescope, rarely more than eight inches in diameter, mounted on an axis so that it can turn only in the plane of the meridian. At the focal plane it is furnished with a reticle

consisting of two horizontal spider lines, one of which is movable, and an odd number of vertical lines, usually greater than five, which are fixed. Attached to the axis on which the telescope turns is a graduated circle whose markings are read with fixed microscopes.

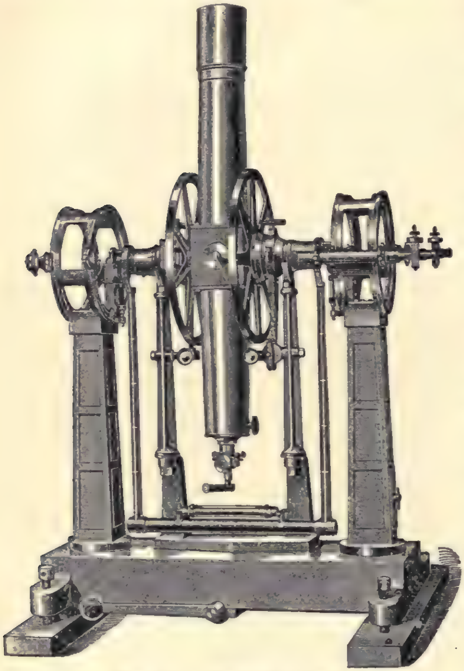


FIG. 38.—Meridian Circle.

In addition to the essentials just given, there are levels, clamps, counterpoises, etc., for getting the instrument in adjustment and keeping it there. Every device of the mechanician's art is used in securing accuracy in the graduation of the circles, uniformity of the pivots on which the telescope turns, freedom from flexure of the tube, and stability of the instrument. The mounting is so delicate that if one lays his hand on one of the supports a minute the heat imparted will throw the whole instrument quite out of adjustment.

If the meridian circle does not have the accurately graduated circle, it is called a *transit instrument*. Transit instruments are constructed so that they may be raised up and their east and west ends interchanged.

When a transit instrument is mounted so that it moves in

the plane of the east and west points and the zenith, it is called a *prime vertical instrument.*

86. Uses of the Meridian Circle. — As has been seen (Art. 23), the whole sky passes the meridian (actually the meridian sweeps across the sky) in about four minutes less than an ordinary day. Let this interval of time defined by the apparent motions of the stars be called a sidereal (star) day. Suppose the observer has a clock keeping sidereal time, its hours being numbered from 0 to 23, and that it is set so that it registers zero when the vernal equinox crosses the meridian. Suppose the sidereal time that any star crosses the meridian is found; this is its right ascension, for the sky turns uniformly. The exact time is found by taking the average of the times at which it crosses the several lines of the reticle. The time of transit is taken to hundredths of seconds of time, and errors greater than one-tenth of a second are inadmissible in good work.

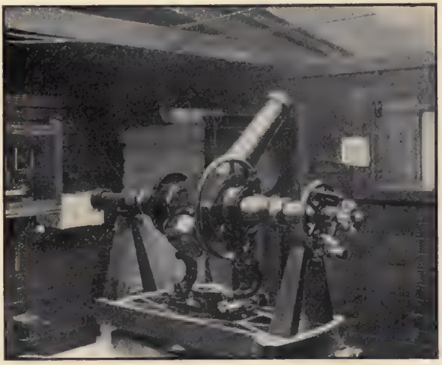


Fig. 39. — Bamberg Transit Instrument in the Students' Observatory of the University of Chicago.

These observations can be used in the converse way. If the sidereal time is unknown and the right ascension of any star is known, its time of transit may be used to set the clock. It is something like a train and an ordinary household clock. When the clock is running, the time of the passing of the train can be predicted; if the clock is not running, it can be set by the passing of the train. In fact, it is from the transits of known stars that every observatory

gets its time; and when the sidereal time is known it is an easy matter to compute the local ordinary time. The running of every railroad train in the country is regulated by time furnished daily from the Naval Observatory at Washington, or from one of two or three other stations.

For the work so far either a transit instrument or a meridian circle may be used; but if the declination is required, a meridian circle must be used. Suppose the readings of the graduated circles are made at both the upper and the lower transits of the pole star. The averages, after the proper corrections are applied, give the readings of the circles when the instrument is pointed at the pole. Suppose now that when the star in question crosses the meridian, the telescope is moved so that it passes along between the horizontal wires of the reticle. When the readings of the graduated circles are made and compared with the readings when the telescope was pointed at the pole, the polar distance of the star is at once found. The declination is 90° minus the polar distance.

These observations can also be used in the converse way. Suppose the declination of the star is known and that its distance from the zenith is measured. Then the latitude of the observer can be found. The distance from the zenith to the equator equals the latitude of the observer. Suppose the declination of the star is $+20°$ and that it is found to be 21° south of the zenith. Then the observer knows that his latitude is $20° + 21° = 41°$.

The explanations which have been given contain the principles which always are involved, but it must be understood that there are many variations from the simple methods described. Many artifices have been introduced for the purpose of obtaining the highest degree of accuracy. These things belong to practical astronomy and can be thoroughly appreciated only by an extended use of the instruments. It was by these most refined methods that Chandler and Küst-

ner discovered, about 1884–1889, that the latitude of a place varies about $0.6''$, corresponding to a distance of only about 60 feet on the earth's surface.

87. Clock, Chronograph, and Photochronograph. — It is evident that in using a meridian circle or transit instrument a clock is a necessary accessory. Astronomical clocks are made to keep sidereal time, and to run with the greatest possible uniformity. It is an unimportant matter if a clock gains or loses, provided it does so regularly, for it takes only a moment to make the required correction; in fact, it is practically impossible to regulate a clock so that it will keep sidereal time exactly.

Uniformity of rate is obtained by eliminating as much as possible the things which introduce irregularities. One of the most important is a change in length of the pendulum with a change in temperature. A very common way of counteracting this effect is to take for the pendulum bob a cylinder filled with mercury. Suppose the temperature decreases; then the rod from which the bob is suspended contracts and tends to make the pendulum shorter. On the other hand, the mercury contracts and settles down, which makes the pendulum longer. When the height and weight of the mercury column are properly related to the length and weight of the pendulum rod, this relation depending on their coefficients of expansion, the effects of slow changes of temperature are eliminated. Rapid changes affect the rod more quickly than the larger cylinder of mercury, and introduce irregularities. The clock is placed in a glass case to insulate it from sudden changes, and in some cases the atmosphere is partially exhausted.

Every precaution is taken to give the pendulum just enough impetus each swing to make up for the motion lost by friction. It is almost needless to say that the clock, as well as every other instrument of precision, is mounted on a separate pier having no connection with other parts of the building.

Observatories are also provided with portable clocks, called *chronometers*, which run on the balance wheel and spring principle, like watches. They are mounted in rings turning on axes at right angles to each other, so that they keep the horizontal position however the box which contains them may be turned. They are now so well constructed that they run with great accuracy.

In former times the observer listened to the clock beats, and, keeping count of the seconds, he estimated the time of transits of the star across the spider lines to tenths of seconds. For about 50 years the times of transits have been registered by means of an instrument called the *chronograph.* It consists of a cylinder a few inches in diameter and about 18 inches long, which is rotated once a minute by clockwork. It carries a sheet of paper, and a fountain pen is fixed so that it traces on it a spiral line as the cylinder turns. The pen is attached to the armature of a small electro-magnet, so that when the current is broken it is pulled quickly to one side by a coiled spring. Electrical connection through the clock breaks the circuit once every second, except at the end of each minute, when two breaks are made, or in some instruments no break at all. The electro-magnet is also in electrical connection with a key in the hands of the observer at the meridian circle. Every time the star crosses a spider line he breaks the circuit by pressing the key, and makes an extra little jog in the line traced by the pen. When the chronograph cylinder is started, the hour and minute are written on the sheet after it has passed a minute mark. Then the time of every subsequent jog, in particular those

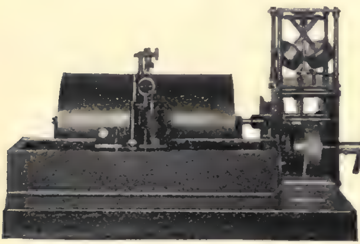


FIG. 40. — Chronograph. *By William Gaertner & Co.*

made by the observer, can be found by counting forward and measuring the fraction of the interval between seconds.

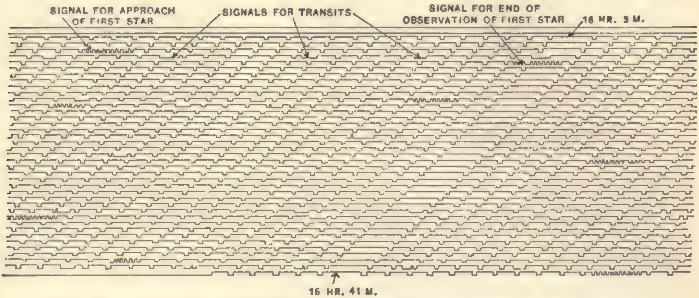


Fig. 41. — Portion of Chronographic Record made at the Students' Observatory of the University of Chicago.

Hough has invented an instrument which records on a tape the times when the star is observed to transit the spider lines in the reticle.

Every observer has a personal equation; that is, he is habitually a little too quick or too slow in pressing the break-circuit key. Several instruments have been designed to make the recording of the transit of a star purely automatic. Such is the *photochronograph*, which consists of an apparatus for holding a photographic plate at the focus of the objective and at intervals of a second making some kind of a break in the continuous trail which the star would otherwise leave. One device is to give the plate a little north and south motion every second by means of electrical connection with the clock, and another is to cover it every second with a screen by the same means. The spider lines are illuminated and photographed on the same plate. When the plate is developed, the images of the lines and the broken star trails appear on it, and from them the time of transit can easily be found.

88. The Altazimuth Instrument. — The meridian circle is not adapted to general observations, for objects can not be

seen through it when they are not in the plane of the meridian. Suppose, however, that the supports of a meridian circle were mounted so that they could be turned around a vertical axis. Then the instrument could be made to turn in the plane of any vertical circle. If the vertical axis carries a graduated circle so that the distance the instrument has been turned from the plane of the meridian can be measured, it is called an *altazimuth* mounting. When the telescope is pointed at any object, the altitude and azimuth are found directly by reading the circles.

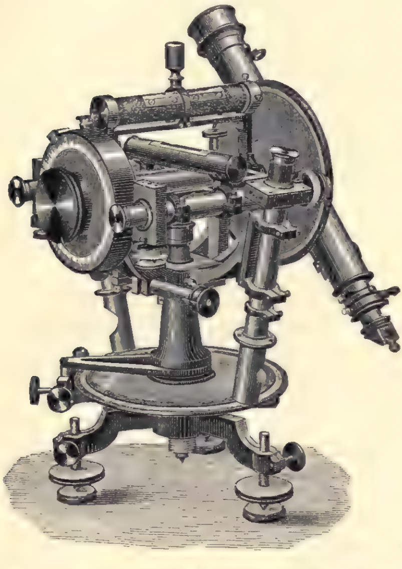


Fig. 42. — Bamberg Altazimuth Instrument.

The thing to be noticed is that the essentials of the mounting are a vertical axis carrying a graduated circle, and another axis at right angles to the first, also carrying a graduated circle. These will be called the *primary* and *secondary* axes of the instrument.

89. Following the Stars with an Altazimuth Instrument at the Earth's Pole. — If an observer were at the earth's pole, he would see the stars move in parallels of altitude in their diurnal motions. If he wished to keep his telescope pointed at the same object, he would keep its altitude fixed, and have it turned by clockwork around its primary axis at the rate at which the stars move. This would be very important if he were taking a long-exposure photograph, or if he wished to study the details of such an object as the moon or a

nebula, or if he desired to make any sort of micrometer measurements.

90. The Equatorial Mounting. — Suppose a platform could be built out from the pole perpendicular to the earth's axis to the point A, Fig. 43. Suppose the observer should take his altazimuth instrument from P out to A, keeping its primary axis parallel to its initial direction. Since the distance $\overline{PA}$ is entirely negligible in comparison with the distance to any celestial object, the motion of A around P would have no effect whatever on the apparent motions of the heavens. Consequently, if the telescope were rotated around its primary axis in the right direction with the angular velocity with which the earth turns, it would continually point at the same object just as it would if it were at P.

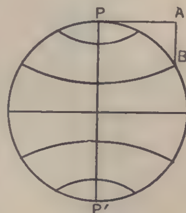


FIG. 43.

Suppose, now, the observer should move his instrument from A to B and rotate it around the primary axis, which is still parallel to its original direction. It would follow the stars just as before. But when it is at B it has quite a different relation to the horizon. The primary axis is parallel to the earth's axis, as before, and the secondary axis is perpendicular to it, but the primary axis is no longer perpendicular to the plane of the horizon. It is now called the *equatorial mounting*. The primary axis is called the *polar axis*, and the other the *declination axis*. The pointing of the instrument is changed in right ascension by rotating it around the polar axis, and in declination by rotating it around the declination axis.

91. Finding an Object with an Equatorial Telescope. — Suppose the right ascension and declination of an object, perhaps invisible to the unaided eye, are given, and that one wishes to view it through an equatorially mounted telescope. The first thing is to set the telescope to the proper declination by

means of the declination circle. There is a clamp provided for the purpose of holding the telescope in this position. The next thing is to find the distance of the body east or west of the meridian, that is, its hour angle.

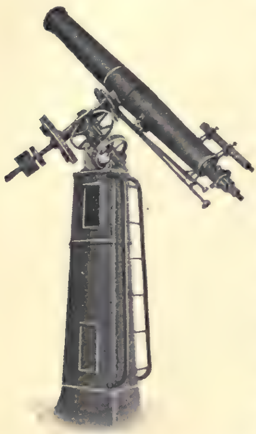


FIG. 44. — Equatorial Telescope by Lohmann Brothers.

As was explained in connection with the treatment of reference points and lines, the hour angle of a body depends upon its right ascension, the time of day, and the day of the year of the observation. An approximate computation of the right ascension of the meridian is made in the two steps previously given, but if one has a sidereal clock it is at once known, for it is the same as the sidereal time. The difference between the right ascension of the meridian and that of the object is its hour angle, which is east or west according as the latter is greater or less than the former.

After the hour angle has been found the telescope is set by the circle attached to the polar axis. When it is clamped and the driving clock set going, the object will be constantly in view. As a matter of fact, the telescope rarely is set accurately at first, the final adjustment being made by the aid of a little telescope with a large field, called a *finder*, attached to the large one.

CHAPTER V

THE EARTH

92. Problems respecting the Earth. — The earth is one of the bodies belonging to the field of astronomical investigations. It is here that astronomy has its closest contact with some of the other sciences. Those problems which can be solved for the other planets also, or which are essential for the investigation of other astronomical questions, are properly considered as belonging especially to the astronomer.

The astronomical problems respecting the earth are divided, though not very sharply, into two classes. The first class consists of those which can be solved, at least by some methods, without regarding the earth as a member of a family of planets; the second class consists of those which involve essentially the relations of the earth to other bodies. The work will be divided on this basis, this chapter being devoted to problems of the first class, and the next to problems of the second class. Then will follow some of the consequences of the results of these two chapters. The methods of solution will constantly overlap; nevertheless, the scheme of division will serve to direct the attention to some real differences, while its partial failures will show the mutual interdependence of different theories.

The problems of the first class are such as the shape of the earth, its size, its density, its general constitution, its atmosphere, and its atmospheric phenomena. The problems of the second class are such as the motions of the earth and planets, the inferences which are drawn from the motions, and the interactions of the planets.

THE SHAPE OF THE EARTH

93. Historical. — It is generally supposed that every one thought the earth was flat until the time of Columbus, but a number of the ancient Greek philosophers believed in its sphericity and gave good reasons for their opinions. It is difficult for us to find out just how completely they convinced themselves of the correctness of their scientific doctrines, for as a people they were much given to theorizing and arguing, oftentimes without appealing to obvious experiments. In this respect they failed to follow what is considered now to be the correct scientific method.

The most ancient philosopher who is known certainly to have maintained that the earth is round is Pythagoras (about 569–490 B.C.). Among the others who advocated its sphericity, the best-known are Eudoxus (407–356 B.C.), Aristotle, (384–322 B.C.), Aristarchus of Samos (310–250 B.C.), and Eratosthenes (275–194 B.C.). Eratosthenes seems to have had the clearest convictions of any of them, for he laid the foundations of mathematical geography and attempted to measure the size of the earth. Notwithstanding these many steps toward the truth, correct ideas perished because of a lack of the scientific spirit, or were crushed out by the mysticism which permeated the thought of antiquity.

It is an undoubted fact that there was no general acceptance of the idea of the globular form of the earth until after Columbus was supposed to have sailed to India, going westward from Spain. The epoch of exploration and discovery which followed his voyages made the theory well known and caused it to be accepted throughout Europe.

94. The Simplest Proof of the Earth's Sphericity. — There are many reasons given for believing that the earth is not a plane, and that it is, indeed, some sort of a convex figure; but there are very few which actually prove that it is really spherical, or of any other particular form. For example, it

has been sailed around, but it might be the shape of a cucumber for all that. Vessels disappear below the horizon, hulls first and masts last, but this only shows its general convexity. The shadow of the earth on the moon at the time of an eclipse seems to be circular, but the observation is very inconclusive because the shadow has no sharp edge (see Art. 222) and its radius is very large compared to that of the moon.

The simplest and most conclusive proof of the globular form of the earth is that *the plane of the horizon* (or the direction of the plumb-line) *changes by an angle which is directly proportional to the distance traveled along the surface, whatever may be the starting point and the direction of travel.* This statement needs some illustration. Suppose the stars were indefinitely remote and that the earth were a plane; then, if one should go along its surface, they always would appear to be in the same direction. If the stars were nearer, like the moon, their directions would apparently change, but the amount would not be proportional to the distance traveled. For example, if a star were directly overhead, a certain distance traveled along the surface of the earth would change its apparent direction by one degree, but ninety times that distance would not make it appear at the horizon.

Suppose the stars were very remote and that the earth were of any convex form other than a sphere. If one should travel along its surface, the apparent directions of the stars would change, but not proportionally to the distance traveled. In fact, the circle is the only oval which is closed and in which the arc traveled is proportional to the change of direction of the perpendicular to it. If the arc described is proportional to the change of direction of the perpendicular to it for every starting point and for every direction of travel, the surface is spherical.

Eratosthenes noticed that the altitude of the pole star was

less when he was in Egypt than it was when he was in Greece. He gave the correct interpretation of the difference, but he did not verify that it was proportional to the distance. Consequently, he had no strict proof that the earth is actually round.

95. The Oblateness of the Earth. — The determination of the earth's form by measuring arcs was not applied at once. It seems to have been somewhat axiomatic to the ancient mind that surfaces are either flat or spherical. When it was found that the surface of the earth is not a plane, it seems to have been supposed that it is essentially spherical.

In 1687 Newton published his great work, the *Principia*, and in it he showed from theoretical considerations, to be explained in the next article, that the earth is bulged out in the equatorial regions. Such a body is said to be *oblate*. Newton's results were not immediately and generally accepted. For example, the Cassinis, who were the leading French astronomers, maintained until about 1745 that the earth is bulged at the poles and drawn in at the equator. Such a body is said to be *prolate*. Newton thought that it is something the shape of an orange, and the Cassinis that it is more like that of a lemon.

In 1671 Picard made a measurement of an arc in France, which, by the way, enabled Newton to verify, in 1682, his theory of gravitation. In the next century the French took hold of the question in earnest. They extended the arc measured by Picard from the Pyrenees to Dunkirk, an angular distance of 9°. The results were published in 1720. They sent an expedition to Peru in 1735 under Bouguer, Condamine, and Godin. By 1745 these men had measured an arc of 3°. In the meantime an expedition to Lapland under the leadership of Maupertuis had measured an arc of 1° near the arctic circle. On comparing these measurements, it was found that the length of a degree measured north and south is greater the farther it is from the equator. This

proves that there is an equatorial bulge, and all later measurements agree with this conclusion.

To see clearly that equal arcs, expressed in angular measure, are longer on an oblate body near the pole than near the equator, consider Fig. 45.

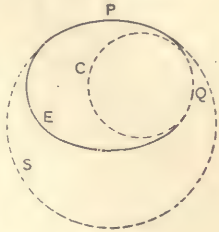


FIG. 45.

The curve E represents a meridian section of the earth. (Of course the flattening is greatly exaggerated in the figure.) The circle C coincides with E near the equator Q, and the larger circle S coincides with it near the pole. A degree of arc on E near P is of about the same length as one on S, while one on E near Q is of about the same length as one on C. Since the circle S is larger than the circle C, a degree on E near P is longer than one near Q.

96. Newton's Proof of the Earth's Oblateness. — Newton's proof of the oblateness of the earth, which anticipated by nearly sixty years the direct verification from observations, is an example of results of one character being obtained from those of quite another character. This proof depends upon the rotation of the earth, and upon the law of gravitation which was derived from the motions of the planets. Hence it is essentially involved with the work of the next chapter.

Let ABC (Fig. 46) be a meridian section of the earth, and $\overline{AC}$ its axis of rotation. Imagine that canals are constructed from the pole A to the center O, and from the equator B to O. Suppose they are filled with water and that the shape of the earth is such that the two columns balance each other. The weight of the water is caused by the attraction of the earth, and it diminishes from the surface to the center, where it is zero. If the earth were stationary, columns of equal length would balance each other.

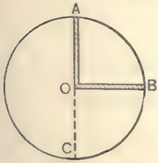


FIG. 46.

The earth is rotating around the axis $\overline{AC}$. Hence the gravity of every unit volume of the column $\overline{BO}$ toward the center is diminished by the centrifugal acceleration due to the rotation of the earth, while that in $\overline{AO}$ is unchanged. Consequently, the column $\overline{BO}$ must be longer than the column $\overline{AO}$ in order that equilibrium may result. The difference in length of the two columns depends upon the rate of rotation of the earth. From a skillful treatment of the question, Newton reached the approximate result that the difference of $\overline{BO}$ and $\overline{AO}$ divided by $\overline{BO}$ equals $1 \div 230$.

97. Pendulum Proof of the Earth's Oblateness. — The time of vibration of a pendulum depends upon the forces acting upon the bob and the distance from the point of support to the center of oscillation. The formula for the time of a single swing is

$$t = \pi\sqrt{\frac{l}{g}},$$

where t is the time, $\pi = 3.1416 \cdots$, l the length of the pendulum, and g the resultant of the earth's attraction and the centrifugal acceleration due to the earth's rotation. If l is measured and t is observed, then the value of g is given by the equation

$$g = \frac{\pi^2 l}{t^2}.$$

If the earth were a sphere, g would vary only because of the centrifugal acceleration, and in a way which could be easily computed; but if the earth were oblate, the decrease of g would be greater near the equator than it would be if the earth were a sphere, both because the attraction would be less and also because the centrifugal acceleration would be greater. Experiments show that g decreases faster toward the equator than it would if the earth were a sphere, and it follows that the earth is oblate.

The attraction of the earth varies with its shape in a very

complicated manner, and the problem of determining its precise form from pendulum observations alone can not be explained here. The method is even more powerful than one would suppose possible, for from pendulum observations alone both the shape and the size of the earth can be found.

98. A Method of measuring Small Quantities. — The variations in g are determined from pendulum experiments instead of by such direct means as the spring balances. Since the principle involved is very important in astronomy, it will be given in some detail.

The variation in g at different parts of the earth's surface is very small, the extreme being about one-half of 1 per cent. If two bodies differed in weight by even a small part of this amount, the difference could be detected and measured with perfect ease with balance scales. But balance scales, which are far more accurate than any others, can not be used in this problem, for the balancing weights and the object weighed are subject to the same relative change of gravity.

The problem is to measure something which is near the limits of observation, and it is solved by devising a convenient method for obtaining the cumulative effects of the quantity in question. In the present case g varies very slightly, but it affects the interval of time of every oscillation of the pendulum. For example, suppose a pendulum will oscillate at one place in a second and in another in $\frac{1}{100,000}$ less time. The difference in one oscillation could not be detected in any way, but in a day (86,400 seconds) the accumulated difference would be $\frac{86}{100}$ of a second, a quantity which is easily measured; and in 10 days it would be more than 8 seconds.

These statements imply that there is some way of measuring time independent of the pendulum. The gain or loss of the pendulum is determined by observations of the transits of stars with a meridian circle or transit instrument.

99. Other Proofs of the Earth's Oblateness. — There are some other proofs that the earth is oblate which are deeply involved in astronomical doctrines and depend upon difficult mathematical processes, but which can be readily appreciated in a general way. As has been said, the oblateness of the earth affects its attraction for other bodies, and conversely their attraction for it.

Consider the motion of the moon around the earth. Let E (Fig. 47) represent the largest sphere that can be cut out of the oblate earth. Now, a sphere attracts an exterior body as though its mass were all at its center of gravity. The equatorial bulge remains to be considered. This is a difficult problem to treat rigorously, but it can be seen that the sum of the attractions of the nearest and farthest parts, A and B, is greater than it would be if they were at the center of the earth. To illustrate it, suppose $\overline{MA} = 1$, $\overline{ME} = 2$, and $\overline{MB} = 3$. Suppose A and B are unit particles. Then, since the attraction (Art. 160) varies as the product of the masses and inversely as the square of the distance, it follows that:

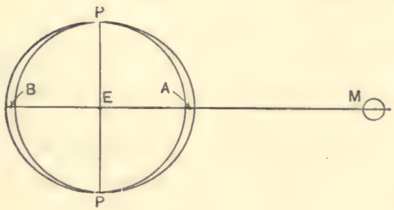


FIG. 47.

$$\text{attraction of } A = \frac{1 \times M}{1^2} = M,$$

$$\text{attraction of } B = \frac{1 \times M}{3^2} = \frac{M}{9},$$

$$\text{sum of attractions of } A \text{ and } B = M + \frac{M}{9} = \frac{10}{9} M,$$

$$\text{attractions of } A \text{ and } B \text{ when at } E = \frac{2 \times M}{2^2} = \frac{M}{2}.$$

It follows that the parts A and B attract the moon more than they would if they were at the center E. Similarly, the appropriate mathematical discussion shows that an oblate body attracts another body in the plane of its equator more than it would if it were spherical.

The moon moves around the earth because of the earth's attraction for it. The fact that the earth is oblate causes certain irregularities in the motion of the moon which have been detected by actual observations. But the oblateness of the earth is so small, and the moon is so far away, that the irregularities produced are very slight. The most striking example of the kind in the solar system is in the motion of Jupiter's smallest satellite, which is near to the very oblate planet Jupiter.

The converse action is more important. The moon's orbit is not in the plane of the earth's equator, and its attraction on the equatorial bulge causes what is called *precession of the equinoxes*, which will be discussed in the next chapter (Art. 147). It is the cause of the wandering of the celestial pole referred to in Art. 47. Theory and observation agree, and the connection was first established by Newton in the *Principia*.

100. Spheroids. — Before discussing the form of the earth, it is necessary to define some geometrical figures.

An ellipse is a plane, closed, oval surface, such that the sum of the distances from any point on its circumference to two fixed points within is constant.

Thus, in Fig. 48, $ABA'B'$ is an ellipse, and $\overline{PF}+\overline{PF'}$ is the same wherever on the curve P may be. The points F and F' are called the *foci*, and C the *center*. The line $\overline{AA'}$ is the *major axis* of the ellipse, and $\overline{BB'}$ is its *minor axis*.

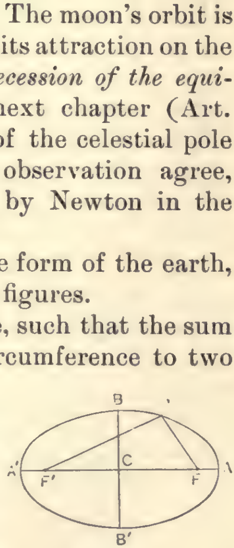


FIG. 48.

The *ellipticity* is $\overline{AA'}-\overline{BB'}$ divided by $\overline{AA'}$. The *eccentricity* is $\overline{FC}\div\overline{AC}$. Suppose P is at B. Then it follows

from the definition of the ellipse that $BF = AC$. From the triangle FCB it follows that

$$\overline{FC} = \sqrt{\overline{BF}^2 - \overline{BC}^2} = \sqrt{\overline{AC}^2 - \overline{BC}^2}.$$

Let $AC = a$, $BC = b$, then the ellipticity is $\frac{a-b}{a}$, and the eccentricity is $\frac{\sqrt{a^2 - b^2}}{a}$.

If an ellipse is rotated around its minor axis, it generates a solid called an *oblate spheroid*. If it is rotated around its major axis, it generates a solid called a *prolate spheroid*. If the figure is like an oblate spheroid, except that the equator is an ellipse instead of a circle, the solid is called an *ellipsoid*.

101. The Actual Shape of the Earth. — Four proofs of the oblateness of the earth have been given. The first, which involves the measurement of arcs, is independent of any theory respecting its motions, mass, or relations to other bodies. The second, which involves the rotation of the earth and the mutual attractions of its parts, is independent of its relations to other bodies, but depends upon the laws of motion. The third depends upon the same things as the second. The fourth, which involves the mutual interactions of the earth and moon, depends, therefore, upon the law of gravitation.

The first proof gives the shape with the accuracy of the observations; the second is not complete without some assumption respecting the variation of density in the earth's interior; the third is rigorous, though mathematically difficult; the fourth simply shows that the earth is oblate and the amount of mass in its bulge. Some theoretical results which are related to Newton's method of proof will be given, and then the results which have been derived from all available data.

In his proof Newton *assumed* that the figure of the earth is an oblate spheroid, and he calculated that the ellipticity of a

meridian section is $\frac{1}{230}$. It was first proved in 1740, by Maclaurin, that, if a body is a rotating homogenous fluid subject to no forces but the mutual gravitation of its parts, an oblate spheroid is one of its figures of equilibrium. When the rotation is not too rapid, there are indeed two oblate spheroids which satisfy the conditions of equilibrium. One is nearly spherical like the earth, and the other is much more flattened at the poles. Thus, A and B (Fig. 49) represent the forms for a certain density and rate of rotation. When the rotation is faster, A is less, and B is more, nearly spherical. For a certain rate of rotation they coincide, and for a greater rate they do not exist. Jacobi showed later that an ellipsoid is also a figure of equilibrium for certain rates of rotation; and, more recently, Poincaré has proved that the number of figures is unlimited, though they are of very special forms. The form nearest a sphere seems to have the greatest stability, and is that which one would expect to find in nature. If the body is made up of a number of separate fluids of different densities, its form can not be exactly an oblate spheroid. The demonstration of this theorem was given by Hamy in 1887.

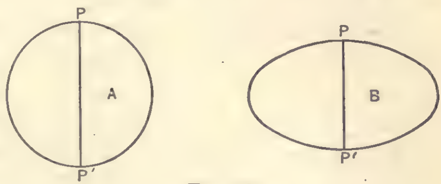


FIG. 49.

It follows from the preceding discussion that we should expect to find that the earth is nearly, though not exactly, an oblate spheroid, and observations of all sorts support this conclusion. Its deviations from an oblate spheroid are so small compared to its size that the observations have not shown with certainty just what they are, except, of course, the continental elevations. At present the slight errors in the observations mask these small differences, though some of the measurements seem to indicate that the equator is not

a circle, but an ellipse with one diameter about half a mile greater than the other. The ordinary method is to treat the earth as an oblate spheroid, and to count all irregularities as deviations from this standard. The spheroid which best satisfies the various observations has, according to Harkness, an ellipticity of $\frac{1}{300}$. In 1866 Colonel Clarke of the English Ordnance Survey found an ellipticity of $\frac{1}{295}$. This is the value generally used, notwithstanding the fact that he found $\frac{1}{293.5}$ in 1878, and Harkness $\frac{1}{300}$ in 1891.

102. Triangulation and Latitude Determinations. — Two classes of observations are necessary to determine the shape of the earth by measurements of arcs. The first class consists of those necessary to find the distance between the two end stations; the second consists of the determination of the latitudes and longitudes of the end stations.

The accurate measurement of a long distance on the surface of the earth is by no means an easy matter. The errors in the measurements down to the time of Picard were very serious, and led to the erroneous conclusion that the earth is a prolate spheroid. The method of obtaining the distances between points which are far apart is a little indirect, but is so characteristic of astronomical processes, and at the same time the results are so far-reaching, that it is worthyof some consideration.

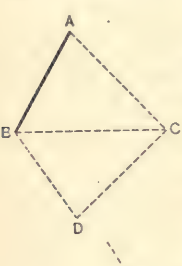


FIG. 50.

Suppose it is desired to find very accurately the distance from A to P (Fig. 50), which is perhaps several hundreds of miles. First, some short distance $\overline{AB}$, say 10 miles, is measured with the very greatest care. The measurement is made along the horizontal whether the surface is smooth and level or not. This distance must be found with an error not exceeding an inch or two. Then a station C is selected which is visible from both A and B, which also must be visible from

each other. The angles BAC and CBA are measured with high-grade surveyors' instruments; and, the two angles and included side of the triangle being known, the sides $\overline{BC}$ and $\overline{AC}$ are computed by plane trigonometry. The work is extended to D in the same way, and continued on to the point P, after which the distance $\overline{AP}$ can be computed.

The importance of measuring $\overline{AB}$ accurately can be inferred from the fact that it finally leads us to the size of the earth. This, in turn, enters directly in the measurement of the distances to the moon, sun, and planets; and the distance from the earth to the sun is the base line which is used for measuring the distances to the stars.

Suppose for simplicity that the stations A and P are on the same meridian; then their angular distance apart is the difference of their latitudes. The latitudes are determined by measuring the altitude of the pole, or something equivalent. As has been explained (Art. 26), the zenith distances instead of the altitudes are always measured except on ships at sea. The zenith is defined by the direction of the plumb line, or the perpendicular to the surface of a basin of mercury, and depends both upon the attraction of the earth and upon the centrifugal acceleration due to its rotation.

If the earth's crust at one of the stations were irregular either in form or density, the direction of the plumb line might be effected so that very discordant results would be found when applied to the whole earth. For example, suppose three short arcs on the same meridian were measured, and that it was found that the middle one was longer in proportion to its angular dimensions than the others; the conclusion would be that, at one of the stations at least, there were local irregularities in the direction of gravity. A more extreme case might arise. The direction of gravity might be changed so much by unusual local conditions that when two stations were near together the one which was farther north actually might have the lesser latitude according to astro-

nomical observations. These local deviations, which are usually $1''$ to $2''$, but which may be sometimes as great as $30''$, are rendered relatively harmless by measuring very long arcs, for the error becomes relatively less the larger the arc measured.

103. The Size of the Earth.—The measurements of arcs give the size of the earth as well as its shape. The earliest-known attempt to measure an arc was made by Eratosthenes, who found 250,000 stadia for the circumference. The length of the stadium which he used is not exactly known, but according to M. Paul Tannery it is probable that his results were not in error by more than 1 per cent. If he used the common Olympic stadium, the error was about 20 per cent. The only other attempt in antiquity to measure the size of the earth was made by Posidonius (about 135–51 B.C.), who found 180,000 stadia for the circumference. No improvements on these results were obtained until Willebrord Snell made a series of measurements in Holland in 1617, which gave the value of a degree as 67 miles.[1] Then followed Norwood, in 1636, in England, and Picard, in 1671, in France, and the measurements of the French in Peru and Lapland previously mentioned (Art. 95).

During the past century the principal governments have carried out extensive surveys of their possessions, especially the coast lines, in various parts of the earth. The longest arcs so far measured are the one stretching from Hammerfest, in Norway, to the mouth of the Danube, and the nearly equal one extending from the Himalayas to the southern end of the Indian peninsula. According to Colonel Clarke's computation of 1866, the equatorial and polar radii are respectively 3963.307 miles and 3949.871 miles; his computation of 1878 gave 3963.296 miles and 3949.790 miles. The differences in these numbers are a fair measure of the possible

[1] This is not an improvement on the results found by Eratosthenes if his work was as accurate as Tannery believes it was.

inaccuracy of the results. The measurements show that the equatorial diameter is nearly 27 miles greater than the polar diameter, and that the total area of the surface is nearly 200,000,000 square miles.

From Clarke's spheroid of 1866, which is generally adopted as a basis for computations, it follows from methods of calculation which can not be explained here that:

At latitude 0° one degree in latitude equals 68.704 miles.
At latitude 30° one degree in latitude equals 68.881 miles.
At latitude 60° one degree in latitude equals 69.230 miles.
At latitude 90° one degree in latitude equals 69.407 miles.

The parallels of latitude are small circles, and the length of a degree in longitude decreases from the equator to the pole. The appropriate computation shows that:

At latitude 0° one degree in longitude equals 69.652 miles.
At latitude 30° one degree in longitude equals 59.955 miles.
At latitude 40° one degree in longitude equals 53.431 miles.
At latitude 60° one degree in longitude equals 34.914 miles.
At latitude 90° one degree in longitude equals 0.000 miles.

104. Different Kinds of Latitude. Since the earth is not a sphere, a perpendicular to its surface (*i.e.* water-level surface) at any point, except on the equator or at the poles, will not pass through its center. This leads to three different kinds of latitude:

(*a*) The *astronomical latitude* is that determined by observing the altitude of the pole, or something equivalent.

(*b*) The *geographical latitude* is the angle between the plane of the equator and the perpendicular to the standard spheroid at the place of observation.

(*c*) The *geocentric latitude* is the angle between the plane of the equator and the line from the center of the earth to the place of observation.

Thus, in Fig. 51, in which the differences are much exaggerated, the full line represents the actual earth, the broken line the standard spheroid, and l_1, l_2, and l_3 the three kinds of latitude of the place of observation A.

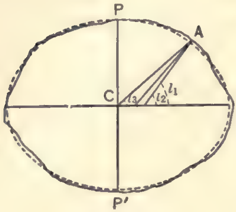


FIG. 51.

The latitudes l_1 and l_2 usually differ but slightly from each other, while l_3 may differ from them by 10′, or a little more.

QUESTIONS

1. With what other sciences is the astronomical work of this chapter most closely related?

2. What are the foundations of mathematical geography which Eratosthenes must have laid?

3. Enumerate all possible reasons for believing the earth is globular. Which of them may be used to find its true shape?

4. The diameter of the earth's shadow at the distance of the moon is nearly three times that of the moon. Draw circles showing how the earth's shadow on the moon shows the shape of the earth.

5. Suppose one always measured arcs of *latitude* and found that they were proportional to their angular values; would he be justified in inferring that the earth is spherical?

6. Are oblate and prolate bodies essentially different in shape, or simply differently placed?

7. Does every body attract an exterior particle as though its mass were all at its center of gravity? As a special example, suppose the body consists of two unit masses at unit distance apart and consider their attraction on a particle anywhere in their line.

8. The acceleration g in our latitude is about 32.16 feet per second; how long would a pendulum have to be to make a swing in one second?

9. Suppose g changes by .005 its value; what is the change in time of one swing?

10. Suppose a change of one second in 10 days can be detected; how slight a variation in g can be found in 10 days?

11. How will an increase of temperature affect the time of swing of the pendulum unless it is in some way compensated?

12. What is the character of the effect of an increase in the atmospheric pressure on the time of the swing of a pendulum?

13. If the observations are not made at the sea level, is it necessary to apply any corrections for altitude?

14. Suppose one should buy goods in San Francisco by weight and sell them in the Klondike region by weight; would he gain or lose if he both bought and sold by balance scales? By spring scales? By spring scales if he received his pay in gold also weighed on spring scales?

15. If the ellipticity of a meridian section of the earth is $\frac{1}{300}$, what is its eccentricity? Suppose one makes a drawing to scale such that the major axis of the ellipse is 8 inches; what is its minor axis? What is the height of the highest mountain on the same scale?

16. What is the equatorial circumference of the earth?

THE DENSITY OF THE EARTH

105. The Density of the Earth. — Since the volume of the earth is known, its mass can be found when its density has been determined; or, conversely, when the mass is known, the density can be found. This is a fairly difficult problem to solve, for, since no direct observations of the earth's interior can be made, it involves the relation between mass and the acceleration of gravitation. It is more difficult than finding the density of nearly any other planet after the density of the earth has been found.

The density of the rock which constitutes that part of the earth's crust which can be examined generally lies between two and three times that of water at its greatest density (water is densest when its temperature is about 39° Fahrenheit). The material in the earth's interior is subject to an inconceivably great pressure, and one naturally would expect that its density would be greater than that of the surface layers. The method of finding the average density of the whole earth consists in first finding its mass by comparing its attraction with that of a body of known volume and density. It is clear that the results depend upon the law of gravitation, which is derived from the motions of the planets. Here

again the work of this chapter is intimately connected with that of the next. The difficult part of the problem from a practical point of view has its source in the fact that the gravitational force is very feeble except when large masses are involved. The observations and measurements which have so far been made indicate that the average density of the earth is about 5.5, or perhaps a little more, on the water standard. The two chief methods of making the measurement will be described briefly.

106. Determination of the Density of the Earth by the Torsion Balance. — Suppose a rod with two small balls at its ends is suspended in a horizontal position by a fine wire, or quartz fiber, as in recent experiments, attached at its middle point. The wire *a*, or fiber, exerts a certain resistance to twisting, or *torsion*, which is measured by displacing the balls, *b* and *b*, Fig. 52, a little from their position of equilibrium and observing the period of their oscillation. The greater the resistance to torsion the shorter the period, and from the period it is possible to calculate the resistance very accurately.

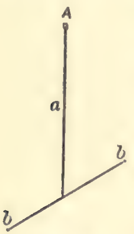


FIG. 52.

The whole apparatus is put in an air-tight chamber and the oscillations are viewed from the outside through a telescope. When the coefficient of torsion has been found, the apparatus is allowed to come to rest. Then some heavy balls are raised so as to be near the balls *b* and *b*, but on opposite sides of the connecting rod. Their attraction for the small balls will cause a slight displacement in the positions of the latter, though the forces are so feeble that it requires several hours for the full effects, which are always very small, to be realized. From the amount of the turning it is possible to compute the attraction which has been exerted. The attraction of the earth is measured by the distance a body will fall in a unit of time. Comparing the two forces

and making allowance for the differences in the distance of the balls b and b from the centers of the large balls and from the center of the earth, the masses of the large balls compared to that of the earth can be found. Then the density of the earth is given by the relation that the mass equals the volume times the density.

The most successful recent experimenter is Boys, who found, in 1894, a mean density of 5.527. The earliest experiment of this kind was by Lord Cavendish in 1798, who reached the conclusion that the earth's average density is 5.5.

107. Determination of the Density of the Earth by the Mountain Method. — The difficulty of the torsion-balance method has its source in the fact that the forces to be measured are exceedingly feeble. There are practical difficulties which prevent the use of large balls having a diameter of more than a few inches. The mountain method consists in getting a greater force by using the attraction of a whole mountain, but other difficulties enter which make this method no better than the other one.

Suppose a single mountain rises abruptly from a fairly level plain, as in Fig. 53. (For purposes of illustration the figure is, of course, greatly out of true proportions.) Consider two stations on opposite sides of the mountain, as A_1 and A_2. If it were not for the mountain, a plumb line at A_1 would hang in the line $\overline{A_1a_1}$, but the attraction of the mountain deflects it to $\overline{A_1b_1}$. A similar deviation takes place at A_2. The problem is to find these deviations.

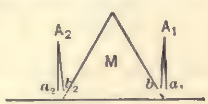


FIG. 53.

Suppose, for simplicity, in the present discussion that A_1 and A_2 are on the same meridian. Then the angle between the lines $\overline{A_1a_1}$ and $\overline{A_2a_2}$ is the actual difference in latitude of the two stations, which can be found, since the size of the earth is known, from the measurement of the distance from A_1 to A_2 by triangulation around the mountain. Now sup-

pose the difference in latitude of the two stations is found by astronomical observations of the elevation of the pole. These observations depend upon the actual directions of the plumb lines $\overline{A_1b_1}$ and $\overline{A_2b_2}$, and the angle found will be the one between these two lines. The difference in the astronomical latitudes minus the difference in the geographical latitudes is the sum of the angles of deviation, $a_1A_1b_1 + a_2A_2b_2$.

From the relations of the two stations to the mountain it can be found how the sum of the deviations is distributed between the two parts. Therefore, suppose the angle $a_1A_1b_1$ is known. The plumb line hangs in the direction $\overline{A_1b_1}$ as a result of the earth's attraction in the direction $\overline{A_1a_1}$ and of the mountain's attraction in the direction $\overline{a_1b_1}$. The forces are to each other, according to the *law of parallelogram of forces*, as the line $\overline{A_1a_1}$ is to $\overline{a_1b_1}$. The attraction of the whole earth depends upon its mass (density times volume) and the distance of A_1 from its center. The attraction of the mountain depends upon its mass and shape, and the distance of A_1 from it. Its volume can be found from measurements, and its density from surface and tunnel samples. Therefore, in the equation which expresses that the attractions are to each other as $\overline{A_1a_1}$ is to $\overline{a_1b_1}$, everything is known except the density of the earth, which this equation furnishes when the deviation has been found from the observations.[1]

There are several weak points in the mountain method. One is that mountains are so irregular in contour and density that it is not easy to compute their masses. Another is that the observations must be made in a rough country

[1] The formula for the deviation of a plumb line suspended at the base of a hemispherical mountain is approximately

$$D'' = 100000 \frac{d_1}{d_2} \frac{r}{R},$$

where D'' is the deviation in seconds of arc, d_1 is the density of the mountain, r its radius, d_2 the density of the earth, and R the radius of the earth. See *Introduction to Celestial Mechanics*, p. 109.

where the earth's crust has been folded, and where rapid variations in density to great depths may considerably change the direction of the plumb line, independently of the direct action of the mountain.

The mountain method of finding the density of the earth was first tried in 1774 by the English Astronomer Royal, Nevil Maskelyne, who selected for his purpose Schehallien, in Scotland, a narrow ridge running east and west. He found a deviation of 12″. The amount of deviation was discussed in connection with the data of the ridge by Charles Hutton, who came to the conclusion that the earth's density is about 4.5. Newton had estimated a century earlier that it is between 5 and 6.

108. The Condition of the Interior of the Earth. — Almost all that is known concerning the condition of the interior of the earth has been found by indirect processes involving the motions of the earth and related phenomena.

There are many reasons for believing that the interior of the earth is very hot, and geologists generally have supposed until recently that it is in a molten condition except for a relatively thin crust. Astronomers and physicists now believe that it is solid throughout, except possibly for small pockets where the pressure is below the normal, and most geologists are accepting this view and are finding that it does not contradict geological phenomena. The temperature is undoubtedly above the melting point of all ordinary material at pressures such as prevail at the earth's surface. But the melting point increases with the pressure, and it is now believed that the pressure increases so fast in the interior compared to the increase of temperature that the point of fusion is never reached. Some of the principal reasons for believing that the earth is solid will be enumerated, without entering into the details of the discussions, which can be given more conveniently in other connections.

(*a*) It has been mentioned (Art. 99) that the attraction

of the moon on the earth's equatorial bulge changes the plane of its equator. This change takes place precisely as though the earth were a solid body ; if it had only a thin shell surrounding a fluid interior, the shell would be made to slide on the fluid part.

(*b*) The moon raises tides on the fluid surface of the earth. If the crust were thin (*i.e.* a hundred miles or so thick), it would yield to the tidal forces, and the water tides with respect to the land surface would be much less than are observed.

(*c*) The axis around which the earth rotates is not quite fixed, but varies with a known period. The period is that which theory requires on the hypothesis that the earth is solid and on the average a little more rigid than steel.

(*d*) Earthquakes produce waves in the earth which go out from the place of disturbance in every direction like sound waves in air. Scientists now have instruments which enable them to measure extremely minute waves, sometimes after they have traveled halfway around, or straight through, the earth. The velocity of a wave depends upon the density and rigidity of the medium through which it travels. From the time it takes a wave to pass through the earth it is found that the interior of the earth is very rigid.

It must not occasion any surprise that the shape of the earth is very nearly that which is needed in order that it may be a figure of equilibrium. For, if it differed very greatly from this form, the strains would be so great that it could not withstand them even with its high rigidity. Our opinions are formed on the basis of experience with small objects, but the leverage for strain increases so much faster than the resisting power in large bodies that matters are quite altered. For example, a glass marble will lie on a rigid plane with very little distortion of figure, but if it were 20 miles in diameter it would flow out at the bottom like

pitch. Or, simpler still, small masses of ice are quite rigid, while glaciers flow much like streams.

THE EARTH'S ATMOSPHERE

109. Nature of the Atmosphere. — The atmosphere is the gaseous envelope which surrounds the earth. Its main constituents are about 79 per cent of nitrogen, about 21 per cent of oxygen, and about .0003 per cent of carbon dioxide. Quite a number of other elements, as argon, neon, and helium, have been found in very minute quantities. Then there is water vapor, which varies greatly in amount at different times. Such things as dust particles and smoke are regarded as impurities rather than true constituents of the atmosphere.

110. Kinetic Theory of Gases. — It will be essential for much which follows to obtain a clear idea of what a gas is.

All of the thousands of substances of which the earth is composed are made up of about 75 different elements, or unit materials. Each of these elements is composed of exceedingly minute particles called *atoms*, which are so nearly alike that they can not be distinguished from each other. The atoms have the property of uniting with each other in various ways, and most substances are the combinations of two or more elements. Thus, water is formed by the union of two atoms of hydrogen and one of oxygen. Water is not hydrogen or oxygen, but a new substance, most of whose properties are entirely different from that of either of the elements out of which it is composed. Hydrocyanic gas is one of the deadliest poisons, yet it is composed of hydrogen, carbon, and nitrogen, which enter very largely into all our food.

A substance is made by the union of atoms in a very intimate way. The atoms form little bodies called *molecules*, which are the smallest amounts of a substance that can exist, for if they are broken up, it becomes something else. It is not

known just how small molecules are, and different kinds probably differ much in size; but the question of their size has been raised by Lord Kelvin and others, and such data as have been obtained indicate that it would take something like five hundred millions of them laid side by side to make a row an inch long.

When a substance is the element itself, two or more atoms (with a few exceptions) are united together to form a molecule of the element. Thus the molecules of oxygen gas are composed of two oxygen atoms. A gas or mixture of gases, and in particular the atmosphere, is made of an inconceivably large number of molecules which are very close together, yet far apart compared with their dimensions. They are free to move around among each other except for collisions, and they do actually move with great velocities, depending upon the weight of the molecule, the temperature, and the pressure. Thus, hydrogen molecules under atmospheric pressure and at the freezing point move, *on the average*, with a velocity of more than a mile per second, and a large percentage move with greater velocities. The higher the pressure and temperature the greater the velocities, but the heavier the molecules the smaller the velocities.

A gas exerts pressure on everything with which it comes in contact. For example, if the air were exhausted from a vessel, the atmospheric pressure on the outside would be so great that it would be crushed unless it were made of very strong material. The pressure of the atmosphere at sea level is about 15 pounds to the square inch, though it varies about half a pound with the condition of the weather. The pressure is the result of the impact of millions of molecules per second on every square inch of surface. This velocity and impact theory of gases is known as the *kinetic theory of gases.*

111. The Escape of Atmospheres. — The height to which a body will rise from the earth depends upon the velocity with

which it starts. If a body were started with sufficiently great velocity, it would pass away to the distance of the moon, or sun, or even to the stars. For an attracting body of any mass and size there is a velocity of permanent escape from it, which is given by the formula [1]

$$V = \sqrt{2gR},$$

where V is the velocity, g the acceleration at the surface, and R the radius. For convenience of reference the velocities of escape for the principal bodies of the solar system are given in the following table:

The velocity of escape for the earth is	6.95 miles per second.
The velocity of escape for the moon is	1.48 miles per second.
The velocity of escape for the sun is	380.00 miles per second.
The velocity of escape for Mercury is	2.45 miles per second.
The velocity of escape for Venus is	6.37 miles per second.
The velocity of escape for Mars is	3.13 miles per second.
The velocity of escape for Jupiter is	37.16 miles per second.
The velocity of escape for Saturn is	22.97 miles per second.
The velocity of escape for Uranus is	13.13 miles per second.
The velocity of escape for Neptune is	13.64 miles per second.

From this table it follows that the earth will lose its atmosphere very slowly, if at all, by the escape of its separate molecules in accordance with the kinetic theory of gases; but the case may be somewhat different with the moon and Mercury.

112. The Height and Mass of the Atmosphere. — The atmosphere extends at least as high as the clouds and highest mountains. The question of where it terminates can be approached either theoretically, or from observations of phenomena which depend upon its existence.

The theoretical discussions show that the atmosphere becomes thinner and thinner until it no longer obeys the law of gases. It probably becomes inappreciable at a distance of 100 miles from the earth's surface.

[1] *Introduction to Celestial Mechanics*, p. 43.

Twilight phenomena furnish a means of finding something about the height of the atmosphere. After the sun sets in the evening, and before it rises in the morning, the sky in its direction is illuminated for a considerable time. It is difficult to tell just when the dawn begins or the twilight ceases, but it is when the sun is not far from 18° below the horizon. The twilight illumination is due to the reflection of the sunlight to the observer, from the upper air, or perhaps from solid particles in the upper air, and it follows from the fact that it is visible until the sun is 18° below the horizon that the atmosphere is at least 40 or 50 miles high.

The duration of twilight depends upon the angle the sun's diurnal path makes with the horizon. When it goes straight down, the twilight lasts an hour and twelve minutes, for the earth turns 15° hourly. If the sun's diurnal path meets the horizon obliquely, the twilight is longer, extending to three or four months near the pole. In our latitude it is about two hours long in the summer and an hour and a half in the winter.

The phenomena of meteors furnish an independent means of finding the height to which the atmosphere extends in sensible quantities. Meteors are small particles of matter which dash into the earth's atmosphere from interplanetary space with high velocities, such as 12 to 45 miles per second. They are invisible until they are made incandescent by friction with the upper air. The heights at which they become visible can be found by two observers at different stations, as A and B (Fig. 54). They both measure the apparent direction of the meteor m, when it first becomes visible. This furnishes the data for solving the triangle and finding the height of m above the earth. Observations of this sort show that the atmosphere is dense enough up to an altitude of 100 miles to make at least some meteors incandescent as they pass through it.

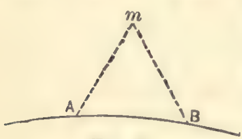


FIG. 54.

The aurora is very probably an electrical phenomenon of the rare upper atmosphere, though it is not yet very well understood. From simultaneous observations by observers at different stations, like those made for meteors, its altitude above the earth can be computed. The southern ends of the streamers are usually over 100 miles high and sometimes as much as 400 miles high. At an altitude of 100 miles the density of the atmosphere can not be much more than one-billionth as great as it is at the sea level.

The atmosphere is nearly all within the first few miles of the earth's surface (half of it is within the first $3\frac{1}{2}$ miles), and forms a relatively thin layer over it. Its total mass is about $\frac{1}{1,200,000}$ that of the remainder of the earth.

113. The Climatic Influences of the Atmosphere. — Next to the sun, the atmosphere has the greatest influence on the climate. One of its most important effects is in equalizing the temperature. In the first place, the direct rays from the sun are absorbed to a considerable extent before they reach the surface, especially if the air is saturated with water vapor. This diminution of the intensity of sunlight is very noticeable in going from a low, moist region to a high, dry one.

The rays which pass through the atmosphere heat up the land and water surface, and at night part of this heat is radiated into space. But the loss is not as great as it would be except for a very interesting circumstance. When the rays come to us from the sun, they occupy a part of the spectrum near the light rays; in fact, the light rays act so much like the heat rays that it is not necessary to distinguish between them in this discussion. But when they are radiated back from the warm (not hot) earth, they are very far below the light rays in the spectrum. Now, the atmosphere is quite transparent to light rays and heat rays near them, but nearly opaque to radiations from cooler bodies like the earth. The result is that the air retains much of the heat which is caught by the earth. It acts much like the glass cover to the gar-

dener's hotbed which admits the rays of the sun almost without sensible absorption, while it keeps the lower spectrum radiations of the soil from escaping.

The water vapor and water particles in the air aid very much in this blanketing process. Gardeners fear frosts only on clear nights after a rain when there is but little water vapor in the air. In high and arid regions the radiation is so rapid that the nights are always cool, however hot the days may be.

The atmosphere equalizes the temperature in another very important way. The air currents carry incalculably large quantities of heat from hotter to cooler regions. The effects can be seen on the western shores of the continents, which are under the influences of almost constant winds from the southwest. If it were not for air currents, both the equatorial and polar regions would be uninhabitable.

114. Effects of the Constitution of Atmosphere. — The effects which have been discussed are not independent of the constitution of the air, as has been mentioned in the case of water vapor. Arrhenius has recently shown that the carbon dioxide in the atmosphere has very important climatic influences. As small as the amount is, he shows that if it were doubled the temperature would be both more uniform and notably higher. If it were lessened, the climate would be more varied and colder. Chamberlin believes that a periodic increase and depletion of the atmospheric carbon dioxide is the primary cause of the great variations which the climate of the earth has undergone. There is abundant evidence that our latitudes have been periodically covered alternately with vast sheets of ice and semi-tropical vegetation. This shows how useless it is to speculate on the climatic conditions of other planets without knowing anything about the constitutions of their atmospheres.

115. Rôle of the Atmosphere in Life Processes. — Life processes are of two kinds, vegetable and animal. Vegetable

substances are largely carbon compounds and water. Since it is the carbon which burns, its abundance is amply established. Plants obtain their carbon from the carbon dioxide of the atmosphere. In the cells of the leaves the energy derived from the sun's radiations in some way separates the carbon and oxygen of carbon dioxide. The carbon is used by the plant, while the oxygen is given back into the atmosphere.

Animals, or at least all the higher forms, depend upon the oxygen of the air for existence. The source of the energy possessed by an animal is very interesting. An animal takes into his system plant cells which contain the carbon. The oxygen from the air unites with this carbon, forming carbon dioxide, and gives up the same amount of energy as was used when the sunlight separated the two elements in the plant. Thus, indirectly the energy used in all our activity comes from the sun.

In discussing the habitability of other planets, at least by any such creatures as are on the earth, the character of the atmosphere must necessarily be treated as an important factor.

116. Twinkling of Stars. — When we look at a star near the horizon, we at once notice that it twinkles, or *scintillates*, especially in the winter time. The phenomenon is purely atmospheric and is due to waves of air of unequal density and refracting power sweeping across the line of sight. When a star is viewed through a telescope, the twinkling is magnified into actual "dancing." If one is watching a star cross the field of a transit instrument, its dancing interferes very much with his estimating the time of its passing the spider lines. Sometimes a star will apparently pass a spider line and then dart back to the other side for an instant.

The clear, cold nights, when the stars "sparkle" and excite the interest of ordinary spectators, are of little value to the astronomer. As was stated in connection with the dis-

cussion of telescopes (Art. 83), the unsteadiness of the air is the worst foe the observer has. The twinkling increases with the distance from the zenith, and it is with much difficulty that valuable observations can be made near the horizon.

117. Refraction of Light by the Atmosphere. — When light passes obliquely from the air into a denser medium, such as glass or water, it is refracted toward the perpendicular to the surface separating the two media. Similarly, when light comes from space into the earth's atmosphere, it is refracted toward the perpendicular; but the density of the air increases from its outer limits to the surface of the earth, and the light is continually bent so that its path is a curve. Thus, in Fig. 55, let L be the light from the star which reaches the observer at A. If it were not for the refraction, he would see it in the direction $\overline{AS}$. But it appears to be in the direction from which the light comes when it enters his eye, and he sees it in the direction $\overline{AS'}$. The angle SAS' is called *the angle of refraction*, and the correction for it must be applied to all observations of position, such as those for obtaining the altitude of the pole and the latitude of the observer. The refraction is zero at the zenith and increases in a rather complicated manner to the horizon, where it averages about $35'$. The amount of refraction depends upon the pressure of the air, its temperature, and the amount of water vapor it contains.

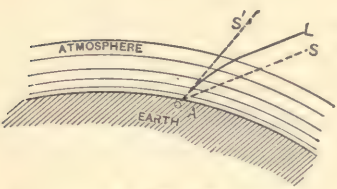


FIG. 55.

The effect of the refraction is to increase the apparent altitude of every object not at the zenith. The apparent diameter of the sun is about $32'$; consequently, when its lower edge is apparently at the horizon, it is really entirely

below the horizon. Thus refraction lengthens the period of sunshine daily by the time it takes the sun to move twice its diameter, or about 1°. If it moved perpendicularly to the horizon, the increase in sunshine would amount to about 4 minutes, but where it rises and sets obliquely, as in our latitudes, the time is considerably longer.

The light from the lower edge of the sun is refracted more than that from the upper. That is, the lower edge is raised apparently more than the upper, and when the sun is near the horizon, it appears flattened in the vertical direction, as any one can verify. Precisely similar remarks apply to the moon.

QUESTIONS AND EXPERIMENTS

1. Can you think of any other way of determining the mass of the earth than by its attraction for some mass?

2. Can the earth's attraction for the moon be used for this purpose?

3. The pressure on matter in the earth's interior equals the weight of the material above it. The weight of a cubic foot of water is about $62\frac{1}{2}$ pounds. What is the pressure per square inch on a body at the bottom of an ocean 5 miles deep? What is the pressure per square inch on a body under 100 miles of matter three times as dense as water?

4. The earth's attraction will cause a body at its surface to fall 16 feet in the first second; how far will a body a mile in diameter cause a body at its surface to fall in the first second? *Ans.* $\frac{1}{20}$ of an inch.

5. How great a deviation will a hemispherical mountain one mile in diameter and of density 3 produce in a plumb line hung at its base?

6. Will the attraction of the mountain introduce any errors in the triangulation which must be made around the mountain in the mountain method of finding the earth's density?

7. What is the law of the parallelogram of forces?

8. If the earth's crust were thin and capable of sliding on a fluid interior, would precession be faster or slower than it is?

9. Do the molecules of a liquid pass around among each other? Do they in case of a solid?

10. Select a place free from artificial lights and see how long the twilight lasts.

11. Suppose the earth's atmosphere is 100 miles deep; draw the earth and atmosphere to scale.

CHAPTER VI

THE MOTIONS OF THE EARTH

THE earth rotates on its axis and revolves around the sun. These facts have been obtained by an enormous amount of work, and after many bitter discussions. A large part of the theories of astronomy depend either directly or indirectly upon them, and no other facts are of more fundamental importance. This chapter will be devoted to the reasons astronomers have for believing in the motions of the earth, and to a discussion of some of the consequences of the motions.

118. The Relative Rotation of the Earth. — The stars have diurnal motions. This shows that either the earth rotates on its axis from west to east, or that all the stars revolve around the earth from east to west in the same period. That is, there is a relative motion which is never denied, and the only problem is whether it is the earth or the stars which move. The ancients generally believed that the earth was stationary and that the stars revolved, though some took the opposite view. It will be seen from what follows that they had no certain means of determining which theory is correct.

119. Historical. — The Greeks were the only people of antiquity who gave much thought to the causes of phenomena, such as the diurnal motions of the heavens. Although Pythagoras believed in the sphericity of the earth (Art. 93), yet he thought it was the fixed center of the universe. The earliest philosopher who is known to have taught that the earth moves was the Pythagorean, Philolaus, who lived

about a century later than Pythagoras (*i.e.* in the fifth century B.C.). His theory was, on the whole, fanciful and permeated with the mysticism of his school of philosophy, yet the conception that the earth may move was a great contribution to thought on the subject, and it never fully perished. Several other Pythagoreans are mentioned as having believed in the motions of the earth. The authority of Philolaus and others was quoted by Copernicus (1473–1543) in his great work on the theory of the solar system. But the clearest statement of the motions of the earth made in antiquity was by Aristarchus of Samos (310–250 B.C.), who believed both in the rotation of the earth and in its revolution around the sun.

Aristotle (384–322 B.C.) recognized the fact that the apparent motions of the stars can be explained either by their revolution around the earth, or by its rotation on its axis. He accepted the former hypothesis as being the true one.

Hipparchus (180–110 B.C.), who was the greatest astronomer of antiquity, and whose valuable discoveries were very numerous, believed in the fixity of the earth. He was followed in this opinion by Ptolemy (100–170 A.D.) and every other astronomer of note down to Copernicus. Copernicus developed the heliocentric theory in detail, and it found quite general acceptance among scientific men. It has been regarded as thoroughly established since the time of Newton (1643–1727), although the direct proofs of its correctness are all more recent.

120. The Laws of Motions. — The natural way of discussing whether any particular body, as the earth, moves or not, is first to set down the laws of motions of bodies in general. Then, supposing the laws are correct, the theory which is finally adopted must agree with them.

A word needs to be said respecting the character of natural laws. A law of conduct prescribed by a state makes it

obligatory for the individual to conform to it. He may, indeed, violate it; but if he does, he is subject to a certain penalty. On the other hand, a law of nature is simply a statement of the way in which natural phenomena succeed each other. A natural law is a descriptive product of the mind and has no influence on events.

The laws of motion, or the statements of the way bodies move, were first given in their completeness by Newton in the *Principia*, although they were partially understood by his predecessor, Galileo.

LAW I. *Every body continues in its state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by exterior forces acting upon it.*

LAW II. *The rate of change of motion is proportional to the force impressed, and the change takes place in the direction of the straight line in which the force acts.*

LAW III. *To every action there is an equal and opposite reaction; or, the mutual actions of two bodies are always equal and oppositely directed.*

Newton called these laws *axioms*, probably because he could not prove them from any simpler principles, although they were certainly suggested to him by experience. They are not axioms in quite the same sense that the whole is equal to the sum of its parts is an axiom, for no normal mind can deny this proposition. But the best thinkers down to Galileo believed something different from Newton's laws, such as that bodies fall without forces being applied to them, or that they tend to move in circles instead of in straight lines. The importance of the laws of motion can be seen from the fact that every astronomical and physical phenomenon involving motion of matter now is interpreted on them as a basis.

The first law of motion is clear as it stands, and involves only the difficulties of defining a straight line and equal intervals of time. In the second law, "the rate of

change of motion" means the product of the mass and the rate of change in velocity. The law might be made to read : *The rate of change of velocity is proportional to the force impressed and inversely proportional to the mass moved*, etc. The remainder of the law is perfectly clear.

The first two laws relate to the motion of a single body when subject to any forces; the third states how two bodies act upon each other. It means essentially that no body can change the motion of another body, whether they are connected by visible bonds or by forces which act across vacant space, without having its own motion (mass times velocity) changed by the same amount but in the opposite direction. This is a little hard to realize, because it is not possible to get two bodies which are subject to no forces but their mutual interactions. If a man stands on the ground and by a rope pulls a weight across its surface, he sees the weight move while he apparently remains still. This is not at all a violation of the law, for the friction of his feet with the ground makes him a part of the earth in the experiment. The body moves, and he and the whole earth together move in the opposite direction to meet it. If he tried the experiment while in a boat, or on smooth ice, the law would be apparently, as well as actually, verified.

The laws of motion have been verified in millions of direct and indirect ways, and there is yet no reason for doubting their correctness. However, it is possible to state them quite differently, and many have preferred these different forms.

THE ROTATION OF THE EARTH

121. Rotation of the Earth proved by its Shape. — Every proof of the rotation of the earth, and of everything else as well, is made on the basis of certain axioms and principles. This is very clearly seen in ordinary geometry, where the axioms are very few and simple, and are set down at the

beginning of the work. It is well to state what axioms are involved in every proof, for then we learn what demonstrations are really of an independent character ; and besides, when results obtained from axioms agree with experience, the axioms are themselves verified to the extent they were involved. Therefore, in enumerating the proofs of the earth's rotation, the principles upon which they depend will be pointed out.

The earth is bulged at the equator because the particles of which it is composed tend to fly away in straight lines in accordance with the first law of motion. The particles are held from flying away by their mutual gravitation, the law of which has been derived from the motions of the planets on the basis of all the laws of motion. It was first shown by Clairaut that the theory and actual measurements are in close agreement. Consequently, we must either accept the rotation of the earth as an established fact, or else deny the truth of at least some one of the laws of motion.

122. Rotation of the Earth proved by the Deviations of Falling Bodies. — If the earth rotates, then the farther a body is from its axis the faster it goes. Let Fig. 56 represent a section of the earth perpendicular to its axis. Let A be the point on the axis and Fm a high tower (of course greatly exaggerated in the figure). Suppose the mass m is dropped from the top. If the earth were not rotating, it would fall in the direction of the plumb line (*i.e.* in the direction of the resultant forces) according to the second law of motion, and would strike the surface at F.

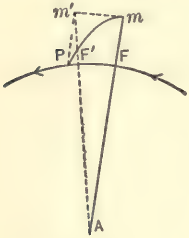


Fig. 56.

Suppose, however, the earth is rotating at such a rate that FA turns to $F'A$ while m is falling to the surface. If it were not for the attraction of the earth, m would go in a straight line to m'. The resultant attraction is at right angles to this

line and does not change the motion of m in this direction, but impresses upon it a new one toward the earth. The result is that it describes the curved line $\overline{mP}$ and strikes the earth at P, a little east of the foot of the perpendicular.

The deviation is zero at the poles and greatest at the equator for a given height. The deviation is small, being in our latitude only about an inch for a fall of 500 feet. Air currents or any little irregularities in the shape of the falling body are sufficient to mask the quantity to be measured. The most successful experiments have been performed in mine shafts and have given quite satisfactory results. They are at least sufficient to compel one to admit either that the earth rotates or that at least one of the first two laws of motion is wrong.

123. Rotation of the Earth proved by Foucault's Pendulum Experiment. — In 1851 a very ingenious and convincing experiment was devised and carried out by Foucault. It depends upon the fact that a pendulum tends to swing constantly in the same plane.

Suppose a pendulum is suspended on a pivot at A (Fig. 57) directly over the earth's pole P, and that it is started swinging in the plane of the meridian m. If the earth were stationary, it would not only continue to swing in the same plane, but also over the same meridian. But since the earth rotates, it will turn around under the pendulum, and the plane in which the pendulum swings will apparently rotate in the opposite direction. At the equator there will be no rotation ; or, as we may say, the period of rotation will be infinite. At intermediate latitudes there will be a rotation and the period will be between a day and infinity. The way the period depends upon the latitude can not be explained here, although it is perfectly well understood.

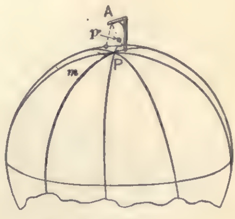


FIG. 57.

Precaution must be taken to avoid external disturbances, but the pendulum experiment is not very difficult to perform, and very often has been successfully carried out. The theory of the pendulum depends upon the first two laws of motion, and because of this experiment one must admit either that the earth rotates or that at least one of the laws is wrong.

124. Other Proofs of the Earth's Rotation. — There are many other more or less complete proofs that the earth rotates, nearly all of which depend upon the same laws of motion. It will be sufficient to enumerate them.

(1) The rotation of the earth may be proved by the gyroscope experiment, which was devised by Foucault. The gyroscope consists of a heavy wheel mounted in gimbals so that it can turn freely in any direction. When the wheel is started spinning it keeps the same plane while the earth turns under it. The heavy wheel plays the same rôle as the pendulum in the pendulum experiment.

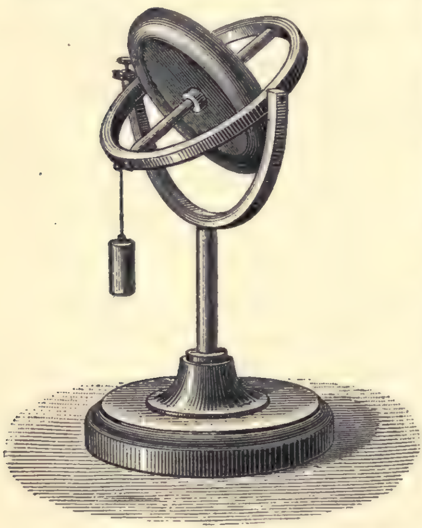


FIG. 58. — Gyroscope.

(2) In the northern hemisphere projectiles deviate to the right and in the southern hemisphere to the left (Ferrel's law).

(3) The direction of trade winds and ocean currents.

(4) The direction of motion in cyclones. (Cyclones, as the term is used here, are not tornadoes, but the large spiral

motions of the atmosphere which take place around the storm centers as they sweep along the earth's surface.)

(5) Analogy with the moon, sun, and a number of the planets which are situated so that their surface markings can be observed. While an analogy is often very convincing, perhaps as a matter of logic it should not be classified as a proof.

125. Rotation of the Earth proved by the Spectroscope. — The spectroscope is an instrument by means of which radial velocities (*i.e.* the relative components of velocity of the observer and object observed in the line joining them) can be measured.

Suppose a star is rising in the east. Then the surface of the earth in the observer's longitude is turning toward it at the rate of 1000 miles an hour at the equator, but with constantly diminishing velocity as the poles are approached. When the star is setting, the same meridian is turning from the star at the same rate. Suppose the radial velocities are measured at the two times, and that the corrections are applied for the motion of the earth around the sun. Then the difference will be twice the velocity of the observer due to the earth's rotation.

The amount to be measured is rather small for spectroscopic processes, yet it is not beyond the power of modern instruments. In measuring the radial velocities of the stars, corrections for the rotation of the earth are regularly applied, and no systematic inconsistencies are introduced, as they would be if the earth did not rotate.

It will be observed that this proof does not in any way depend upon the laws of motion, although it does involve the validity of certain spectroscopic principles which can more profitably be discussed at another place (Art. 327).

It is clear from the preceding discussion that the rotation of the earth is no longer open to question. That the earth rotates is not only supported by many individually strong

and very certain proofs, but the fact that this conclusion forms a link in a chain of scientific doctrine which has connections with nearly every physical theory establishes it as firmly as any fact we accept.

126. The Uniformity of the Earth's Rotation. — It follows from the laws of motion, as the appropriate mathematical discussion shows, that if the rotation of the earth is not influenced by exterior forces, or by any change of form and size, it will forever continue to turn at a perfectly uniform rate. One way of answering the question whether the rotation is uniform or not is by investigating what forces there are which can change the rotation. Another is by comparing it with something which is known to keep time perfectly. No clock can be made to run accurately enough for any such tests. The only alternative is to use the motion of some other body as a standard. But the motion of this body would conform to the laws of motion, and the question of its uniformity would be the same as that of the rotation of the earth. Therefore no better answer can be obtained than that found by discussing the forces which may change the earth's rotation.

The earth is rotating in the luminiferous ether and a considerable quantity of meteoric matter. The latter, if not the former, offers some resistance to its motion, and causes it to rotate more slowly, just as friction of the air slowly diminishes the rotation of a top. But this resistance is exceedingly slight and will produce no measurable results in thousands of years. Probably this cause does not change the length of the day a second in 1,000,000 years.

The sun and moon generate tides in the earth which move around it in the westward direction. Since the earth rotates eastward, the impact of these waves on the shores and the friction they encounter in passing along the oceans retard the rotation of the earth. The earth is not perfectly rigid (no body is), and there are body tides in it, apart from

those in the water and atmosphere, which also retard its rotation. These tidal effects are very minute, especially if the earth is very elastic. Although it is not possible to calculate just how effective they are, yet it seems safe to say that they will not change the length of the day a second in many thousands of years.

The earth gradually loses its interior heat and this allows it to shrink a little. Earthquakes generally are caused by some large mass of matter settling, and the rocks furnish abundant evidence of shrinkage of the earth and wrinkling of its surface layers. If the size of the earth decreases, its rotation increases; for it follows from the laws of motion that the moment of momentum of a body, which is the sum of the products of the masses, distances, and angular velocities of its parts, is a constant whatever changes it may undergo as a consequence of internal reactions. The angular velocities of all parts of the earth are the same. The masses of the different parts do not change; therefore, if the earth contracts, every particle is nearer the axis (except those originally in the axis), and the angular velocity must increase to keep the sum a constant. It is not known just how fast the earth is contracting, but it certainly contracts slowly, and according to Woodward's investigations, the influence of its shrinking on the length of the day does not amount to so much as a second in 20,000,000 years.

The attractions of the other bodies for the earth do not change its rate of rotation. It is something like the attraction of the earth for a balance wheel, which spins as though the earth did not attract it.

Thus there are two known causes operating to increase the length of the day and one to shorten it. Their effects are very small, and can not be computed with any considerable degree of accuracy. It is not certainly known whether the day is increasing or decreasing in length, but it is probably increasing exceedingly slowly.

There are many temporary and nearly balancing influences which slightly affect the rotation of the earth. Among these may be mentioned the evaporation of vast quantities of water and its precipitation in other latitudes, the elevation and subsidence of portions of continents, and the alterations of surface produced by erosion. In fact, anything which changes the distance of any mass from the earth's axis affects the rotation. But the masses moved are so insignificant when compared to the whole earth that the results of their influences are entirely negligible.

127. The Variation of Latitude. — It was mentioned in connection with the discussion of the condition of the interior of the earth, in Art. 108, that the earth's axis of rotation is not quite fixed. There is no particular reason why it should not oscillate around its axis of figure, if it is given the proper disturbance, something as a top wabbles when it spins. But if there is such an oscillation, it has a perfectly definite period, depending upon the size, mass, distribution of density, and rigidity of the earth, just as the wabbling of a top has a perfectly definite period. On the hypothesis that the earth rotates as though it were a rigid body, Euler (1707–1783) long ago showed that the period of oscillation of the axis, if it oscillates, must be 305 days. The German astronomer, Peters, sought evidence from observations of an oscillation of this period, but found none.

About 1884–1885 observations of extraordinary precision by Chandler, at Cambridge, and by Küstner, at Berlin, gave unmistakable evidence that the latitude does vary to the extent of nearly $0.6''$, which corresponds to a wandering of the terrestrial pole in a curve having a diameter of nearly 60 feet. Other observers in various parts of the world soon verified these results. The subject has been deemed of so much importance by astronomers that observatories have been established in various parts of the world, as Maryland, California, the Sandwich Islands, and Japan, by interna-

tional coöperation, for the special purpose of studying this question.

In 1891 Chandler took up the discussion of the results which had been obtained, together with the records of earlier observations of latitude. By a laborious but splendid piece of work he succeeded in showing that the very complicated motion of the pole is the resultant of two simpler motions. One is an annual elliptical motion around the axis of figure as a center, with a maximum radius of about 14 feet and a minimum radius of about 4 feet, and the other is a circular motion of period 428 days, with a radius of about 15 feet. They are both from west to east. Recent discussions show that the law of areas holds in the elliptical motion, that the ellipse rotates from east to west about 5° yearly, and that there is also another very slight variation whose period is about 436 days. The question is so delicate that these finer results may be modified somewhat.

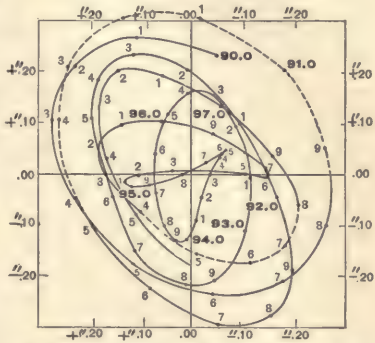


FIG. 59. — Diagram of Wandering of Pole from 1890 to 1898 as determined by Chandler from Observations of Latitude.

The problem is to account for the periods. The annual period suggests that this motion is due to some annual disturbance, such as the precipitation of water or snow in certain latitudes, or to air currents and ocean currents ; but the cause is not certainly known. The motion with a period of 428 days may be the indirect effect of an annual disturbance. One can see how it may be by considering a pendulum of such length that it would naturally oscillate in seconds. Suppose it were given a little disturbance, always of the same nature, every two-thirds of a second.

It would have a motion with a period of two-thirds of a second, and also another with its natural period of one second, and the combined effect would be very complicated.

Euler showed that the natural period of oscillation of the earth around its axis of rotation, *on the hypothesis that it is rigid*, is 305 days. He supposed it would not make much difference if it were not rigid. To test the matter Newcomb assumed, instead of perfect rigidity, that the earth has the rigidity of steel and computed the period of oscillation. He found the surprising result that in admitting this degree of elasticity, which in small spheres such as we are familiar with seems very slight, the period of the variation is increased to 441 days, which is a little too long. He concluded, therefore, that the average rigidity of the earth as a whole lies between perfect rigidity and that of steel; or, that *the earth is a little more rigid than steel.* The same results were reached independently by S. S. Hough. Woodward has shown that any yielding of the earth to the strains which result when the axis of rotation differs from the axis of figure will increase the period of oscillation.

QUESTIONS AND EXPERIMENTS

1. Copernicus believed in the rotation of the earth. Which of the proofs of it enumerated in the text could he have given?
2. What is an axiom? Is an axiom ever wrong?
3. Are the laws of motion ordinary axioms, or direct inferences from experience?
4. In what way does the construction of a railway around a curve depend upon the laws of motion?
5. What is a force? Can you use your definition consistently in the laws of motion?
6. Fill a pail about half full of water, tie a small rope to the bail, twist it up, and observe the bulging effects of centrifugal acceleration when it untwists.
7. How would it be possible to determine one's latitude by the gyroscope experiment?

8. Suppose a gyroscope could be kept running indefinitely. Suppose a ship carries one turning in the plane of its meridian when it leaves a port on the equator. If it travels along the equator, can the angular distance traveled be determined from the gyroscope?

9. Prove Ferrel's law of deviation of projectiles and wind and ocean currents.

10. In proving the rotation of the earth by means of the spectroscope, is it necessary to make corrections for the motion that the observed star may have from or toward the earth? If the earth's motion around the sun were uniform and in a straight line during the observations, would it be necessary to correct them for it?

11. What effect does the evaporation of water in the equatorial regions and its precipitation in higher latitudes have upon the rotation of the earth?

12. Several of the largest rivers in the world flow southward from middle latitudes in the northern hemisphere, and carry into the sea vast quantities of sediment. What effect, if any, does this have upon the rotation of the earth?

13. Does it change the rotation of the earth to take stone from a quarry and make it into a high building? If so, how?

THE REVOLUTION OF THE EARTH

128. Apparent Motion of the Sun. — It was stated in connection with the discussion of the ecliptic (Art. 24) that the sun apparently moves eastward among the stars in a great circle. But the motion is not quite uniform, as was known even as early as the time of Hipparchus. The sun's apparent diameter varies a little, and in such a way that it is the greatest when its angular motion is the fastest, and the smallest when its angular motion is the slowest.

The apparent motion implies either that the sun goes around the earth, or that the earth goes around the sun; and as far as mere appearances go, one hypothesis is quite as satisfactory as the other. From the changes in the apparent size of the sun it follows either that the sun's diameter actually changes, or that its distance from the earth changes. If its diameter is constant, it appears larger the nearer it is to the earth.

129. Apparent Motions of the Planets. — When defined by their motions, the planets are the bodies which travel among the stars, generally eastward and within a few degrees of the sun's apparent path, the ecliptic. They are divided into two classes by their motions. One class consists of those which are apparently never very far from the sun. They are Mercury and Venus. The other class consists of those which may be anywhere along the ecliptic, wherever the sun may be. The apparent motions of the two classes of planets are somewhat different and will be discussed separately. The bodies of the first class are called the *inferior planets*, and those of the second class the *superior planets*.

The apparent motions of the inferior planets with respect to the sun are vibrations back and forth past it near the ecliptic, but very rarely being so near to the sun's path that they pass in front of or behind it. The motion westward is made more quickly than that eastward. The difference is relatively greater in the case of Venus, which has the larger motion, than it is in the case of Mercury. The motion is not uniform, but is fastest when the planet is near the sun. It is very slow near the ends of the oscillations, and for a few days just at the ends there is almost no apparent motion with respect to the sun.

With respect to the stars the motion is also eastward and westward, but the westward component is much smaller than

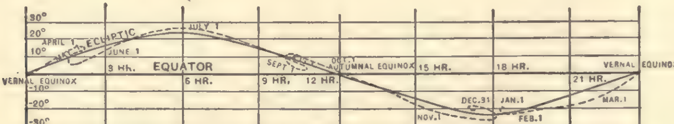


Fig. 60. — Apparent Motion of Mercury during 1905.

the eastward. For example, Mercury goes eastward about 125° and then westward about 10°. The northward and southward motions sometimes cause them to make loops during their westward motion.

It is not necessary to discuss the motions of the superior planets directly with respect to the sun. Their motions with respect to the stars are very much slower than those of the inferior planets, and they go in an eastward direction among the stars except when they are opposite to the sun, when they move westward for a time. Those planets which have the greatest motion eastward also have the greatest motion westward. On account of the northward and southward motions the apparent motions among the stars are often loops during the westward excursions.

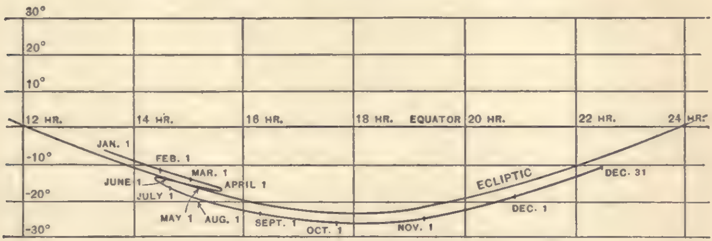


FIG. 61. — Apparent Motion of Mars during 1905.

The determination of the actual motions of the planets, including the earth, is a question of the highest importance, and one which has been settled only on the basis of a vast amount of observational data and after a long evolution of ideas on the subject. The vital relations of these questions to astronomical doctrines demand their careful consideration.

130. Historical. The Ideas of the Pythagoreans. — Astronomy was cultivated by the Chaldeans and Egyptians, but little is known of the progress made in it until after the acute and imaginative Greeks turned their attention to it. Thales (seventh century B.C.) is credited with having introduced the Egyptian astronomy into Greece, but so far as known his own contributions were entirely unimportant. Then followed a school of philosophers known as the Pythagoreans, after its founder Pythagoras (sixth century B.C.), who made

many and important contributions to the philosophy of the science, but very few to its data.

Pythagoras believed that the earth is round, but that it is immovable and at the center of the universe. He supposed the stars, moon, sun, and planets move around the earth on crystalline spheres. One of his followers, Philolaus (fifth century B.C.), ascribed to the earth a motion of rotation and a revolution around a central fire. The sun was supposed to reflect light from this central fire. The theory was altogether fanciful and in accord neither with any observations nor with what would now be called common sense.

The great contribution of the Pythagoreans came from the very weakness of their method. By giving free scope to the imagination without being hampered too much by known phenomena, they introduced ideas which have been the most important stimuli in the investigations which have led to the truth. It was a bold thing for them to advance the theory that the earth is a sphere resting on nothing, and that it moves. Such ideas could be originated only by men having keen imaginations and the ability for abstract reasoning. These conceptions seem simple to us only because we have been taught them from earliest childhood.

131. Eudoxus, Aristotle, Aristarchus. — After the Pythagorean school, Greek astronomy became less purely speculative and more scientific. Eudoxus (409–356 B.C.) was the first astronomer of note in this transition period, and the new spirit had become dominant in the work of Aristarchus (310–250 B.C.). This revolution of method was one of the great contributions of the Greeks to scientific inquiry.

Eudoxus tried to construct a theory of celestial motions out of uniform circular motions which should agree with observations. The details of his scheme will be omitted, for they gradually evolved into the great work of Ptolemy, to be discussed presently. Aristotle believed in the sphericity of the earth, but that it did not move. He advanced the very

conclusive argument against its revolution that there are no apparent displacements of the stars as should result from it. He discussed the causes of the phases of the moon and several other phenomena with great keenness.

Aristarchus was one of the first astronomers of the celebrated school founded at Alexandria by Alexander the Great. He is the first astronomer who worked out on scientific lines the fact that the apparent motions of the stars and sun could be explained by supposing that the earth rotates and revolves. He overcame the objection of Aristotle by supposing that the fixed stars are indefinitely remote.

132. Apollonius, Hipparchus, Ptolemy. — The Alexandrian school was famous for the development of mathematics as well as of astronomy. It was in it, about 300 B.C., that Euclid developed the science of geometry. Nothing was more natural than that the astronomy of this school should become less physical and more mathematical. Eudoxus and his followers supposed that the stars, moon, sun, and planets were set in rotating crystalline spheres. Apollonius (latter part of third century B.C.), a famous mathematician, started a new era by proposing to consider the motions of the heavenly bodies as being purely geometrical, instead of trying to explain them by introducing all sorts of physical devices. In this way the problem became stripped of much of the dead material which had previously cumbered it, but it does not appear that Apollonius made any great progress in applying his ideas.

Hipparchus (180–110 B.C.) is universally conceded to have been the greatest astronomer of antiquity. Almost all we know of his work is through that of his disciple and admirer, Ptolemy, who lived nearly three centuries later. His main lines of activity lay in four directions. (1) He developed trigonometry, without which precise astronomical calculations can not be made. (2) He made a more extensive and more accurate series of observations than any other astrono-

mer until the time of the Arab, Albategnius (850–929 A.D.). (3) He systematically and critically compared his observations with earlier ones in order to discover any changes that might have taken place. (4) He employed a geometric scheme of eccentrics and epicycles to represent the motions of the heavenly bodies.

Ptolemy (100–170 A.D.) was the first astronomer of note after Hipparchus and the last great astronomer of the Alexandrian period. From his time till that of Copernicus not a single important advance was made in the science of astronomy. From Pythagoras to Ptolemy was 700 years, from Ptolemy to Copernicus 1400, and from Copernicus to the present time 400 years. Ptolemy's was the crowning work of the first period, and that of Copernicus the first of the new; or, perhaps better, that uniting the old with the new, which may be considered to have begun with Kepler (1571–1630). The work of Ptolemy is preserved in the *Almagest* (*The Greatest Composition*). Only those parts of its thirteen books which refer to the explanation of the motions of the moon, sun, and planets will be mentioned here.

133. The Ptolemaic Theory. — Ptolemy supposed that the earth is a fixed sphere situated at the center of the universe. He supposed that the sun and moon move around the earth in circles, but that the earth is not exactly at their centers. Thus, suppose C, Fig. 62, represents the circle described by the moon or sun. The earth E is at a little distance from the center, the amount being determined by the observations. Such motion is called *eccentric motion.* Hipparchus explained the motions of the sun and moon by supposing they move in eccentrics with uniform speed. Ptolemy improved the theory by supposing that the angular motion is uniform with respect to P, a point symmetrically opposite to E with respect to O. It can be shown mathematically that the motion can be much more accurately represented in this way than by a simple eccentric.

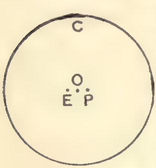


FIG. 62.

The greatest contribution of Ptolemy was the theory of the motions of the planets. The motions of the inferior planets were explained by supposing that they revolve uniformly in small circles around centers which are always in the line joining the earth and sun. Thus, in Fig. 63, E represents the earth, supposed to be fixed, S the sun, which was supposed to travel along its circle in the manner described above, and O a point on the line between the earth and the sun, and at the center of the circle C around which the planet was supposed to move uniformly. The circle C was supposed to be slightly inclined to the plane of the sun's motion. The point O, the size of C, the inclination of C, and the rate of motion along C were selected so that the apparent motions of the inferior planets were very satisfactorily represented.

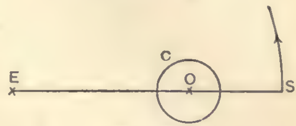


FIG. 63.

The motions of the superior planets were explained by supposing that they revolve uniformly in small circles whose centers travel uniformly in large circles around the earth. The large circles were supposed not to lie exactly in the plane of the sun's motion, and the plane of the small circle was supposed to be appropriately inclined to that of the large circle. The small circle was called an *epicycle*, and the large circle a *deferent*.

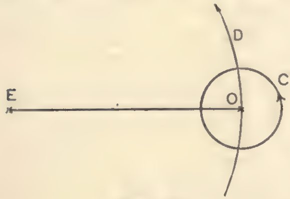


FIG. 64.

Thus, in Fig. 64, E is the fixed earth, D the deferent, and C the epicycle, whose center O moves along D. By choosing C of the proper size, and the proper rates of motion along D and C, the apparent motions of the superior planets were very satisfactorily represented.

134. Copernicus. — After Ptolemy astronomy was cultivated only by the Arabs until about the fifteenth century,

when great interest in the subject sprang up in Germany. The first great astronomer after this long stationary period was Copernicus (1473–1543). The invention of printing (about 1440) had made the work of the Greek philosophers much more accessible, and independent thinkers found that there were reasons for doubting the ultimate authority of Aristotle. Especially were the suggestions that the earth might be in motion becoming known. Copernicus was not a great, or even a skillful, observer, but he devoted years of his life to a contemplation of the apparent motions of the heavenly bodies with a view to discovering their real motions. He gradually became convinced that the simplest explanation is that the sun and stars are fixed, and that the earth both rotates on its axis and revolves around the sun. It must be understood that he did not prove the correctness of this view, but that he showed that it would account for all the apparent motions more simply than the hypothesis that the earth is fixed.

The chief merit in the work of Copernicus consists in the faithfulness and minute care with which he showed that the heliocentric theory would satisfy observations as well as the geocentric theory.

135. Tycho Brahe. — The immediate follower of Copernicus was Tycho Brahe (1546–1601), who rejected the Copernican system for theological reasons, and because he could observe no apparent annual motions of the stars due to the motion of the earth. He was a very energetic and painstaking observer, and his great contribution to astronomy consisted of the excellent series of observations which he made. For example, he found the length of the year with an error less than one second of time.

Tycho Brahe adopted a system in which the earth was placed at the center of the universe. The moon and sun were supposed to revolve around the earth, and the other planets around the sun.

136. Galileo. — Between Tycho Brahe and Newton (1643–1727) there were two great astronomers, Galileo (1564–1642) and Kepler (1571–1630), who, to a remarkable extent, but in quite different directions, led to the complete overthrow of the Ptolemaic system and prepared the way for the *Principia.* Galileo was the first distinguished Italian astronomer. He performed two services of great importance for astronomy. The first was his vigorous and unanswerable defense of the Copernican system (though he did not rigorously prove its correctness), which was much aided by his discoveries with the telescope. His great work, *Dialogue on the Two Chief Systems of the World*, the Ptolemaic and Copernican, published in 1632, was written in a most ingenious fashion. It represented a discussion between a follower of the Ptolemaic system and one of the Copernican system, in the presence of a neutral third person, Simplicio. At every point the Copernican won, and Simplicio was compelled to agree that he was right. In this way the relative merits of the rival doctrines were brought into sharp contrast in a way that every one could appreciate.

Galileo's other contribution to astronomy did not attract so much of popular interest, but it was equally important. He made the first serious start in experimental dynamics, and discovered such facts as that the time of oscillation of a pendulum is independent (nearly) of the arc through which it swings, that bodies of all weights fall a given distance in the same time, and that the first law of motion describes how bodies move when they are not acted upon by any forces. He put into constant practice those principles of scientific investigation which were only advocated by his great English contemporary, Francis Bacon (1561–1627).

137. Kepler. —Kepler (1571–1630) is one of the most interesting characters in the whole history of science. His most noteworthy mental characteristics were his mysticism, his unwearying industry in the face of many troubles and

almost universal failure of his theories, and his absolute simplicity and honesty.

Kepler was a pupil of Tycho Brahe, who assigned to him the task of working out the theory of the motion of Mars. Tycho Brahe had made a very accurate series of observations of this planet and shortly after his death the records were turned over to Kepler. The latter tried combination after combination of eccentric and epicycle in his attempts to explain them, and succeeded in reducing the differences between theory and observation to less than 8′, which is only about two-thirds the distance between Mizar and Alcor. The discrepancies were very small, yet he did not believe that they were due to errors by Tycho Brahe, although his measurements of position were necessarily all made without the aid of a telescope. Accordingly he took up the subject again, trying one system of epicycles after another, but always failing to secure satisfactory results. Finally he made the happy guess that the orbit of Mars is an ellipse, and he found that this curve answered the requirements when the sun was supposed to be at one of its foci. Without strictly proving it, he stated that the orbits of all the planets are ellipses with the sun at a focus of each, and that the orbit of the moon is an ellipse with the earth at one of its foci.

The next problem was to find how Mars moves in its ellipse, and after an immense amount of computation he showed that it moves so that the radius from it to the sun sweeps over equal areas in equal intervals of time, whatever their length. Thus, in Fig. 65 the shaded sectors are all of the same area, and a planet describing this orbit would move over the arcs which subtend them in equal intervals of time.

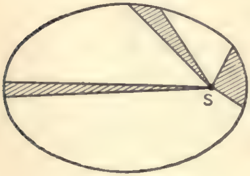


Fig. 65.

The laws of the motion of Mars, the fruits of eight years'

computation, were published in 1609 in a book of considerable dimensions. In this work, with childlike candor, Kepler explained how he had taken up hypothesis after hypothesis with hope and enthusiasm, and how his computations, which were given in considerable detail, had compelled him to abandon them, apparently without regret. There is no other example in which a discoverer has exposed with equal freedom both his successes and his failures.

In 1618 Kepler discovered a third law, called the *harmonic law*, which relates the distances and periods of the planets. It is that the squares of the periods of any two planets are to each other as the cubes of their respective mean distances from the sun.

138. Kepler's Laws of the Planetary Motions. — The really valuable results obtained by Kepler are contained in his following three laws of planetary motions: —

LAW I. *The orbit of every planet is an ellipse with the sun at one of its foci.*

LAW II. *Every planet moves so that the radius from the sun to it sweeps over equal areas in equal intervals of time, whatever their length.*

LAW III. *The squares of the periods of any two planets are to each other as the cubes of their respective mean distances from the sun.*

One thing must be pointed out which is generally overlooked, and that is that Kepler's work was entirely devoted to the explanation of relative motion. His theory would have satisfied observations in precisely the same way if he had supposed that the earth is stationary, that the sun revolves around it in an ellipse, and that all the other planets revolve around the sun in ellipses. The shape of the orbit depends simply upon the distances and directions of the sun and earth from each other, whichever is regarded as fixed.

QUESTIONS

1. Show by a diagram that the apparent motion of the sun is satisfied as well by the hypothesis that the earth goes around the sun as that the sun goes around the earth.

2. Enumerate the steps in the evolution of the Greek astronomy. Which required the greatest imaginative powers? Which the greatest mathematical powers? Which agrees most nearly with our ideas of the scientific method?

3. Show by diagrams that the scheme of eccentrics of Hipparchus and Ptolemy will give results in general agreement with the motion of the sun as described in Art. 128.

4. Show by diagrams that Ptolemy's scheme of epicycles for the inferior planets will give results in general agreement with their motions as described in Art. 129.

5. Show the corresponding thing for the superior planets.

6. On these plans does it make any difference how far away the planets are supposed to be, provided their orbits are on the same relative scale?

7. Under the Copernican theory that the planets all revolve around the sun in circles, can the general phenomena of their motions be represented without using epicycles? Discuss the question in detail with diagrams for both inferior and superior planets.

8. Was Tycho Brahe's scientific objection to the Copernican theory logically valid?

9. Prove from the laws of motion that bodies of all weights will fall the same distance in the same time.

10. What is an ellipse?

11. Show that all apparent motions are satisfied by supposing the sun moves around the earth in an ellipse, and that the other planets revolve around the sun in ellipses, as well as by Kepler's theory.

12. In what respects would such a theory seem less reasonable than that of Kepler?

139. Revolution of the Earth proved by the Laws of Motion. — It can be shown by measurements, which can not be explained at present, that the volume of the sun is more than a million times that of the earth. It is apparent at once that, unless the sun is inconceivably rare, its mass is vastly greater than the earth's. According to the laws of motion,

which can be verified by laboratory experiments, two bodies which are both free to move, revolve around their common center of gravity. Therefore if the earth and the sun move in accordance with these laws, they will revolve around their center of gravity, which is a point very near the center of the sun.

Newton made the assumption that the laws of motion are universally true, he derived the law of gravitation, and he showed that every motion of the whole system could be consistently explained. Hence it follows that the earth moves or that at least some one of the laws of motion is wrong. The Newtonian theory correlated such a vast body of phenomena that it was universally accepted long before there were any conclusive independent proofs of the motion of the earth.

140. Revolution of the Earth proved by the Aberration of Light. — After the development of the Copernican theory, astronomers, beginning with Tycho Brahe, made continual attempts to detect an annual apparent displacement of the stars due to the motion of the earth. In 1725 James Bradley, later third Astronomer Royal of England, took up the problem. He failed to find the quantity sought, but his observations showed another annual motion whose explanation he discovered in 1728. This phenomenon is known as the *aberration of light.*

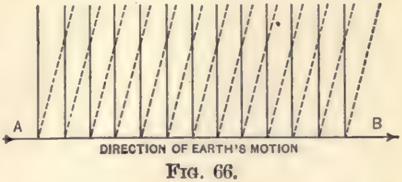


FIG. 66.

In Fig. 66, let $\overline{AB}$ be the direction of the earth's motion, and suppose, for simplicity, that the light from the star, represented by full lines, strikes it perpendicularly. The velocity of light is very great, but not infinite; consequently it seems to come to the observer slantingly, as represented by the broken lines. The reason is nearly the same as that which causes rain falling vertically

to appear to descend obliquely when one rides through it. The displacement of aberration is in the direction in which the observer is going, and depends both upon his own velocity and that of light. It is said that the true explanation was suggested to Bradley by his observing, while sailing on the Thames, the changes in the direction of the wind vane as the direction of the boat was changed. The sailors informed him that the direction of the vane depended upon the motion of the boat as well as upon the direction of the wind.

The most recent and best observations show that the aberrational constant is $20.47''$, with a possible error of $0.01''$ or $0.02''$. This is the value of the angular displacement of a star when its light comes perpendicularly to the direction of the earth's motion. A star at the pole of the ecliptic apparently describes a circle with this radius; a star on the ecliptic is displaced in a straight line by this amount each side of its mean position; a star between the ecliptic and its pole describes an ellipse whose length is twice the aberrational constant. The velocity of light can be measured in many ways, and it turns out that the aberrational constant is just what it should be on the theory that the earth moves around the sun.

Consequently it must be admitted either that the earth revolves around the sun, or that every star, irrespective of its distance, has such an actual motion that its apparent motion is the same as it would be if the earth moved, and besides, that the theory of aberration is wrong.

The rotation of the earth also produces a very slight aberration, amounting to $0.31''$ at the earth's equator. This may be considered as being an independent proof of the earth's rotation.

141. The Relation of the Aberrational Constant to the Velocity of Light and the Velocity of the Earth. — Let the line V, Fig. 67, represent the velocity of light, and v the velocity

of the earth. Then the angle a is the angle of aberration. From the triangle it follows that

$$a'' = 206{,}265 \frac{v}{V}.$$

If the earth moves around the sun, it follows from its size and the length of the year, that its average velocity must be about 18.5 miles per second, and physical experiments show that the velocity of light is 186,330 miles per second. Hence the formula gives

FIG. 67.

$$a'' = 20.47''.$$

The agreement of this result with that found by observation proves conclusively the revolution of the earth around the sun.

The relation may be used in quite a different way. The problem of measuring the distance to the sun, upon which the theoretical velocity of the earth depends, is one presenting many practical difficulties of a formidable character. As a matter of fact a'' and V can be measured at least as accurately as v. Consequently, by transposing V and the numerical factor, the value of v can be found, and from it the distance to the sun.

142. Revolution of the Earth proved by the Parallax of the Stars. — If an object which is not indefinitely remote is viewed from two points which are not on a line passing through it, its apparent direction from the two places is different. This difference in direction with respect to a specified distance apart of the points of observation is called the *parallax*. The difference in direction of a star as seen from two places on the earth's orbit, separated by the distance from the earth to the sun, is the parallax of the star. Or, in other words, it is the angle subtended by the radius of the earth's orbit at the distance of the star.

Notwithstanding a great many attempts to find parallaxes

of the stars no one succeeded, because of their inconceivably great distances, until between 1830 and 1840. It should be mentioned, however, that errors had misled several earlier observers to suppose that they had succeeded in the quest. There are now about 40 stars known whose parallaxes are so large that it has been found possible to measure them. It follows, therefore, either that the earth revolves around the sun, or that 40 stars have small actual motions which give them the same apparent motions as they would have if the earth did move.

143. Revolution of the Earth proved by the Spectroscope. — If the earth revolves around the sun, its radial velocity with respect to any star not at the pole of the ecliptic varies during the year. For simplicity, consider a star in the plane of the ecliptic. At one time of the year the earth is moving toward it; six months later it is receding from it at nearly the same rate. If the radial velocities at the two epochs are measured by the spectroscope, they will differ by twice the velocity of the earth, whether the star has a radial velocity or not.

Observations of radial velocities, particularly in the last ten years, have abundantly verified the motion of the earth. They are made, however, for more important reasons, which will be discussed at the proper place. They do not depend upon the laws of motion or the parallaxes of the stars, but they are related to the aberration, because, like it, they depend upon the velocity and character of light waves.

It follows that no scientific theory is more firmly established than that the earth revolves around the sun, and it must be regarded as one of the permanent acquisitions of science.

144. The Shape of the Earth's Orbit. — Kepler found that the orbit of Mars is an ellipse, and he supposed that all the other orbits are of the same character. It can be proved very simply for the earth by observing the apparent

angular motion of the sun from day to day and its apparent angular diameter. Since the apparent diameter of a body varies inversely as its distance, the orbit may be plotted to any desired scale by first laying down lines making angles with each preceding one equal to the corresponding angular motion of the sun, and then laying off on these lines distances which are inversely proportional to the apparent diameter of the sun. This could not have been done with sufficient accuracy to prove anything before telescopes came into use. It must be understood that the method is not now of any particular importance. It simply illustrates the principles involved.

The earth's orbit is very nearly round, the eccentricity being only 0.01677. Half the length of its major axis is 93,000,000 miles, and is called the *mean distance* of the earth from the sun, but it is not the average distance. The average distance depends upon the eccentricity, and is always a little greater than the mean distance.

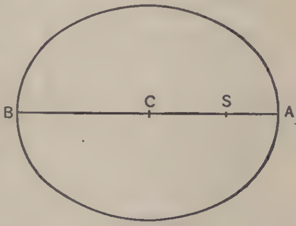


FIG. 68.

Since the eccentricity $e = \overline{CS} \div \overline{AC}$, it follows that $\overline{CS} = 1{,}560{,}000$ miles. The point A is called the *perihelion* (Greek for "around the sun"), and B the *aphelion* (Greek for "away from the sun"). When the earth is at perihelion it is more than 3,000,000 miles nearer the sun than it is when it is at aphelion. The eccentricity of the ellipse shown in the figure is greatly exaggerated.

145. The Obliquity of the Ecliptic. — The angle between the plane of the equator and the plane of the ecliptic is called the *obliquity of the ecliptic.* The celestial equator is defined by its being 90° from the celestial pole, and the ecliptic is defined by the apparent path of the sun. Suppose the declination of the sun is observed daily. Its greatest

declination north or south, which it has when it is 90° from the equinoxes, is the obliquity of the ecliptic.

Thus, in Fig. 69, the arc $\overline{EN}$, or $\overline{WS}$, measures the angle between the two planes when E and N are 90° from the vernal equinox, ♈.

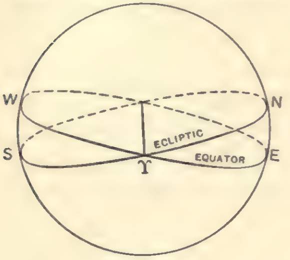


FIG. 69.

Eratosthenes was one of the first to attempt to measure the obliquity of the ecliptic, and he obtained the value 23° 51′. Modern observations have shown that it is about 23° 27′.

146. Precession of the Equinoxes. — The precession of the equinoxes is a phenomenon depending upon the earth's rotation, but it is more conveniently discussed at this place because of its connection with the topics now under consideration.

When Hipparchus compared his observations of some of the stars with those which had been made more than a century earlier by Aristyllus and Timocharis, he found that their latitudes were unchanged, but that their declinations had varied. He interpreted this as meaning that the position of the equator had changed, and from a discussion of the differences in the declinations and right ascensions and observations of the sun, he inferred that the obliquity of the ecliptic remains practically constant, while there is a precession, or retrograde motion, of the equinoxes along the ecliptic at the rate of 36″ a year. This can easily be seen from Fig. 70. Let S be the position of a star, E_1 the position of the equator at the first epoch, and E_2 its position at a later epoch. The longitude and latitude at the first epoch are $\overline{♈_1B}$ and $\overline{BS}$ respectively, and at the second epoch $\overline{♈_2B}$ and $\overline{BS}$ respectively. That is, the longitude changes because of a change in the point from which it is counted, while the

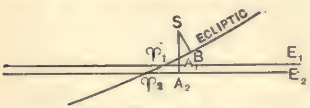


FIG. 70.

latitude remains constant. The right ascensions and declinations at the two epochs are respectively $\overline{\Upsilon_1 A_1}$, $\overline{A_1 S}$, and $\overline{\Upsilon_2 A_2}$, $\overline{A_2 S}$.

The precession of the equinoxes is justly considered as being one of the great discoveries of Hipparchus, although the rate he found was wrong. Modern observations show that precession is at the rate of 50.2″ annually, from which it follows that the equinoxes will make a complete revolution in 25,800 years.

It can be seen from Fig. 69 that the precession arises from the fact that the plane of the equator revolves westward while keeping the same inclination to the ecliptic. Otherwise stated, the pole of the equator describes a circle with a radius of 23° 27′ around the pole of the ecliptic as a center. But the pole of the equator is defined by the axis of the earth. Therefore the earth's axis describes a double circular cone whose angle is twice 23° 27′. At the time of Hipparchus the pole of the equator was about 12° from Polaris, while now it is within 1.25° of this star. As was stated in Art. 47, in 12,000 years it will be very near Vega (Alpha Lyræ).

147. Cause of Precession of the Equinoxes. — As was seen in the last article, the precession is due to a rotation of the earth's axis around a mean position; or, that which is equivalent, to a backward rotation of the plane of the earth's equator in such a way as to make a constant angle with the plane of the ecliptic. The cause of this change was first explained by Newton in the *Principia*.

The precession of the equinoxes is produced by the attraction of the moon and sun on the earth's equatorial bulge, about four-fifths of the result being due to the action of the moon. For simplicity, consider the effects of the moon's attraction alone. Let Fig. 71 represent the earth, with the equatorial bulge much exaggerated. Let E be the largest sphere which can be cut out of the oblate earth. The moon's attraction has no direct effect on the rotation of this part.

For simplicity, consider only those parts of the equatorial bulge which are on the side of the earth toward the moon and on the opposite, and call them A and B respectively. It is easy to see that the moon tends to bring A down into the line joining the moon and the center of the earth. It can be shown that the moon also tends to pull B up into the same line produced. If the earth were not rotating, it would be tilted over into a position of equilibrium. Suppose, however, it is rotating so that A moves *from* the reader as he looks at the diagram perpendicularly to the paper. Now a particle at A has a velocity perpendicular to the plane of the paper, and it is also subject to a force toward the line $\overline{EM}$. The result is that it tends to move a little to the right of the perpendicular to the paper. But the line of motion of this point and the center of the earth define the plane of the equator, which therefore has a tendency to revolve in the retrograde direction. The force acting on B and its motion give a tendency to rotation of the equatorial plane in the same direction.[1]

Fig. 71.

The equatorial bulge is so small, the earth is so large and rotates so fast, and the forces involved are so feeble that the precession is very slow. Besides, it is very irregular, for it is zero when the sun and moon are on the celestial equator, and is greater the farther they are from it. The moon crosses the equator twice every month and the sun crosses it twice a year. The disturbances due to the moon and sun vary and combine in a very complicated way, and depend also upon the varying distance of these bodies.

148. Nutation. — The moon's orbit is inclined to the plane of the ecliptic about 5°, and the precession of the equinoxes which the moon produces is with respect to its orbit instead

[1] For a more complete, though quite elementary, explanation see *Introduction to Celestial Mechanics*, p. 237.

of with respect to the ecliptic. The line of the intersection of the plane of the moon's orbit with that of the ecliptic rotates backward on the ecliptic, completing a revolution in 18.6 years. The inclination of the planes to each other remains constant except for short periodic changes. The amount of the precession of the equinoxes depends upon the position of the plane of the moon's orbit, and this introduces a variation in the precession with a period of 18.6 years, called the *nutation*. It causes a sort of nodding (nutation) of the position of the pole with respect to its mean path through an arc whose maximum value is about 9.2″.

Nutation was discovered by Bradley from the same observations that led him to the knowledge of aberration. However, he did not publish his conclusions until he had extended his observations over the full 18.6-year period from 1727 to 1747. D'Alembert, a famous French mathematician, gave an exposition of the theory in a treatise on precession and nutation which he published in 1749.

The motion of the plane of the moon's orbit is due to the sun's attraction. Thus, the sun affects the earth's axis of rotation directly, and also indirectly, through its first disturbing the orbit of the moon, which produces a direct precession. The coördination and explanation of such complicated phenomena as these illustrate the value and far-reaching connections of the laws of motion and Newton's law of gravitation.

149. The Seasons. — The seasons are due to the varying amounts of light and heat received from the sun at different times of the year. Every one has observed that the days are longer, and that the sun's rays strike the earth more nearly perpendicularly, in the summer than in the winter. The problem here is to explain the law of these changes from the motions of the earth which have been discussed.

The sun travels (apparently) very slowly along the ecliptic, on the average a little less than a degree a day, or 360° in 365.25 days. Consequently its diurnal motion, which is due

to the rotation of the earth, is for a day very nearly in a declination circle. In Fig. 72, *NESW* is the plane of the horizon, and these four points are the cardinal points. The curve *B* is the equator, half of which is above the horizon, which it intersects in the east and west points. The earth is at *O*, and the line $\overline{OP}$ is in the direction of its axis. When the sun is at one of the equinoxes, its diurnal motion is in *B*, and the days and nights are of equal length whatever the latitude of the observer. When the sun is north of the celestial equator, its diurnal motion is in a circle parallel to *B*, as *C*. In this case it is above the horizon more than half of the 24 hours. Similarly, when it is south of the equator, it moves in a circle parallel to *B*, as *A*, and is less than half of the 24 hours above the horizon. This figure shows clearly why the daily periods of sunshine vary in length.

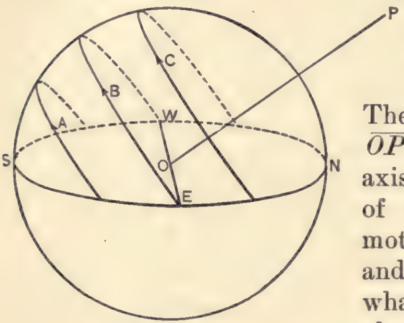


FIG. 72.

The variation of the angle at which the sun's rays strike the surface is illustrated in Fig. 73. For the northern hemisphere the seasons summer, autumn, winter, and spring are when the earth is at *A*, *B*, *C*, and *D* respectively. When the earth is at *A* the rays fall perpendicularly on the circle 23° 27′ north of the earth's equator. This is called the Tropic of Cancer, because the sun is at this time in the sign of the zodiac called Cancer. There is an area whose radius is 23° 27′, bounded by the Antarctic circle, on which the sun does not shine.

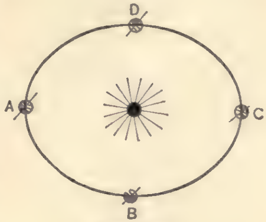


FIG. 73.

When the earth is at *C*, conditions are reversed with respect to the northern and southern hemispheres. The circle on which the sun's rays fall perpendicularly is the tropic of Capricorn, and the unilluminated part of the earth is the Arctic zone.

When the earth is at *B* or *D*, the sun is on the celestial equator and the days and nights are of equal length all over the earth.

The sun's motion northward and southward is not uniform. Suppose that part of the celestial sphere containing the equator and the ecliptic is represented on a plane, Fig. 74. Let *V* be the vernal equinox and *A* the autumnal equinox, and suppose for simplicity that the sun travels uniformly along the ecliptic. Consider the northward and southward motions. It is evident from the figure that the change in declination is most rapid when the sun is at one of the equinoxes. There are two points, *S* and *W*, at which the declination does not change, and near which it is for some time nearly stationary. These points are called the summer and winter *solstices* (places where the sun stands still).

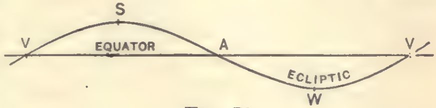


Fig. 74.

The chief reason why the sun's rays are less effective when they strike the surface obliquely is that those filling a given prism are spread out over a larger surface. Thus, the prism of rays in Fig. 75 would cover the surface *ABCD* striking perpendicularly, and the greater surface *abcd* striking obliquely. The more they are spread out the less effective they are in heating the surface. This is illustrated by the difference in the intensity of the sun's rays at midday and near sunset.

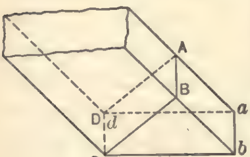


Fig. 75.

Of course, when the rays descend obliquely they pass through a greater thickness of atmosphere and are more absorbed, but this is not very important until they become quite oblique, for most of the heat absorbed by the atmosphere is near the place where it otherwise would have struck the surface.

150. Relative Amount of Sunlight in Different Latitudes. — For an observer at the earth's equator the celestial equator *B*, Fig. 72, cuts the horizon perpendicularly, and every circle parallel to it, as *A* or *C*, is bisected by the horizon. The result is that, neglecting refraction, any point on the equator receives 12 hours of sunlight every day in the year. At the earth's poles the sun shines half the year, but in this case uninterruptedly. The result is that the equator and poles receive the same number of hours of sunlight in a year. It can be shown that the same thing is true for every intermediate latitude, that what is gained daily at one time is lost at another. It follows that as far as the length of sunlight in a year is concerned, the result is the same as though the equator and ecliptic coincided.

One might be led to suspect that every place receives the same amount of light and heat it would if the equator and ecliptic coincided, but such is not the case. To show that this statement is true, consider, first, points on the equator. Since the period of sunshine is every day the same, the most light and heat are received daily per unit area when the sun passes directly overhead. But when the equator and ecliptic are inclined to each other this happens only twice a year, while it would happen every day if they coincided. Consequently, the earth's equator receives less heat than it would if the obliquity of the ecliptic were zero. Since the angle between the equator and the ecliptic does not diminish the amount that the whole earth receives annually, it follows that some latitude receives more heat than it would if the obliquity of the ecliptic were zero.

Consider a point at the earth's pole. If the ecliptic and the equator were coincident, the sun would be always on the horizon and the level surface would receive infinitely little light and heat. But with the present obliquity the pole receives a very appreciable amount when the altitude of the sun is near 23° 27′. In intermediate latitudes the subject becomes very complicated because of the varying altitudes of the sun and lengths of the days. But the result is that the obliquity of the ecliptic causes the polar regions to receive more heat, and the equatorial regions less, than they would otherwise.

Another interesting fact is that, other things being equal, the highest temperatures do not occur at the equator. For, compare the conditions at the equator with those at a point in the equatorial zone, but near one of the tropics. At the equator every day has 12 hours of sunshine, and the sun passes near the zenith only a few times. On the other hand, near the tropics every day is longer than 12 hours for six months, and the sun passes near the zenith many days in succession, because the sun's motion in declination is slow when it is near the solstices.

The obliquity of the ecliptic has two general effects. *The first is to cause variations in the climate at any one place, and the second is to decrease the differences in climate in different latitudes.*

151. The Lag of the Seasons. — From the astronomical point of view, March 21 and September 23, the times at which the sun passes the two equinoxes, are corresponding seasons. The middle of the summer is when the sun is at the summer solstice, June 21, and the middle of the winter when it is at the winter solstice, December 21. But from the climatic standpoint March 21 and September 23 are not corresponding seasons, and June 21 and December 21 are not the middle of summer and winter respectively. The climatic seasons lag behind the astronomical.

The cause of the lag of the seasons is very simple. On June 21 any place north of the tropic of Cancer is receiving the most heat daily that it gets any time in the year. On account of the blanketing effect of the atmosphere, less is radiated than is received; consequently the temperature continues to rise. But after that date less heat is received daily; on the other hand, more is radiated daily, for the hotter a body gets the faster it radiates. In a few weeks the loss equals, and then exceeds, that which is received, after which the temperature begins to decrease. The reasons are corresponding for all the other seasons.

If there were no atmosphere and if the earth radiated heat as fast as it acquired it, there would be no lag in the seasons. In high altitudes, where the air is thin and dry, this condition is partially realized and the lag of the seasons is less, though the phenomenon is very much disturbed by the great air currents which do so much to equalize temperatures.

152. Effect of the Eccentricity of the Earth's Orbit on the Seasons. — The earth moves around the sun in an ellipse in such a way that Kepler's first two laws are fulfilled. It follows that it makes the half revolution including the perihelion in less time than it does the remainder. It is found from observations that the earth is at the perihelion point on December 31, though the date may vary a day because of the leap year.

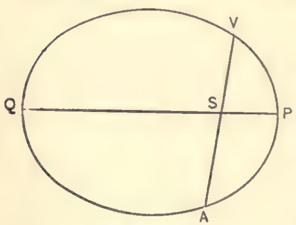


FIG. 76.

Thus, in Fig. 76, P is the perihelion point, Q the aphelion point, and $\overline{AV}$ the line of intersection of the plane of the equator and the plane of the ecliptic. The sun is apparently at the vernal equinox when the earth is at V, and at the autumnal equinox when the earth is at A. The angle PSV is about 80°, and ASP 100°. Since the area of the sector APV is less than that of the sector VQA, it fol-

lows from Kepler's law of areas that the earth passes over the arc APV in a shorter time than it does over VQA.

Consider the actual seasons. The earth is at V on March 21 and at A on September 23 (the dates may vary a day because of the leap year). The interval from March 21 to September 23 is 186 days, and the interval from September 23 to March 21 is 179 days.[1] Therefore in the northern hemisphere the summer is over seven days longer than the winter, while the opposite is true in the southern hemisphere. But to counterbalance this the distance to the sun is greater during the summers of the northern hemisphere than it is during those of the southern hemisphere. Likewise the winters in the northern hemisphere are shorter than those in the southern hemisphere, but the greater amount of heat received daily, because of the nearness to the sun, makes up for the deficiency in length of the season. It is a fact, although it can not be proved by elementary processes, that corresponding latitudes in the northern and southern hemispheres receive the same total amount of heat annually, and also during any corresponding seasons or parts of seasons.

Although the eccentricity of the earth's orbit does not cause a difference in the whole amount of heat received in corresponding latitudes in the two hemispheres, yet it does make a difference in the way the heat is distributed throughout the year. A place at a given latitude in the northern hemisphere receives less heat daily during its summer than is received by a place of the same southern latitude during its summer. In the winters the northern latitude receives more heat daily than is received by corresponding southern latitudes. The result is that the variation in the daily amount of heat received is less in the northern hemisphere than it is in the southern.

Consider a point on the earth north of the tropic of Cancer and another having an equal southern latitude. The northern

[1] The two intervals are very nearly 186¼ and 179 days respectively.

one receives the greatest amount of heat daily at the summer solstice and the southern one at the winter solstice. But the amount of heat received varies inversely as the square of the earth's distance from the sun. The earth is near its aphelion at the summer solstice and near its perihelion at its winter solstice. Using the numbers previously given (Art. 144), it is found that the greatest amount of heat daily received at the southern station is nearly 7 per cent greater than the greatest amount daily received at the northern station.

Thus, other things being equal, the climate of the southern hemisphere would have greater extremes than that of the northern. But, unless the unequal distribution of heat has some effect on the loss of heat by radiation into space, the average temperatures in the two hemispheres would be the same. One can not expect to find these theoretical results very exactly verified, for the climate is greatly affected by local conditions such as ocean currents and mountain ranges. Since the southern hemisphere is much more largely covered with water than the northern, the temperature would be more uniform, other things being equal. Probably the effects of the large water surface more than offset those resulting from the varying distance of the earth from the sun.

153. Effect of a Change in the Eccentricity of the Earth's Orbit. — As will be explained in the next chapter (Art. 169), the attractions of the other planets for the earth change the eccentricity of its orbit for many thousands of years in one direction, and then for many thousands of years in the other. While this is going on the length of the major axis remains constant except for small short-period variations. The effects that these changes have on the amount of heat received by the earth is a question which can be solved only by a mathematical discussion.

It can be shown[1] that the amount of heat and light received per unit area by any planet is proportional to the recip-

[1] See *Introduction to Celestial Mechanics*, p. 143, problem 8.

rocal of the product of the major and minor axes of its orbit. If the major axis remains constant while the eccentricity increases, then the minor axis decreases, for the ellipse becomes more flattened. Hence, when the eccentricity of the earth's orbit increases, the amount of heat received becomes greater; and, in agreement with the discussion of the preceding article, the seasons in one hemisphere become more uniform and in the other more extreme. The attractions of the other planets cause the major axis of the earth's orbit to rotate (Art. 169), and it follows from this that in the course of time the earth will pass its perihelion at all the various seasons. Consequently, in the long run, both hemispheres have the extreme seasons.

Geologists have not come to any general agreement respecting the cause of the ice ages. The English geologist Croll advanced the theory, and supported it in a book devoted to the discussion, that they have been due to the very unequal distribution of heat throughout the year at the epochs of great eccentricity of the earth's orbit. He held this view notwithstanding the fact that the total amount of heat received would have been greater at such times, as has been explained above. A fatal objection to the theory is that the intervals between the successive ice ages would have been, if this were the cause, very much longer than the geological data indicate.

QUESTIONS

1. Is the apparent position of the sun affected by aberration?

2. Would sound from a distant source suffer an aberration if the hearer were riding rapidly on a train? Sound travels at the rate of about 1000 feet per second. If there is an aberration of sound, what would be its amount for a hearer riding perpendicularly to its direction at the rate of 30 miles per hour?

3. Does the motion of the star observed also produce an aberration?

4. The parallax of the nearest known star (Alpha Centauri) is 0.75″. How far away would an object an inch in diameter have to be put to subtend this angle?

5. When the parallax of a star is less than 0.1″, it can not be measured with certainty. What is the lower limit to the distances of the stars whose parallaxes can not be measured?

6. The effect of parallax is to make a star apparently describe a small orbit. How do the apparent motion and position in this orbit compare with those in its aberrational orbit?

7. Could a star be selected in such a position that the aberrational and parallactic displacements would be entirely distinct from each other even if they were of equal amounts?

8. Show that in proving the motion of the earth by means of the spectroscope it makes no difference whether the star observed is moving or not.

9. Are the longitudes of all stars affected in the same way by precession? Are the right ascensions and declinations of all affected in the same way?

10. Draw three figures corresponding to Fig. 72 showing the effects of a change of the observer's latitude on the diurnal circles of the sun.

11. Does the oblateness of the earth have any effect at all on the width of the Arctic and Antarctic zones?

12. What effect would an increase in the amount of carbon dioxide in the atmosphere have on the amount of lagging of the seasons?

154. The Period of an Inferior Planet. — The period of a planet may be defined in two ways. One is the time it takes the planet to move from any relative position with respect to the sun as seen from the earth to the same relative position again. The simplest case would be from conjunction with the sun to conjunction again, with motion in the same direction; or, if the moon were considered, from new moon until new moon again. Such a period is called a *synodic period* (Greek "with" and "way," meaning in the same way, or direction). Another period is the actual time it takes the planet to revolve around the sun; or, that which would be an apparent revolution with respect to the stars as seen from the sun. Such a period is called a *sidereal period* (stars, or constellation, period).

Since the synodical period depends upon the motion of the earth as well as that of the planet, while the sidereal period depends upon the motion of the planet alone, it is clear that

the latter is what one would use ordinarily in speaking of the time of revolution. Of the two, the synodical period can be more easily found from observations, and from it the sidereal period can be computed, as will be shown.

Let Fig. 77 represent the orbits of an inferior planet and the earth. Suppose the sun, the planet, and the earth are in the same line, with the planet between the earth and the sun, as SP_1E_1. The planet is then said to be in *inferior conjunction.* It moves faster than the earth and apparently passes by the sun from east to west. After a certain interval of time the earth will have moved on to E_2 and the planet around to P_2. The planet is now said to be in *superior conjunction.* In this case it apparently passes the sun from west to east. After a still longer interval the earth and planet are at E_3 and P_3, and the planet is in inferior conjunction again. The time that it has taken the earth to go from E_1 to E_3 is the synodic period of the planet.

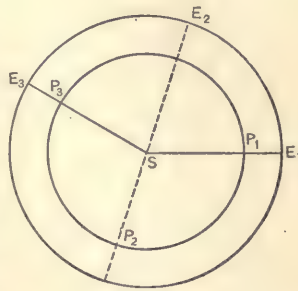


FIG. 77.

Let E represent the period of the earth expressed, say, in days. Let S and P be the synodic and sidereal periods of the planet expressed in the same units. Then $\frac{1}{E}$ is the fraction of a whole revolution that the earth moves in one day. Similarly, $\frac{1}{P}$ is the fraction of a whole revolution that the planet moves in one day. The difference $\frac{1}{P} - \frac{1}{E}$ is the fraction of a revolution gained by the planet on the earth in one day. But the planet gains a whole revolution on the earth in the synodical period of S days. Therefore the fraction

of a revolution gained in one day is $\frac{1}{S}$. Equating this to the previous expression for the same thing, we have

$$\frac{1}{S} = \frac{1}{P} - \frac{1}{E}.$$

This is the relation connecting the synodic period, the sidereal period, and the year. When two of these quantities are known, the third can be found. Thus, when S and E are known, it is found, by clearing of fractions, that P is given by the equation

$$P = \frac{S \times E}{S + E}.$$

There is a simple and direct way of finding the sidereal period, but it is not of much value. The orbits of all the planets are inclined to the plane of the ecliptic. The interval of time between the instant at which the planet crosses the plane of the ecliptic, say, from south to north, until it crosses the same way again is a sidereal period. This is not, however, absolutely exact, for the places where the planet crosses very slowly change.

155. The Period of a Superior Planet. — A superior planet has both a synodical and a sidereal period, defined the same as in the case of an inferior planet. It has but one conjunction with the sun, which is called simply "conjunction," although it is a superior conjunction. When it is directly opposite to the sun, it is said to be in *opposition*. Then the earth and the planet are the nearest together that they ever get.

Let the same notation as before be used for the periods. But in this case the earth moves faster than the planet, so that $\frac{1}{E} - \frac{1}{P}$ is the fraction of a revolution gained by the earth in one day. Hence the equation connecting the length of

the year, the synodical period, and the sidereal period is

$$\frac{1}{S}=\frac{1}{E}-\frac{1}{P}.$$

When S and E are known, P is given by the equation

$$P=\frac{S\times E}{S-E}.$$

Since the sidereal periods of the distant superior planets are very long, it is not easy to find them by direct observations. But the greatest synodical period is only a little over two years in length, and therefore the indirect process of finding the sidereal period from the synodic is more convenient. It should not be inferred that astronomers have to wait a whole synodical period in order to find the sidereal period of a planet. The powerful mathematical processes now in use enable them to solve the problem with considerable accuracy from three observations of apparent direction separated by intervals of a few days.

156. The Relative Distance of an Inferior Planet. — The relative distances from the sun of all the planets of the solar system can be found without knowing any actual distance. This enables us to draw a map of the whole system in the correct proportions. Hence, when one actual distance is known, all the others can be found.

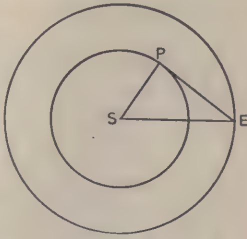


FIG. 78.

In Fig. 78 S, E, and P represent the sun, earth, and an inferior planet. The angular distance of the planet from the sun as seen from the earth is called its *elongation*. When the planet has its greatest elongation from the sun, the triangle SPE is right-angled, with the right angle at P. The angle at E is observed; consequently all the angles of the triangle are known. If

one side were known, the others could be computed by trigonometry. To get the relative distance assume that $\overline{SE}$ has an arbitrary value, as 1. Then $\overline{SP}$ will be a certain fraction of it which can easily be computed. For example, if the angle at E were 30°, the distance $\overline{SP}$ would be $\frac{1}{2}$. Without the use of trigonometry one can draw a triangle having the proper angles and find the relative distances by measurement.

157. The Relative Distance of a Superior Planet. — The relative distance of an inferior planet was found from its greatest elongation; the relative distance of a superior planet can be found from the same principle a little differently applied.

Suppose the planet is in opposition at a certain epoch, as at P_1 with the earth at E_1 (Fig. 79). After a certain interval of time the earth will have its greatest elongation as seen from the planet. As seen from the earth the planet will be apparently 90° from the sun, which determines the time when the relations are fulfilled. Suppose the planet is at P_2 and the earth at E_2. Suppose the period of the planet has been determined previously; then the angle P_1SP_2 through which the planet has traveled is known. The angular motion of the planet as seen from the earth is QE_2P_2, which equals the angle at A. Now, the angle at P_2 equals the angle P_1SP_2 minus the angle at A, and is therefore known. Having computed the angle at P_2, the relation between $\overline{SP_2}$ and $\overline{SE_2}$ can be found by trigonometry precisely as in the case of the inferior planets.

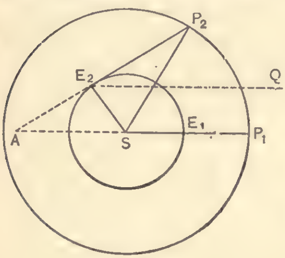


FIG. 79.

The earth has its greatest elongation from the sun as seen from the planet when the planet as seen from the earth is 90° from the sun, or *in quadrature*. This determines the time to make the observations upon which the computation

is based. At this time the apparent position of the planet is farthest behind that which it would have as seen from the sun, the angle being precisely that at P_2.

158. The Elements of an Orbit. — The position of a planet in space at any time depends upon the position, shape, and size of its orbit, and the time it was at some position in its orbit, as at the perihelion point. These quantities, which enable one to compute the position of a planet at any time, are called the *elements* of its orbit.

If a planet were started in the vicinity of the sun (*i.e.* in the solar system), its path would depend upon the point at which it was started and the velocity and direction of projection; for, if any one of these things were changed, the orbit would be different. Let us see how many things the orbit depends upon as defined in this way. The point from which the body is projected depends upon three things; for example, the right ascension and declination as seen from the sun and the distance from the sun. The direction of projection depends upon two things, just as the direction to a star involves two coördinates. The velocity of projection is a single thing. There are, therefore, six things upon which the path of the body depends; and, since any point in an orbit may be thought of as being a starting point, the position always depends upon six distinct things. Consequently an orbit has six elements.

The plane of a planet's orbit is defined by its position with respect to the plane of the ecliptic, and it takes two things to express this relation. One is the line in which the two planes intersect, called the *line of nodes*, and the other is the angle between the two planes, called the *inclination of the orbit.* The place, or node, at which the planet crosses the plane of the ecliptic from south to north is called the *ascending node* (symbol, ☊), and the other, the *descending node* (symbol, ☋). The elements given are the longitude of the ascending node, which is the angular distance along the

ecliptic from the vernal equinox to the ascending node, and the inclination of the orbit. The symbols are ☊ and i respectively.

The next element to be defined is the direction that the major axis of the ellipse has in the plane of the orbit. It is defined by the angle obtained in starting from the ascending node and counting forward in the direction of motion of the planet in the plane of motion to the perihelion point. The angular distance is called the *longitude of the perihelion* from the node, and its symbol is ω (Greek letter omega).

In Fig. 80, $\overline{AB}$ is the line of nodes, and $\overline{SV}$ the line to the vernal equinox. The elements ☊, i, and ω are shown in the diagram. It must be remembered that ☊ is counted in the plane of the ecliptic, and ω in the plane of the orbit. Sometimes in place of ω the element π (Greek letter pi) is used, which is defined by the equation

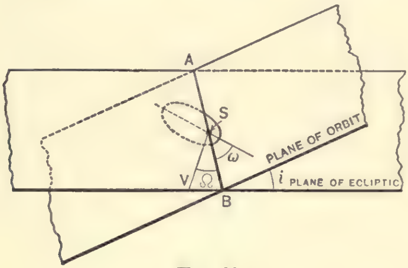


FIG. 80.

$$\pi = ☊ + \omega.$$

The next element is the half major axis, denoted by a, which defines the size of the orbit.

The shape of the orbit is defined by its eccentricity, denoted by e.

The position of the body in its orbit is defined in two ways. One is to give the time at which it is at the perihelion point. This time is denoted by T. The other is to give its longitude at an arbitrary epoch. This is denoted by ϵ (Greek letter epsilon).

The six elements of a planet's orbit are, therefore, ☊, i, ω, a, e, T (or ϵ).

One remark remains to be added. The motion of a planet

depends to some slight extent upon its mass, for its attraction moves the sun. Therefore, to get the motion of the planet with respect to the sun with great accuracy, the mass of the planet (or something else equivalent) must also be known.

QUESTIONS

1. Is the synodic period of an inferior planet as seen from the earth the same as the synodic period of the earth as seen from the planet?

2. The sidereal periods of all the planets are longer the farther they are from the sun; how do the synodic periods of the inferior planets depend upon their distance from the sun? How is it in the case of the superior planets?

3. Suppose the synodic period and the year are of the same length. What is the sidereal period in the case of an inferior planet? In the case of a superior planet?

4. Suppose there is a superior planet whose sidereal period is 5 times that of an inferior planet and whose synodical period equals that of the inferior planet. What is their common synodical period and what are their sidereal periods?

5. In applying the method of Art. 157, is it necessary to make the observations when the planet is in quadrature?

6. Suppose the periods of the earth and an inferior planet are known. Show that their relative distances are defined by a triangle whose angles can be found from observing the time of conjunction and the elongation at any other time.

7. If a body moves subject to no forces (*i.e.* in a straight line), how many elements has its orbit?

CHAPTER VII

THE LAW OF GRAVITATION

159. Necessary Preliminary Work. — Since a law of nature is a description of the way certain natural phenomena succeed each other, it follows that it must be based upon preliminary experiments and observations. The law of gravitation was deduced from two distinct classes of results, both of which have been to some extent discussed. In the first place, it was derived from the planetary motions, which were discovered by Kepler. They were the final results of observations and discussions which were begun in prehistoric times, and whose history has been sketched. In the second place, it depended essentially upon the laws of motion (Art. 120), which seem to have been unknown until the time of Leonardo da Vinci and Galileo, and which were first stated in their generality by Newton. The results of the first class were obtained from an immense number of observations of the heavenly bodies and most laborious computations; those of the second class, from experiments on the motions of bodies at the surface of the earth, and from a rare insight into physical phenomena.

The work which led to results of the first class demanded especially patience and skill in observing, while that which secured the results of the second class depended more upon the imagination and intuition. The proof of the law of gravitation from them involved the rarest powers both of invention and of mathematical deduction. In fact, every discovery in science depends upon two things, observation and intuition. Unless based upon observations, a theory

has no value. Unless the results of observations are unified by a theory, they are of little value.

160. Discovery of the Law of Gravitation. — Isaac Newton (1643[1]–1727), the discoverer of the law of gravitation, was born the year after Galileo died. In 1665 he graduated from Cambridge, having shown in the last years of his university course extraordinary talent in mathematics and natural science. He at once entered upon a period of unparalleled intellectual activity. In the first two years he had discovered the binomial theorem, laid the basis of his work on fluxions, which has grown into the modern calculus, developed his theory of colors, and conceived the idea that the moon is held in its orbit by forces of the same nature as those which cause bodies to fall at the surface of the earth. During these years he was in retirement at his home in Woolsthorpe, because of the plague which was raging in England. He afterward described this as being the most productive period of his life.

At the very beginning of his work Newton must have been in possession of the first law of motion, which he very probably discovered independently for himself, and which constituted the first step in his discovery of the law of gravitation. In accordance with it the moon must be acted upon by exterior forces, for it does not describe a straight line. Its motion is like that of a projectile, only the velocity is so great that it goes entirely around the earth without hitting it. This is illustrated in Fig. 81, where E represents the earth, and P a high elevation on it. Suppose a body is projected horizontally. For a certain initial velocity it will strike the surface at A; for greater velocities, at B and C; and

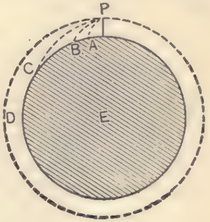


FIG. 81.

[1] Dec. 25, 1642, Old Style, or Jan. 4, 1643, according to the present calendar.

for a sufficiently great velocity it will go entirely around the earth in the curve *D*. The orbit of the moon is like the curve *D*, except it is very far from the earth.

It is said that the young man Newton was one day lying under an apple tree when an apple fell, and that it then occurred to him that the moon, which was visible in the sky beyond, fell from the straight line in which it tended to move because of the same force as that which made the apple descend. This second step was a distinct addition to the idea that the moon revolved around the earth because it was acted on by *some* force, for the unity of the forces was perceived. The third step in the discovery was to find how the force varies with the distance.

If the moon is held in its orbit by a force directed toward the earth, it is reasonable to suppose that the planets describe their orbits under the influence of forces directed toward the sun. Assuming that their orbits were circles, Newton proved, from Kepler's third law of motion (Art. 138), that the forces which act upon them vary inversely as the squares of their distances from the sun. He then applied the same theory to the moon. He knew that the acceleration of gravity at the earth's surface is about 32 feet per second, and that the distance of the moon is about 240,000 miles. It was not a difficult problem to find from these data and the size of the earth and period of the moon, whether the earth's attraction at its surface and at the moon vary inversely as the squares of the distances. At the time Newton carried out this computation the size of the earth was not very accurately known, and he found considerable disagreement in the results. Accordingly, he laid the subject aside for 14 years, when, learning of Picard's measures in France, he returned to the question and verified the relation for the earth and moon.

The next stage in Newton's work on the law of gravitation began in 1679, when he proved that *if a body moves sub-*

ject to any force constantly directed toward a fixed point, then the radius vector from that point to the body describes equal areas in equal intervals of time. And conversely, *if a body moves so that the law of areas is fulfilled for any point taken as an origin, then it moves subject to a force which is constantly directed toward that point.* Consequently, it follows from Kepler's second law of planetary motions that the forces to which the planets are subject are constantly directed toward the sun.

At the same time Newton proved the important result that *if a body moves in an ellipse around a central force situated at one of its foci, then the force in different parts of the orbit varies inversely as the square of the distances from the focus.* Hence it follows from Kepler's first law that the force to which any planet is subject varies inversely as the square of its distance from the sun. Kepler's third law shows that the accelerations of all the planets would be the same if they were at the same distance from the sun.

Newton was very indifferent about gaining a reputation and feared being drawn into controversies, so he did not publish immediately any of his results. In the meantime Hooke, Wren, and Halley had been struggling with the problems of central forces, and Halley, at least, had proved from Kepler's third law, regarding the orbits as circles, that the forces toward the sun vary from planet to planet inversely as the squares of their distances. He discussed with Hooke and Wren what the orbit of a body moving under such a force necessarily would be, but none of them could solve the problem. Later in the same year (1684) he visited Newton, obtained from him the correct results, recognized the brilliancy of his genius, and became the sincere and devoted follower of the great astronomer to the end of his life. He prevailed upon Newton to prepare his results for publication in book form. The great work, *Philosophiæ Naturalis Principia Mathematica* (*The Mathe-*

matical Principles of Natural Philosophy), commonly called simply the *Principia*, was composed in 1685 and 1686, and appeared in July, 1687.

It is impossible to give in a single paragraph any adequate conception of the great discoveries contained in the *Principia*, or of the exceptional talents required to develop them. In an introduction of about twenty pages Newton defined mass, momentum, inertia, forces, the laws of motion, and referred to his experiments in support of them. Then in Book I he treated of the motions of bodies in general on the basis of his general principles. In the eleventh section he gave the basis for his treatment of the motion of the moon, which is one of the most difficult problems in celestial mechanics. Airy has said that it is the greatest chapter on physical science ever written. However that may be, it is certain that his results have scarcely been extended by the geometrical methods which he employed in the more than 200 years which have elapsed since their publication. The reason is due, at least partly, to the fact that investigators have universally adopted the more powerful methods of analysis introduced by the French, but whose foundations Newton himself laid. In Book II he treated of the motions of bodies in resisting media and related problems. In Book III he applied his mathematical results to the phenomena of the solar system. After showing that the moon is attracted by the earth, the planets by the sun, and the moons of Jupiter by Jupiter, he proceeded in Proposition VII to the general law of gravitation that *every particle in the solar system attracts every other particle with a force which is proportional to the product of their masses and which varies inversely as the square of their distance apart.* Newton confessed his ignorance as to the cause of this attraction, and it is not yet known.

161. Value of the Law of Gravitation. — The value of a general principle depends upon the number of known phe-

nomena it coördinates, and upon the ability it gives one to make predictions. Let us consider the law of gravitation in these respects. Newton showed in the *Principia* how every known phenomenon of the motions, shapes, and tides of the bodies of the solar system could be derived from it. It became in the hands of his successors one of the most valuable means of discovery; and, if we are ever able to find out with certainty what has been the evolution of the solar system and what it will be in the future, we shall use the law of gravitation as an essential principle in the discussion. The full meaning of such statements can be completely understood only with a thorough knowledge of the problems of physical science.

The law of gravitation was undoubtedly Newton's greatest discovery, and the importance of his investigations can be inferred from the statements of competent judges. The brilliant German scholar, Leibnitz (1646–1716), who was Newton's bitterest rival, said, "Taking mathematics from the beginning of the world to the time when Newton lived, what he had done was much the better half." The great French mathematician, Lagrange (1736–1813), one of the masters of celestial mechanics, wrote, "Newton was the greatest genius that ever existed, and the most fortunate, for we can not find more than once a system of the world to establish." The English scientist and writer on the history of science, Whewell (1794–1866), wrote in his *History of the Inductive Sciences*, "It [the law of gravitation] is indisputably and incomparably the greatest scientific discovery ever made, whether we look at the advance which it involved, the extent of the truth disclosed, or the fundamental and satisfactory nature of this truth." Let us compare with these magnificent and deserved eulogies Newton's own estimate of his efforts to attain to the truth: "I do not know what I may appear to the world; but to myself I seem to have been only like a boy playing on the seashore, and divert-

ing myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me." There is every reason to believe that this is the sincere and unaffected expression of a great mind which realized the magnitude of the unknown as compared to the known.

VERIFICATION OF THE LAW OF GRAVITATION

162. Character of Orbits described by Bodies moving according to the Law of Gravitation. — General principles are discovered by inductions; they are verified by deducing their consequences and comparing them with observations. The consequences which flow from the law of gravitation are obtained by such difficult mathematical processes that they can be explained here only in general terms.

The first problem, which was solved by Newton in the *Principia*, is to find what the character of the curve must be if the force to which the body is subject varies inversely as the square of the distance. The answer is that the orbit always will be a conic section, however the body may be started. The curves are called conic sections because they may all be obtained by plane sections of circular cones.

163. The Conic Sections. — The simplest conic section is the circle, which is the intersection of a cone and a plane perpendicular to its axis. When the plane is inclined a little from the perpendicular to the axis, an *ellipse*, whose properties have been described (Art. 100), is obtained. If the intersecting plane is parallel to one side (an element) of the cone, a curve, called a *parabola*, is obtained. If the intersecting plane is inclined to the axis less than an element, so that it intersects both branches of the double cone, the *hyperbola* is obtained.

A parabola is a curve such that the distances of every point on it from the point (focus) F and the line L are

equal. It extends to infinity in the direction opposite to *L*. One may think of a circle as being an ellipse whose foci have united at the center; a parabola may be thought of as being an ellipse one of whose foci has been removed to infinity. Its two arms become ultimately parallel.

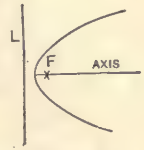


FIG. 82.

The hyperbola consists of two sets of symmetrical branches extending to infinity. It is defined by the property that the difference of the distances from any point on it to two fixed points (foci) *F* and *F′* is constant. One may think of it as being an ellipse, one of whose foci has been removed to infinity along the major axis and brought back from the opposite direction.

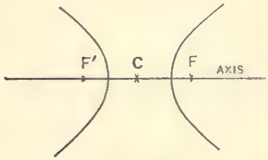


FIG. 83.

164. Character of the Orbit depending on the Initial Conditions. — The conic that a body moves in when subject to a given force depends entirely upon its initial position, direction of motion, and velocity. Suppose, for example, that a number of bodies are started with a series of velocities at a given distance from the sun, and at right angles to the line joining their initial position with the sun.

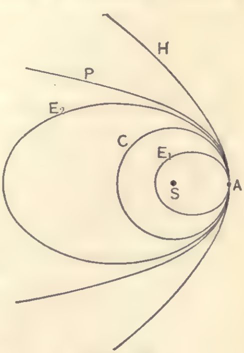


FIG. 84.

In Fig. 84, *S* represents the sun, which is the center of force, and *A* the point of projection. For a certain velocity the body will describe an ellipse E_1; for a *single* greater velocity, the circle *C*; for a still greater velocity, an ellipse E_2; for a *particular* greater velocity, the parabola *P*; and for a still greater velocity, an hyperbola *H*. There are besides two

limiting cases. First, if there were no initial velocity, the body would fall straight to the sun. The orbit would be considered as being an ellipse in which the minor axis had shrunk to zero. Second, if the initial velocity were infinite, the motion would be in a straight line perpendicular to the line $\overline{SA}$. This would be considered as being the limit as the branches of the hyperbola became straighter. If the body were started in any other direction than at a right angle to the line joining its initial position and the sun, the orbit would also be a conic section, its character depending upon the initial velocity as before.

The orbits of all the bodies of the solar system are conics. The planets' orbits are all ellipses, and the orbits of all their satellites are either ellipses or circles. The orbits of the comets are either elongated ellipses, parabolas, or in some cases barely possibly hyperbolas. Consequently there is thus far perfect agreement between observations and the Newtonian law of gravitation.

The erroneous opinion often is held that the orbits of the planets would be circles except for their mutual disturbances. The statements made above show the fallacy of this idea. It would be most remarkable if their orbits were circles, for there is but a single initial condition which will give a circular orbit through a given point, while an infinity of velocities will give ellipses through the point. One might as well expect to find all the trees in a natural forest growing in rows running north and south.

Sometimes it is thought that the mutual attractions of the planets keep the system in a sort of balance, and that if one planet were removed, the remainder would fall into the sun. Although they do influence the motions of each other to some extent, the idea is entirely false. If all the planets except the earth were in some way destroyed, no one but an astronomer or a somewhat attentive observer of the sky would notice any difference.

165. Motion in Orbits. — The law of areas completely describes how the planets move in their orbits, but it does not show how the motion in a conic depends upon the law of gravitation. In fact, this dependence can not be completely explained by elementary processes, but the general reasons can be easily exhibited.

It follows from the law of areas that the nearer a planet is to the sun the faster it goes, and that the velocity at any point is inversely proportional to the perpendicular from the sun to the tangent at that point. Since the actual velocity is greater while the body is nearer, it follows that the angular velocity is also greater. The mathematical treatment shows that it varies inversely as the square of the radius.

Let us see in a general way how the motion takes place. At the point *A*, Fig. 85, the planet is moving at right angles to the radius from the sun; consequently, its direction of motion is changed by the attraction of the sun, while its instantaneous velocity remains the same. When *A* is the aphelion point, as represented in the figure, the acceleration produced by the attraction

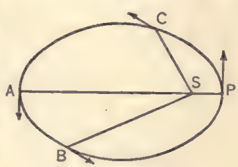


FIG. 85.

of the sun is greater than the centrifugal acceleration due to the motion of the planet in this part of its orbit. Hence the planet is drawn inside of the circle through *A*. After this, as at *B*, the motion is not perpendicular to the radius, and the sun's attraction has two effects. One is to change the direction of motion as before, and the other is to increase the velocity. The sun's effectiveness in changing the direction of motion is decreased, both because the attraction is not perpendicular to the line of motion, and also because the centrifugal acceleration due to the increased velocity is greater. On the other hand its attraction, and, therefore, its effectiveness in changing the direction of motion, is increased because the dis-

tance to the planet has decreased. For a time the last influence predominates and the radius makes a more and more acute angle with the tangent; and then the smallness of the angle and the centrifugal acceleration conspire, notwithstanding the increased attraction of the sun, to make the angle greater. The velocity is constantly increased, and the angle between the radius and tangent increases until it is 90° again at the perihelion point P. The velocity is so great at this point that the centrifugal acceleration is greater than the acceleration of the sun's attraction. Consequently, the body goes outside of the circle through P, and the radius makes an angle greater than 90° with the tangent. The velocity is decreased; the angle between the radius vector and the tangent to the orbit increases for a while, then it decreases and approaches 90° as the body approaches A. A rigorous discussion shows that the planet will pass through A again with the same velocity and direction of motion as it had at the preceding revolution. The motion in the second half revolution is precisely the opposite of that in the first half.

Suppose the orbit is a parabola and that P is the perihelion point. The velocity of the body at P is greater than it would be if it were moving in an ellipse. In fact, the velocity at a given perihelion is least in the circle, and increases with the eccentricity of the ellipse. It is greater still in a parabola, being $1.4\cdots$ times that in a circle, and this velocity is exceeded by that in the hyperbola, where it increases indefinitely as the curve approaches a straight line. Thus, in the case of the parabola the attraction of the sun is not sufficient to destroy the motion of the body away from it. The direction of motion is changed until it ultimately coincides with that from which it came, and the velocity theoretically becomes zero at infinity. In the case of the hyperbola the velocity at P is still greater, the attraction of the sun does not change the direction of motion to

that from which it came, and theoretically the body approaches infinity with a finite velocity.

166. Other Laws of Force. — We can not avoid asking whether the system would not be just as satisfactory if the law of gravitation were different. For example, the attraction might vary inversely as the third, or any other power of the distance. The appropriate mathematical investigation shows that the motion under every other law of this kind (except one) would be in spirals except for very special conditions which the mutual attractions would at once destroy. Every planet would wind its way either in to the sun and be melted, or out to the regions of perpetual night and frigidity. The exception would occur if the force varied directly as the distance. In this case the orbits of all the planets would be ellipses, with the sun at their centers instead of at their foci, and they would all have the same period. The moon would not belong to the earth in the sense that it does now, for the sun's attraction for it would be greater than that of the earth and would largely determine its motion. The earth's gravitation could not control the loose material on its surface, and it would move away in response to the greater forces exerted by the sun, while the distant stars would be even more dangerous. This law, therefore, from our present point of view would be a failure.

It is not known why the attraction varies inversely as the square of the distance. Perhaps it will be found sometime that gravitation is due to a wave motion of some sort through the ether, or a more subtile medium, and that its velocity exceeds that of light as the velocity of light exceeds that of sound. If such a thing were the cause, its intensity would vary inversely as the square of the distance, just as the intensity of light or sound varies inversely as the square of the distance from its source. An investigation by Laplace showed that if gravitation is propagated with a finite velocity it must

be at least six million times that of light, and a more recent discussion by Lehmann-Filhès has confirmed these conclusions.

167. Gravitation among the Stars. — The Newtonian law of gravitation is usually stated as holding throughout the universe, and it probably does; but the proof is of no such absolutely certain character as that which is given for the solar system.

At the time of Newton the law of gravitation was believed to prevail among the stars because it was thought they were suns much like ours. The spectroscope has greatly strengthened this opinion by showing that the stars contain a number of familiar elements.

The law was proved for the solar system from the motions of the planets. In the last 100 years, and particularly in the last 25 years, many cases of two stars revolving around their center of gravity have been found. It is impossible to determine the relative orbits with precision, but the results so far obtained show that the attraction is either the Newtonian law or one in which the intensity of the force varies with the direction of the bodies from each other as well as with their distances apart. It seems entirely unreasonable to suppose that the attraction depends upon the direction, and if it does not the Newtonian law must prevail. Although the proof is not absolutely conclusive, astronomers unhesitatingly adopt the theory that the law of gravitation is universal in its application.

168. Perturbations. — If the planets were subject to no forces except the attraction of the sun, their orbits would be strictly ellipses, but they attract each other just as they are attracted by the sun. These mutual attractions cause them to depart sensibly from elliptic motion, and these deviations are called *perturbations*. The moon's orbit around the earth is much more greatly perturbed by the sun than the orbits of the planets are by each other.

The problems of the mutual perturbations of the planets

and of the perturbations of the moon are of the most difficult character, and have taxed to the utmost the powers of mathematicians. One source of the difficulty can be quite easily understood. Suppose P and Q are two planets whose orbits would be as shown in Fig. 86 if they did not disturb each other's motion. But the planet Q pulls P out of the elliptic orbit, the direction and amount depending upon the distance of the two bodies apart, their direction from each other, and the position of P in its orbit. All these quantities vary in a most complicated way, and it is necessary to find the result of these continuous and varying influences. In a similar way P pulls Q out of its elliptic orbit. The problem could be solved if it ended here, but these perturbations produce more complicated secondary results, which are particularly exemplified in the case of the moon. Since P and Q have departed from their orbits, it follows that their distances and mutual relations have changed; consequently the perturbing forces have changed. The result is that the perturbations first considered become the source of new perturbations, and so on indefinitely.

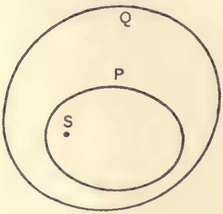


FIG. 86.

The problem is considerably simplified in the solar system because the masses of the planets are very small compared to that of the sun. The sun is so much greater than any planet that it almost absolutely controls the planetary motions. The first perturbations are small and the secondary perturbations are generally negligible, except in the case of the moon, where the work must be carried much further. If the masses of the planets had been large, say one-twentieth that of the sun, the perturbations would have been so great that Kepler's laws would not have been approximately true. With this foundation for the discovery of the law of gravitation removed, it is doubtful if the great Newton, or any of

the illustrious mathematicians who have thus far appeared, could have extracted from the intricate tangle of phenomena this master principle of celestial motions.

The planets do not move exactly in ellipses, or in any simple curves. In fact, they probably never revolve twice in precisely the same orbits, yet it has been found convenient both in the mathematical analysis and in popular description to speak of them as always moving in elliptical orbits, but in ones whose positions, shapes, and dimensions constantly change. It is something like considering motion in a circle as always being in a straight line whose direction changes continually in the same way. The problem of perturbations is to determine how the elements of the variable elliptic orbits change with the time.

169. Variations of the Elements of the Planetary Orbits. — The planets are occasionally near each other in their orbits for short times, and then very far separated for longer intervals. When they are near each other, their mutual perturbations are greatest. This approach and recession gives rise to perturbations which oscillate one way and then the other after a few revolutions of the planets at most, and they are called *short-period perturbations*, or inequalities. They are always small, and because of their oscillating character their effects on the system are not important. They affect all the elements of the orbits. The law of gravitation is verified by the fact that observations agree with the hundreds of short-period perturbations which theory shows should exist.

There is another class of perturbations called *long-period inequalities*. They are like the short-period inequalities except that in this case the elements vary in one direction for hundreds, and in some cases even for thousands, of years. They arise when the planets are in conjunction at intervals of only a few revolutions in almost the same parts of their orbits. The most remarkable case of this kind in the solar

system is that of Jupiter and Saturn, which have a long inequality of about 900 years. The irregularity had been noted by Halley (who, of course, did not know its period), but it was first explained by Laplace in 1784. The whole deviation of Jupiter is less than half a degree, and that of Saturn about three-quarters of a degree.

There is a third class of disturbances known as *secular perturbations*, because they continue indefinitely in one direction. This is not strictly true, for, notwithstanding the form in which they appear in the ordinary analysis, part of them are only certain phases of extremely long oscillations, as Laplace and Lagrange showed at the end of the eighteenth century. The result of greatest importance is that the major axes of the planets' orbits have no direct nor secondary secular perturbations, from which it can be shown that the periods of revolution around the sun have no secular variations.

The eccentricities of the planetary orbits change in one way very slowly for immense periods of time, and then in the other. The periods of these oscillations for the various planets differ greatly, and the eccentricities of some orbits are increasing while those of others are decreasing. At present the eccentricity of the earth's orbit is decreasing, and it will reduce to about 0.003 in 24,000 years. Then it will increase for 40,000 years, when it will be about 0.02.

The inclinations of the orbits of the planets to the plane of the ecliptic oscillate in the same general way as the eccentricities. The variations are small and take place in very long periods.

The nodes and perihelia change continuously in one direction. The nodes of the orbits of all the planets move in the retrograde direction, and all the perihelia advance except that of Venus. Their periods of revolution are counted by tens and hundreds of thousands of years.

On looking over these results one is struck by the fact that

all those elements whose substantial constancy is necessary in order that the system may maintain its general characteristics oscillate through narrow limits, while those whose variations make only unimportant changes in the system proceed in one direction indefinitely. Clearly if the major axes continually increased or decreased, the changes would be disastrous to the earth. If the eccentricities increased indefinitely, the perihelia would approach nearer and nearer to the sun; the orbits would cross and collisions might ensue. If the inclinations were to change greatly, the mutual interactions of the planets would be entirely altered, although one would not say that there would be any direct effects which would be disastrous to any planet. The positions of the nodes and perihelia are of little consequence for the general characteristics of the system, and they are the two elements which change continually in one direction.

170. Variation of the Elements of the Moon's Orbit. — The perturbations of the moon's orbit are in a general way much like those of the planetary orbits, but the disturbances are much greater. In addition to this the moon is so near to us that much smaller inequalities can be observed.

In the orbits of the planets the perturbations are small because the disturbing masses are small compared to that of the sun; in the moon's orbit the perturbations are fairly small because the disturbing body, the sun, is so remote compared to the moon's distance from the earth. But these two simplifying circumstances operate quite differently. While the secondary perturbations of the planetary orbits are generally insensible, it is not at all the case in the orbit of the moon, where perturbations of many orders must be computed.

The sun's attraction makes the length of the month nearly an hour longer than it would otherwise be. All the elements undergo short-period perturbations, but none of them has secular inequalities except the perigee (corresponding to the perihelion), which continually advances, and the nodes, which

continually regress. The perigee makes a revolution in 8.855 years, and the nodes in 18.600 years.

There are a number of important perturbations which are not described in terms of varying elliptic elements. The simplest of these is the *variation*, which was discovered from observations by Tycho Brahe about 1590. It is a sort of elongation of the moon's orbit in a line at right angles to that joining the earth and sun. Its period is one month and it displaces the moon nearly 40′, which is considerably more than its diameter. The greatest inequality of all, known as the *evection*, depends upon the eccentricity of the moon's orbit, and has a period of 31.8 days. At its maximum effect it displaces the moon in geocentric longitude through an angle of about 1.27°. It was discovered by Hipparchus and was carefully observed by Ptolemy. In all these perturbations, theory and observations agree.

171. The Annual Equation. — There are a number of perturbations, most of which are secondary, which prove the validity of the law of gravitation in the most certain manner.

It was stated in the last article that the sun increases the moon's period of revolution round the earth. This effect is greatest when the earth is near perihelion, and its variation as the earth's distance from the sun varies, makes a slight change in the lengths of the months throughout the year. Theory and observation in this complicated interaction are in perfect accord.

172. Lunar Perturbations due to the Attractions of the Planets. — It is perfectly clear that the planets disturb the moon just as the sun disturbs it, but to a much smaller extent. Indeed, the only direct perturbation of sensible magnitude is due to Venus, and has a period of 273 years.

There are, however, indirect effects of the greatest interest. As seen from the planets the earth and moon appear very near together, and the principal effects of the attractions of the planets are in disturbing them in their common motion

around the sun. Now, when the earth's distance from the sun is changed in this way, the sun's disturbing effects on the moon are changed also. This gives rise to perturbations of the motion of the moon which are due in an indirect way to the attractions of the planets first changing the motion of the earth. A law which correlates and explains such complicated phenomena as these must be considered as having been most firmly established.

173. The Secular Acceleration of the Moon's Motion. — The perturbation here considered is of the same general character as those treated in the last article, though it is somewhat more involved.

In the early part of the eighteenth century Halley found from a comparison of ancient and modern eclipses that the mean motion of the moon is gradually increasing. It was not explained by the theory of gravitation until 1787, when Laplace showed that it is due to the fact that the attractions of the planets for the earth are causing the eccentricity of its orbit gradually to decrease ; this diminishes the average disturbing effects of the sun. The perturbations by the sun increase the month (Art. 170). When they become less the month becomes shorter and the moon's motion is accelerated.

See how complicated the question is. The small unbalanced effects of the attractions of the planets for the earth cause the eccentricity of its orbit to oscillate with a period counted by tens of thousands of years. While the eccentricity is decreasing, the disturbing effects of the sun are changed so that the moon's motion increases ; when the eccentricity begins to increase, the moon's motion will become slower. These questions are involved in the theory of eclipses, and it is only by means of the law of gravitation that such phenomena can be predicted for long periods.

174. Unexplained Phenomena. — Notwithstanding the thousands of triumphs of the law of gravitation, there are two or three small irregularities of motions which have not yet been

shown to follow from it. The most remarkable example is that of the motion of the perihelion of Mercury's orbit, which the observations show is about 41″ per century greater than that found from the mathematical discussions. The cause of this disagreement is not known. It may have its source in the imperfections which still exist in the theory of perturbations, though this does not seem at all probable. There may be some disturbing cause yet unknown. If there were a series of small planets nearer to the sun than Mercury is, or a dense ring of meteors, the motion could be explained ; but then theory shows that there should be other irregularities of motion which have not been observed. It may be that the law of gravitation is not exactly true. For example, it may depend upon the velocities of the attracting bodies. Since Mercury moves the fastest, the effects would first become sensible in the case of this planet. Possibly this very slight residual will lead to an important discovery, just as the discrepancy of 8′ between theory and observation led Kepler to the discovery of the elliptical form of the orbits of the planets. However, the history of such disagreements points away from this hypothesis. Time after time astronomers have suspected that the law of gravitation is not perfectly exact, only to find later that the fault was in the imperfections of their methods.

The next most interesting case is the secular acceleration of the moon's motion described in Art. 173. Laplace made a mistake in his calculations and his theory agreed with the observations. It was first corrected by Adams. As corrected, the theoretical acceleration is about 6″ per century while the observations show that it is from 8″ to 12″ per century. The observational amount depends upon ancient eclipses, the records of which are very imperfect, and it may very well be that most of the disagreements arise from the errors in the early observations, or in an incorrect identification of the eclipses mentioned in the ancient chronicles. Another expla-

nation is that the extra $2''$ to $6''$ is only an apparent acceleration due to a slackening of the rotation of the earth from tidal retardation (see Art. 351).

The lunar theory as a whole is not quite satisfactory, for if the constants are determined so that it satisfies the observations for one period, it will fail to a sensible extent for both earlier and later dates. The disagreements undoubtedly have their origin largely in imperfections in the mathematical treatment of this most difficult problem.

175. The Stability of the Solar System. — A question of the highest interest is whether the solar system will preserve indefinitely its present general form. If it does, it will be called stable.

This is a question for mathematical treatment, and it follows from the discussion given in Art. 169 that so far as the theory goes it indicates that the planetary motions are stable. It is generally, though quite erroneously, stated that it has been rigorously proved that the orbits of the planets will never change greatly. The weakness lies in the fact that the expressions for the perturbations are known to be valid only for comparatively short periods of time ; they may be quite misleading when applied for indefinite ages. Besides, the conclusions are based on primary and secondary effects, and it is not known with certainty that the perturbations of higher orders do not contain the evidences of very remote but sure disaster.

It is very probable that the best planetary theories will fail in time, just as there are disagreements in the lunar theory after comparatively short intervals. The correct statement is that the mathematical theory shows with certainty that the system will not be greatly changed by the mutual attractions of its members for many thousands of years, and there is nothing whatever to prove that it is not permanently stable. But the geological and biological evidences of the great age of the earth are just as good as the

mathematical, though they do not, of course, give any information about the way in which the elements of its orbit change.

The theory which has just been discussed is developed on the hypothesis that the planets are homogeneous spheres, subject to no forces whatever except their mutual attractions. The facts are that they are of irregular density and oblate, and are subject to a multitude of smaller influences, among which may be mentioned growth by addition of meteors, resistance due to meteors, and complex tidal interactions.

It is not certain that ordinary matter endures forever. In fact, some of the recent discoveries respecting the divisibility of atoms and the properties of the element *radium*, and other substances giving similar phenomena, suggest that it is perhaps in a state of transition, though at an extremely slow rate, just as the mountains are dissolved by air and water and carried down into the sea.

No positive statement can be made respecting the stability of the solar system, but it seems to accord best with all we know to suppose that it is undergoing what seems in the light of our limited experience a very slow evolution, and that its present condition is transitory and but one stage in a great sequence of events.

176. Historical. — Newton discovered the law of gravitation and was the first one to apply it to the interpretation of the phenomena of the solar system. His first problem was to treat of the motion of one body around the sun when disturbed by no other forces. This is called the problem of two bodies. The geometrical processes he employed have been superseded by those of analysis.

Newton next treated the case where there are three bodies mutually attracting each other. He applied his methods to computing the motion of the moon around the earth as disturbed by the sun. He succeeded in explaining all the inequalities known at his time, though that relating to the

motion of the moon's perigee was not published in its correct form in the *Principia*. The more exact solution was found in his papers about twenty-five years ago.

The Newtonian theory was speedily and enthusiastically accepted in Scotland and England, but was not received with favor in continental Europe until after its ardent advocacy by Voltaire, after his visit to London in 1727. The Scotch and English clung to Newton's methods and made no progress. On the Continent the much more powerful methods of analysis were being developed, and when the law of gravitation was once accepted many of its consequences were speedily derived. In the eighteenth century the development of celestial mechanics was almost entirely the work of five men: Euler (1707–1783), a Swiss, born at Basle; Clairaut (1713–1765), born at Paris; D'Alembert (1717–1783), also a native of Paris; Lagrange (1736–1813), born at Turin, Italy, but of French parentage; and Laplace (1749–1827), son of a French peasant of Beaumont, in Normandy. The work of these men in celestial mechanics and mathematics was one continuous series of triumphs. At the time of Laplace it seemed as if there were no more difficulties to conquer, and early in the nineteenth century he gathered together all that was known on the subject in his monumental work, *Mécanique Céleste* (*Celestial Mechanics*). It is needless to say that new problems immediately arose, and that this field, as well as every other, is practically inexhaustible.

During the nineteenth century the discoveries in the consequences of the law of gravitation were much better distributed, though on the whole the French were in the lead. Mentioning only the more prominent of those whose work is finished, France had De Pontécoulant (1795–1874), Leverrier (1811–1877), Delaunay (1816–1872), and Tisserand (1845–1896); England had Airy (1801–1892) and Adams (1819–1892); Germany had Gauss (1777–1855) and Hansen (1795–

1874); Sweden had Gyldèn (1841–1896); and America had Peirce (1809–1880). It is impossible to give any popular description of the great achievements of these men, or of the rapid advances that are still being made.

TIDES

177. The Tide-raising Acceleration. — The tides on the earth are due to the attractions of the sun and moon. It will be sufficient in explaining the cause of the tides to consider the action of the moon alone.

Before proceeding to the direct discussion, a preliminary theorem is necessary. *If two bodies are subject only to equal parallel accelerations, their relative positions are not changed.* The truth of this follows at once from the laws of motion, but it is made plainer by an illustration. Suppose a large body and a small body are dropped from a great height at the same time, and that the small one is started five feet below and ten feet east of the large one. If the effects of the air are neglected, they will keep the same relative positions until one of them strikes the bottom, for the earth's attraction accelerates them both the same. Suppose the mass of the large body is ten times that of the small one. Then the earth attracts it ten times as much, but the acceleration is the same, for it takes ten times the force to give it the same acceleration.

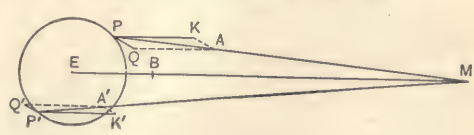


FIG. 87.

Let E (Fig. 87) represent the center of the earth, and M the moon. (The distance of the moon is greatly diminished to render the figure clear.) Consider the tendency of the moon to displace the particle P on the surface of the earth. Let $\overline{EB}$ represent the acceleration of M on E (the solid earth) in

direction and amount. In the same units let $\overline{PA}$ represent the acceleration of M on P in direction and amount. Since P and M are nearer together than E and M, it follows that $\overline{PA}$ is greater than $\overline{EB}$.

Let the acceleration $\overline{PA}$ be resolved into two components so that one of them shall be equal and parallel to $\overline{EB}$. It is $\overline{PK}$ in the figure. The other component is found by using $\overline{PA}$ as a diagonal and $\overline{PK}$ as a side, and completing the parallelogram. It is $\overline{PQ}$ in the figure. By the law of the parallelogram of forces $\overline{PA}$ is exactly equivalent to $\overline{PK}$ and $\overline{PQ}$, and conversely. By the preliminary theorem, $\overline{EB}$ and $\overline{PK}$ being parallel and equal do not tend to change the relative positions of E and P, and therefore cause no tide. The remaining acceleration $\overline{PQ}$ cannot be paired with any other, and is the tide-raising acceleration.

The part of the figure with accents is drawn from precisely the same principles. $\overline{P'K'}$ is parallel and equal to $\overline{EB}$, and $\overline{P'Q'}$ is the tide-raising acceleration.

Suppose figures are constructed for points all the way around the earth. The lines representing the tide-raising accelerations will be as given in Fig. 88. The method of drawing them is the geometrical counterpart of the rigorous mathematical treatment of the subject, and may be relied upon as giving the full explanation of the reason for the tides.

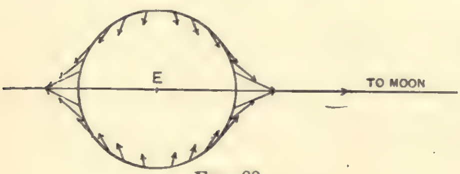


FIG. 88.

If one desires actually to compute the positions of the lines, it can easily be done. In Fig. 87 take the distance $\overline{EB}$ arbitrarily. By the law of gravitation the accelerations are inversely as the squares of the distances. Therefore

$$\overline{EB} : \overline{PA} = \frac{1}{\overline{EM}^2} : \frac{1}{\overline{PM}^2};$$

whence

$$(1) \qquad \overline{PA} = \left(\frac{\overline{EM}}{\overline{PM}}\right)^2 \cdot \overline{EB}.$$

Having located the point A, the line $\overline{PQ}$ is found by constructing the parallelogram as described above.

This exposition shows at once that there is an almost equal tendency for a tide to be raised on the side of the earth toward the moon, and on the opposite side. The difference can very easily be computed for the two points in the line $\overline{EM}$. The distance $\overline{EM}$ is about 60 times the radius of the earth. Suppose P is between E and M and take $\overline{EB}$ equal to unity. Then

$$\overline{PQ} = \overline{PA} - \overline{EB} = \overline{PA} - 1.$$

From equation (1) it is found that

$$\overline{PA} = \left(\frac{60}{59}\right)^2 = 1.0342.$$

If P' is opposite to the moon, then

$$\overline{P'Q'} = \overline{EB} - \overline{P'A'} = 1 - \overline{P'A'};$$

and from equation (1)

$$\overline{P'A'} = \left(\frac{60}{61}\right)^2 = 0.9674.$$

Therefore

$$\overline{PQ} = 0.0342,$$
$$\overline{P'Q'} = 0.0326,$$

from which it follows that $\overline{PQ}$ is less than five per cent greater than $\overline{P'Q'}$. The differences in other corresponding points are still less, and entirely vanish 90° from the line $\overline{EM}$.

178. Relative Magnitudes of the Tides raised by the Sun and Moon. — It follows from the masses of the sun and moon

and their distances from the earth, that the sun accelerates the earth about 175 times as much as the moon does. Since its distance is 93,000,000 miles, equation (1) of the last article becomes in the same units as were used in the case of the moon,

$$\overline{PQ} = \overline{PA} - \overline{EB} = \overline{PA} - 175,$$

$$\overline{PA} = \left(\frac{93{,}000{,}000}{92{,}996{,}000}\right)^2 \cdot 175 = 175.0151.$$

Therefore

$$\overline{PQ} = 0.0151,$$

or less than half of that due to the moon. Consequently, the tides raised by the sun are less than half as great as those raised by the moon. The small relative difference in its distance from the center and surface of the earth more than offsets its greater attraction.

179. Variations in the Tides. — The tides far from coast lines in the ocean, as measured on small islands, are two or three feet high, but they vary greatly from a number of causes. In shallow water along the coast, especially where the tidal waves converge into a bay or mouth of a river, the tides are many feet high. Agreement between theory and observation is more easily secured where the tides are not so greatly modified by local conditions.

The most important cause of variation in the tides is the change in the relative directions of the moon and sun from the earth. When the moon and sun are in a line with the earth, either on the same side or opposite sides, their effects add and produce the highest tides, called *spring tides*. When they are 90° from each other their effects partially destroy each other and produce the lowest tides, called *neap tides*.

The tides in any latitude vary with the declinations of the sun and moon. For example, their respective tides are less in the northern hemisphere when they are over the equator than when they are north of it.

The tides vary with the changing distances of the sun and moon. The eccentricity of the moon's orbit is rather large (about 0.05) and its distance varies considerably. The percentage of change of the tide-raising force is nearly three times the percentage of change in distance. Consequently, there are important monthly and yearly variations due to changes of distance.

All these varying influences combine in a most complicated fashion, yet by means of the law of gravitation these ever-changing phenomena are shown to succeed each other according to fixed laws, and they can be predicted with perfect certainty.

180. The Lag of the Tides. — Suppose the earth were a viscous mass instead of a solid covered with fluids, the water and atmosphere. Suppose the earth rotated faster than the moon revolves in the direction indicated by the arrow in Fig. 89. It would have a tendency to carry the tidal waves A and B on beyond the line $\overline{EM}$. But the moon would tend to keep them in the line $\overline{EM}$. The result would be that they would travel along the surface of the earth in the direction opposite to its rotation, that is, from east to west.

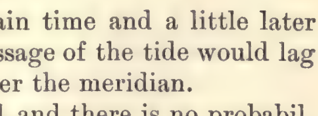


Fig. 89.

Now, consider a point on the earth's surface. The rotation would carry it under M at a certain time and a little later under the tide A. That is, the passage of the tide would lag behind the passage of the moon over the meridian.

If the earth is not perfectly rigid, and there is no probability that it is, there are tides in its solid parts like those just described. But the tides in the fluid surface are quite different. When a tidal wave is raised, it runs around the earth with a velocity depending upon the depth of the ocean. The motions of the waves are interfered with by the continents, they meet from various directions and interfere with one

another, sometimes adding together and sometimes destroying one another, and they are greatly modified by the ocean beds. The subject of water tides is so complicated that a satisfactory theory for a station could scarcely be constructed without any observations; but when observations have once shown the effects of local conditions, the variations due to other causes can be predicted from the principles discussed in the last article.

SPECIAL PROBLEMS

181. The Masses of the Planets. — There are a number of important special problems whose solutions depend on the law of gravitation, but which are not in a very direct way verifications of it. They will be given here.

The methods of measuring the density and mass of the earth have been given. In principle, they consist in comparing the attraction of the earth with that of a body of known mass and dimensions; and in carrying out the comparison the law of gravitation is involved. The same principle is employed in determining the mass of any heavenly body, but the problem is practically much simpler, for the comparison can be made with the whole earth.

Let m_1 and m_2 represent the masses of two bodies revolving around their common center of gravity as a consequence of their mutual attractions, P their period of revolution, a their mean distance apart, and k a constant depending upon the units employed. Then P is determined by the equation[1]

$$(1)\qquad P = \frac{2\pi a^{\frac{3}{2}}}{k\sqrt{m_1 + m_2}}.$$

In this equation m_1 may represent the earth and m_2 the moon. Consider two other bodies and distinguish the two cases by giving the masses, P, and a accents. Then it follows that

$$(2)\qquad P' = \frac{2\pi a'^{\frac{3}{2}}}{k\sqrt{m'_1 + m'_2}},$$

[1] See *Introduction to Celestial Mechanics*, Art. 89.

for π is the constant 3.1416 ⋯, and k is the same when the same units are employed. Squaring equations (1) and (2), and dividing (1) by (2), it is found that

$$(3) \qquad \frac{P^2}{P'^2} = \frac{a^3}{a'^3}\,\frac{m_1' + m_2'}{m_1 + m_2};$$

whence

$$(4) \qquad m_1' + m_2' = \left(\frac{P}{P'}\right)^2 \left(\frac{a'}{a}\right)^3 (m_1 + m_2).$$

When the period and mean distance, P' and a', have been determined from observations, the sum of the masses is given by the important equation (4). It is by means of this equation that the masses of the stars have been determined so far as they are known.

Suppose m_1 and m_1' both represent the mass of the sun, which may be taken as unity, and that m_2 and m_2' are the masses of two planets. Then equation (3) becomes

$$(5) \qquad \frac{P^2}{P'^2} = \frac{a^3}{a'^3}\,\frac{1 + m_2'}{1 + m_2}.$$

Since m_2 and m_2' are very small, the equation is nearly true when the last factor is taken as unity. With this simplification it is the mathematical expression for Kepler's third law (Art. 138). The equation shows what corrections to the law are necessary.

The masses of all the planets which have satellites are determined directly from equation (4). The masses of those which do not have satellites are determined with much greater difficulty from their perturbations of other planets, or of comets which pass near them.

182. Surface Gravity of the Heavenly Bodies. — The gravity at the surface of a body depends upon both its mass and its size. Suppose the bodies are spheres, which will be accurate enough for present purposes. Since a sphere attracts exterior bodies as though its mass were all at its center, it fol-

lows that the gravity, or weight, of a unit mass at its surface is given by the formula

$$(1) \qquad G = \frac{k^2 m}{R^2},$$

where G is the gravity, m the mass of the sphere, R its radius, and k a constant depending upon the units employed. Consider another body using the same units, and distinguish it by accenting G, m, and R. Then

$$(2) \qquad G' = \frac{k^2 m'}{R'^2}.$$

Dividing (2) by (1) it is found that

$$(3) \qquad \frac{G'}{G} = \frac{m'}{m}\left(\frac{R}{R'}\right)^2.$$

When the mass of a body has been determined by the method of the preceding article, and its radius from observations, its surface gravity is given directly by (3).

It is frequently simpler to use radii and densities instead of radii and masses. Let the densities be d and d' respectively. Then $m = \frac{4}{3}\pi d R^3$, $m' = \frac{4}{3}\pi d' R'^3$, and equation (3) becomes

$$(4) \qquad G' = \frac{d' R'}{dR} \cdot G.$$

QUESTIONS

1. Give an example of a theory not based upon observations which has been shown to be false.

2. Give an example of known phenomena which have not yet led to a satisfactory theory.

3. Enumerate the steps Newton took in the discovery of the law of gravitation.

4. What proofs did Newton have of the motion of the earth?

5. What are the conclusions which may be derived from Kepler's second, first, and third laws?

6. How many circular, elliptic, parabolic, and hyperbolic orbits may there be with a given perihelion point?

7. Are the perturbations to be considered as blemishes on the system, like imperfections in a complicated machine?

8. In what instances has the law of gravitation so far failed to explain observed phenomena?

9. The mass of the moon is about $\frac{1}{80}$ that of the earth; what is the ratio of the attraction of the moon to that of the earth for a body at the earth's surface on the line $\overline{EM}$ (Fig. 87)?

10. By Art. 177 the tide-raising force at this point is 0.0342 of the moon's attraction for the earth; how much, therefore, does the tide-raising force decrease the weight of bodies under these circumstances?

11. If the sun and moon are over the earth's equator the two tides of a day are nearly equal. Discuss the question with diagrams if they are north or south of the equator.

12. Draw diagrams showing how the tides produced by the sun and moon add when these bodies are in a line with the earth.

13. What is the surface gravity of the sun if its radius is 109 times that of the earth, and its density one-fourth as great?

14. What is the surface gravity of a body 10 miles in diameter having the same density as the earth?

15. What would be the earth's surface gravity if its diameter were twice as great and its mass the same?

16. What would be the earth's surface gravity if its diameter were twice as great and its density the same?

CHAPTER VIII

TIME

183. Definition of Equal Intervals of Time. — If a person observes a number of events, he is conscious that they are distinguished from each other, though they be of the same nature, and he explains it by saying that they occurred at different times. Moreover, the separate events of the same series would be distinguished from each other by another observer in precisely the same way. For example, if some one repeats a series of letters, different observers will agree on their order. That is, we can arrange events in a *unique order* and thus measure the interval between two of them by the number of others which have intervened.

It is supposed that there is another element in time besides order. When any one observes two events, he is conscious that they are separated by an interval in which other events may have taken place, but different observers generally will not agree on the interval. It may be that the feeling that an interval has elapsed is induced by the consciousness that other events have occurred between them, such as something which can be seen or heard, or, perhaps, such physiological processes as the beating of the heart. The observer whose mental acts in the interval most exceeded the normal would give the highest estimate of the length of the interval. It is clear that each observer must be compared with his own normal, for he is accustomed to call the average interval required for a certain number of changes of consciousness a certain name, as a minute.

There is no evidence that a minute, or a day, seems of the same length to every one, although every one gives the intervals the same names. It may well be that a week seems as long to an intellectually active man as a month does to the sluggard of low intelligence. One can see how his own estimates vary under different conditions. If he travels for a few days, especially when he is young, amid unfamiliar surroundings, on his return home he seems to have been absent for weeks. On the other hand, as he becomes older and new experiences are less frequent and the mental processes less active, time seems to have passed more rapidly.

It will be found that the estimates of intervals of time made by different observers agree more nearly the more fully they base them upon counts of simple phenomena. It will be perceived at once that this implies that they have supposed that the simple events observed have occurred at equal intervals, or, perhaps in rare cases, in unequal but simply related intervals. This being the basis of our ideas of intervals of time, and the only way of measuring them, it remains to develop a method for practical applications. To do this it is only necessary to select some event which is repeated an indefinite number of times in such a manner that it will occur after, in the sense of order, any event of another kind, and such that it can be universally observed. Newton's first law of motion, which affirms that *a body subject to no forces moves uniformly in a straight line*, is a definition of equal intervals of time which fulfills the conditions; or rather, certain consequences of it, to be discussed in the next article, fulfill the conditions. That is, two intervals are equal by *definition* if a moving body which is subject to no forces passes over equal distances in them.

The definition of equal intervals of time by the first law of motion agrees, on the average, with the experience of any individual. It may be due to the fact that, as physiologists teach us, every mental act is accompanied by physical changes

in the brain. If this is so, it is only reasonable that the phenomena of consciousness should correspond with the laws of motion of material bodies.

184. Practical Measure of Time. — It follows from the laws of motion that a rotating body which is subject to no exterior forces turns at a uniform rate. No body in such a state is known, but the attraction of a distant body, like that of the sun for the earth, does not change the rate of rotation except through the very slight retardation due to the tides. Consequently, for practical purposes, the rotation of the earth may be taken as measuring equal intervals of time. Of course, many other celestial motions might be used, but they would all be attended with practical disadvantages and no compensating advantages. Any artificial timekeeper, as a clock, would run in accordance with the same laws, but more irregularly because of the impossibility of entirely removing disturbing influences.

There is one feature of the method of using the rotation of the earth to define equal intervals of time which should be mentioned. It must be shown that it is possible to determine when the earth has turned through equal angles. There is a theoretical difficulty which arises from the fact that no fixed direction in space is known. If no heavenly body were visible, it would be impossible for us to find out directly whether the earth rotates or not. The rate of rotation of the earth is actually determined from the apparent motions of the stars, which must be assumed to be in constant directions from us. The assumption is not an entirely new one, for it follows from the laws of motion and the distances of the stars that they could not make daily revolutions around the earth. Consequently, the method of measuring time by the rotation of the earth depends upon the laws of motion in an indirect way as well as the direct. The corresponding difficulty in applying the first law of motion is that no fixed point in space is known from which to measure distances.

185. Sidereal Time. — Sidereal time has been discussed in Art. 23 in connection with the description of the method of cataloguing the stars, but the essentials will be repeated for the sake of comparison with the other kinds of time to be mentioned. Sidereal time depends upon the rotation of the earth with respect to the stars. The interval between the passage of a star across a meridian and its next passage across the same meridian is a sidereal day, which is divided into 24 equal hours. The hours are numbered up to 24. The sidereal time is zero when the vernal equinox crosses the meridian.

Sidereal time agrees exceedingly closely with time as defined by the first law of motion. The agreement would be perfect if the rotation of the earth were not disturbed by external forces, but, as was stated in Art. 126, these disturbing forces have wholly insignificant effects, except possibly when the rotation of the earth is compared with other celestial motions. Even here there seems to be but one place where a change in the rate of its rotation is even suggested, and that is in connection with the exceedingly difficult problem of the acceleration of the moon's motion.

186. Solar Time. — Solar time is defined by the rotation of the earth with respect to the sun. Because of the earth's revolution around the sun, the sun appears, as seen from the earth, to move eastward among the stars, completing a revolution with respect to them in a year.

In order to compare sidereal time and solar time, suppose a star and the sun are on the meridian at the same instant; after a certain interval the earth will have turned so that the star is again on the meridian, but in the meantime the sun will have moved eastward nearly a degree. It will take the earth nearly 4 sidereal minutes to turn through this remaining degree and bring the meridian up to the sun. The interval between the passage of the sun across the

meridian and its next passage is a *solar day*, which is nearly 4 minutes longer than the sidereal day.

187. Variations in the Lengths of Solar Days. — When compared with the sidereal day the solar days vary in length for two principal reasons, which will now be explained.

The earth's orbit is elliptical, and the motion of the earth is such that the law of areas is fulfilled; therefore its angular motion with respect to the sun varies. The sun's apparent angular motion among the stars is just the same as the earth's angular motion as seen from the sun. Therefore the sun's apparent eastward motion is not uniform. Since this is what causes the difference in length between the sidereal and a solar day, it follows that the difference will vary; or, the length of the solar day varies when compared to that of the sidereal day. When the sun apparently moves fastest, the difference is greatest; and when the sun apparently moves slowest, it is least.

Figure 90, in which the eccentricity of the earth's orbit is exaggerated for the sake of clearness, shows the reason of the difference in length of the solar days. Suppose the earth moves from the perihelion point P to P' in a sidereal day. The line which is in the plane of the meridian has made a complete rotation, but it must turn on through an angle equal to PSP' in order to point to the sun, as it will at the end of the solar day. Now consider the motion of the earth at the aphelion A. In a sidereal day it will have moved from A to A'. It must still turn through the angle ASA' in order to complete a solar day. This angle is less than PSP', and the solar day is correspondingly shorter than the one when the earth was at perihelion. It follows from the character of the earth's motion that, so far as this cause is concerned, the lengths of the solar days constantly decrease while the earth is moving from perihelion to aphelion.

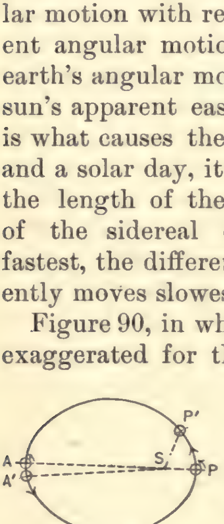


FIG. 90.

The other reason that solar days vary in length is that the sun does not move eastward along the equator. Suppose, for simplicity, that its motion along the ecliptic is uniform, and consider the effect of the inclination of the ecliptic to the equator. In Fig. 91, V represents the vernal equinox and A the autumnal equinox. The hour circles are drawn at intervals of 15°, and equal spaces are marked off along the ecliptic by the dotted lines. Since the diurnal motion of the sun is along the equator or a circle parallel to it, it is the eastward motion of the sun in right ascension which makes a solar day longer than the sidereal.

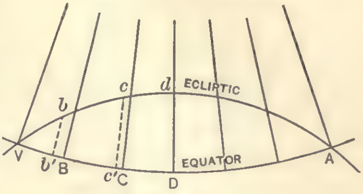


FIG. 91.

Consider the motion near the vernal equinox. While the sun is moving 15° from V to b its actual eastward motion in right ascension is only Vb'. When the sun is near the solstice d, 15° takes it from c to d, and its eastward motion in right ascension is $c'D$. That is, when the sun is near one of the equinoxes, it moves the slowest in right ascension, because a considerable part of its motion is either northward or southward; and when it is near one of the solstices, its eastward motion along the equator is most rapid, because its orbit nearly coincides with a declination circle. As far as this factor alone affects the solar days, it tends to make them longest while the sun is near the solstices, and shortest while it is near the equinoxes.

The first cause tends to make the solar days longest on January 1 and to decrease gradually both ways from this date for six months. The second cause tends to make them longest on December 21 and June 22, and shortest on March 21 and September 23. The two effects combine and give the following results: the longest day in the

year is December 22 (it may vary by a day from this date because of the leap year), which is 4 m. 26.5 sec. longer than the sidereal day when expressed in sidereal time. Thus, the day which has the least time of sunlight for positions north of the equator, and which in ordinary speech is said to be the shortest day in the year, is actually the longest from the time the sun is on the meridian until it is on the meridian again. From December 22 the solar days constantly decrease in length until March 26, which is only 3 m. 38.0 sec. longer than the sidereal day. Then they increase in length until June 20, which is 4 m. 9.5 sec. longer than the sidereal day. From June 20 the solar days again decrease in length until September 17, the shortest day in the whole year, which is only 3 m. 35.2 sec. longer than the sidereal day. Then the solar days constantly increase in length until December 22. The difference in length between the longest day and the shortest day in the year is therefore about 51.3 sec. of sidereal time. This is rather small, but its cumulative effects are quite noticeable, as will be seen in Art. 189.

It would seem to be a simple matter to assume that all solar days are of the same length, especially as the variation from one to the next is very slight. Nothing but a very accurate clock would show in ordinary affairs any disagreement, and with this it would not be important. But if the astronomer should attempt to use the rotation of the earth with respect to the sun as defining equal intervals of time, he would become involved in irregularities of motions which he would be utterly unable to bring into simple agreement with any theory. This illustrates the extreme sensitiveness of astronomical theories to even slight errors.

188. Mean Solar Time. — Our ordinary activities are dependent upon the periods of sunshine. Hence, for practical purposes it is quite desirable to have a unit of time based in some way upon the rotation of the earth with respect to the sun. On the other hand it is undesirable to have

a unit of variable length. Consequently the mean solar day, which is the average length of all the solar days of the year, is introduced. In sidereal time its length is 24 hr. 3 m. 56.556 sec.

The mean solar day is divided into 24 mean solar hours, the hours into 60 minutes, and the minutes into 60 seconds. In short, this is the time in ordinary use. Mean solar days are all of the same length with the same approximation that sidereal days are of the same length, and ordinary time-pieces are made to keep mean solar time as nearly as possible. It would be very difficult, if not impossible, to construct a clock which would keep true solar time with any high degree of accuracy.

189. The Equation of Time. — The difference between the mean solar time and the true solar time is called the equation of time. It is taken with such an algebraic sign that when it is added to the true solar time the mean solar time is obtained.

It follows from the way in which the lengths of the solar days vary that the equation of time is zero on April 15, June 14, September 1, and December 24. The maximum numerical values of the equation of time between these dates are: February 11, + 14 m. 30 sec.; May 14, − 3 m. 50 sec.; July 26, + 6 m. 15 sec.; and November 2, − 16 m. 20 sec. These dates may vary by a day because of the leap year, and the amounts by a few seconds because of the shifting of the dates and the perturbations of the earth's motion.

Some interesting results follow from the equation of time. For example, on December 24 the equation of time is zero, and it follows from the numbers given above that the solar day at this time is about 30 sec. longer than the mean solar day. Consequently, the next day the sun will be about 30 sec. slow; that is, the noon is shifted about 30 sec. with respect to the sun. As the sun is just past the winter solstice the period from sunrise to sunset is increasing, but very

slowly, the exact amount depending on the latitude. For our latitude the gain in the forenoon resulting from the earlier rising of the sun is less than the loss from the shifting of the solar time of the noon. Consequently, almanacs will show that the forenoons are getting shorter at this time of the year, although the whole period between sunrise and sunset is increasing. The difference in the length of the forenoon and afternoon may accumulate until it amounts to as much as half an hour.

190. Standard Time. — The mean solar time of a place is called its *local time.* All places having the same longitude have the same local time, but places having different longitudes have different local times. In going around the earth, a distance of nearly 25,000 miles at the equator, the difference in local time is 24 hours; consequently, at the earth's equator 17 miles in longitude give a difference of about one minute in local time. In latitudes 40° to 45° about 13 to 12 miles in longitude give a difference of one minute in local time.

If every place along a railroad extending east and west should keep its own local time, there would be endless confusion and great danger in running trains. In order to avoid this it has become customary for all places which do not differ more than half an hour in local time from that of some convenient meridian to use the local time of that meridian. Thus, while the extreme difference in local time of places using the same time is about an hour, the error in either of them is only about half an hour. In this manner a strip of country in our latitudes about 750 miles wide uses the same time, and the next strip of the same width an hour different, and so on. The local time of the standard meridian of each strip is the *standard time* of that strip.

Standard time is at present in use in nearly every civilized part of the earth. The British Islands, Belgium, and Holland use as standard time the local time of the meridian

which passes through the Royal Observatory at Greenwich. France uses as standard time the local time of the meridian passing through the Paris Observatory. This meridian is 9 m. 21 sec. east of Greenwich; consequently the time used in France is 9 m. 21 sec. fast compared to that used in England. Germany, Italy, Switzerland, and Sweden use the local time of the meridian 1 hour east of Greenwich. Japan uses the local time of the meridian 9 hours east of Greenwich.

The United States and British America are of such great extent in longitude that it is necessary to use 4 hours of standard time. The eastern portion uses *Eastern Time*, which is the local time of the meridian 5 hours west of Greenwich. This meridian runs through Philadelphia, and in this city local time and standard time are identical. At places east of this meridian it is later by local time than by standard time, the difference being nearly one minute for 12 miles. At places west of this meridian, but in the Eastern Time division, it is earlier by local time than by standard time. The next division going westward is called *Central Time*. It is the local time of the meridian 6 hours west of Greenwich. This meridian passes through St. Louis. The next division is called *Mountain Time*. It is the local time of the meridian 7 hours west of Greenwich. This meridian passes through Denver. The last division is called *Pacific Time*. It is the local time of the meridian 8 hours west of Greenwich. This meridian passes about 100 miles east of San Francisco.

If the exact divisions were used, the boundaries between one time division and the next would be 7.5° east and west of the standard meridians. As a matter of fact, the boundaries are quite irregular, depending upon the convenience of railroads. The change in time is nearly always made at the end of a division, for obviously it would be unwise to have railroad time change during the run of a given train crew.

As a result, the boundaries of the several time divisions as used are very irregular, and vary in many cases very strikingly from the standard divisions.

FIG. 92. — Standard Time Divisions in the United States.

191. Distribution of Time. — The accurate determination of time and its distribution are of much importance. There are several methods of determining time, but the one in common use is to observe the transits of stars across the meridian, and thus obtain the sidereal time; then, from the mathematical theory of the earth's motion, to compute the mean solar time. It might be supposed that it would be easier to find the time by observing the transit of the sun, but it is not so. In the first place, it is much more difficult to determine the exact time of the transit of the sun's center than it is the time of the transit of a star, and it occurs but once in 24 hours while many stars may be observed; in the second place, it gives true solar time instead of mean solar

time, and the computation is as difficult in this case as it is when the other method is used.

It remains to explain how time is distributed from the places where the observations are made. In most countries the time service is under the control of the government and the time signals are sent out from the national observatory. Thus, in the United States the chief source of time for railroad and commercial purposes is the Naval Observatory at Georgetown Heights, Washington, D.C. There are three high-grade clocks keeping standard time at this observatory. At night their errors are found from observations, and, after applying the corrections for them, the mean of the three clocks is taken as giving the true standard time for the succeeding twenty-four hours. At five minutes before noon, eastern time, the Western Union Telegraph Company and the Postal Telegraph Company suspend the ordinary business and throw their lines into electrical connection with the standard clock at the Naval Observatory. The connection is arranged so that the sounding key makes a stroke every second during the 5 minutes until noon except the twenty-ninth of each minute, the last 5 seconds of each of the first four minutes, and the last 10 seconds of the fifth minute. This gives many opportunities of determining the error of a clock at any point. To simplify matters clocks are made which are automatically regulated by these signals, and there are at present more than 30,000 of them in use in this country.

The time signals are sent out from the Naval Observatory with an error usually less than two-tenths of a second, but frequently this is considerably increased where a system of relays must be used to reach great distances.

These noon signals also operate time balls in eighteen ports in the United States. This device for furnishing time, chiefly to boat captains, consists of a large ball being dropped at noon, eastern time, from a considerable height at conspicu-

ous points, by means of electrical connection with the Naval Observatory.

Time for the extreme western part of the United States is distributed from the Mare Island Navy Yard in California ; and, besides, a number of college observatories furnish time to particular railroad systems. Nearly every observatory regularly determines time for its own use.

192. Civil and Astronomical Days. — The civil day begins at midnight, for then business is ordinarily suspended and the date can be changed with the least inconvenience. The astronomical day of the same date begins at noon twelve hours later, for an astronomer would not like to change the date in the midst of a set of observations. It is true that many observations of the sun and some other bodies are made in the daytime, but by far the greater amount of observational work is done at night. The hours of the astronomical day are numbered up to 24 just as in the case of sidereal time.

193. Place of Change of Date. — If one should start at any point and go entirely around the earth westward, the number of times the sun would cross his meridian would be one less than it would have been if he had stayed at home. Since it would be very awkward to use fractional dates, he would most simply obtain the correct date at the end by arbitrarily changing his time one day forward at some point in his journey. That is, he would omit one date and day of the week from his reckoning. On the other hand, if he had gone around the earth eastward, he would give two days the same date and day of the week.

The change is usually made at the 180th meridian from Greenwich. It is a particularly fortunate selection, for this meridian scarcely touches any land surface at all, and then only small islands. One can easily see how troublesome matters would be if the change were made at a meridian passing through a thickly populated region, say at the

meridian of Greenwich. On one side of it people would have a certain day and date as Monday, December 24, and on the other side a day later, Tuesday, December 25.

The place of change of date does not strictly follow the 180th meridian from Greenwich, for settlers going eastward from Europe carried one date with them, while those going westward from Europe and America arrived in the same longitude with a different date.

194. The Sidereal Year. — The sidereal year is the time required for the sun apparently to move from any position with respect to the stars as seen from the earth to the same position again. Perhaps it is better to say that it is the time required for the earth to make a complete revolution around the sun, directions being determined by the positions of the stars. Its length in mean solar time is 365 da. 6 hr. 9 m. 8.97 sec., or just a little more than $365\frac{1}{4}$ days.

195. The Anomalistic Year. The anomalistic year is the time that it takes the earth to move from the perihelion of its orbit to the perihelion again. If the perihelion point were fixed, this period would equal the sidereal year; but the perihelion point moves forward, completing a revolution in about 108,000 years, and the consequence is that the anomalistic year is a little longer than the sidereal year. It follows from the period of its revolution that the perihelion point advances about 12″ annually. Since the earth moves about a degree daily, on the average, it takes it about 4 m. 40 sec. to move 12″. The actual length of the anomalistic year in mean solar time is 365 da. 6 hr. 13 m. 48.09 sec.

196. The Tropical Year. — The tropical year is the time it takes the sun to move from a tropic to the same tropic again; or, better for practical determination, from an equinox to the same equinox again. Since the equinoxes regress about 50.2″ annually, the tropical year is about 20 minutes shorter than the sidereal year. Its actual length in mean solar time is 365 da. 5 hr. 48 m. 45.51 sec.

The seasons depend upon the sun's place with respect to the equinoxes. Consequently, if the seasons are always to fall at the same time of the year, the tropical year must be used. This is, indeed, the one in common use, and, unless otherwise specified, it is always meant by the term *year*.

197. The Calendar. — In very ancient times the calendar was based largely on the motions of the moon, which determined the times of religious ceremonies. The moon does not make an integral number of revolutions in a year, and it was often necessary to interpolate a month in order to keep the seasons in place. The whole matter was in the hands of the priests, and great confusion prevailed.

The week was another division of time used in antiquity. The number of days in this period was undoubtedly based upon the number of known celestial bodies, other than the stars. Thus, Sunday is the sun's day; Monday, the moon's day; Tuesday, Mars's day; Wednesday, Mercury's day; Thursday, Jupiter's day; Friday, Venus's day; and Saturday, Saturn's day. The origin of the names of the days of the week, when traced back to the tongues from which English has been derived, show the above meanings.

In the year 46 B.C. the Roman calendar, which had fallen into a state of great confusion, was reformed by Julius Cæsar under the advice of the Alexandrian astronomer Sosigenes. The new system, called the Julian calendar, was entirely independent of the moon, and made three years of 365 days each, and then one, the *leap year*, of 366 days. The extra day was added in February. This mode of reckoning, which makes the average year consist of $365\frac{1}{4}$ days, was put into effect at the beginning of the year 45 B.C.

It is seen from the length of the tropical year given above that this system of calculation involves a small error, averaging 11 m. 15 sec. yearly. In the course of 128 years the Julian calendar gets one day behind. To remedy this small error,

in 1582, Pope Gregory XIII introduced a slight change. Ten days were omitted from that year, and it was decreed that three leap years out of every four centuries should henceforth be omitted. This again is not quite exact, for the Julian calendar gets behind three days in $3 \times 128 = 384$ years instead of 400 years, yet the error does not amount to a day until after more than 3000 years have elapsed.

To simplify the application, every year whose date number is exactly divisible by 4 is a leap year unless it is exactly divisible by 100; those years whose date-numbers are divisible by 100 are not leap years unless they are exactly divisible by 400, when they are leap years. Of course, the error which still remains could be further reduced by a rule for the leap years when the date number is exactly divisible by 1000, but there is no immediate need for it.

The calendar originated and introduced by Gregory XIII, and known as the Gregorian calendar, is now in use in all civilized countries except Russia and Greece, although it was not adopted in England until 1752. At that time eleven days had to be omitted from the year, causing considerable disorder, for the people imagined they were in some way being cheated out of that much time. The Julian calendar is now thirteen days behind the Gregorian calendar. The Julian calendar is called Old Style (O.S.) and the Gregorian, New Style (N.S.).

198. Days of the Week on the Same Date of Successive Years. — An ordinary year of 365 days consists of 52 weeks and one day, and a leap year consists of 52 weeks and two days. Consequently, in succeeding years the same date falls one day later in the week except when a 29th of February intervenes, when it is two days later. In 4 years, unless it is one of the exceptional periods containing no leap year, the date is changed 5 days in the week; and in 28 years the date is changed 35 days, or back to the starting point, and the relation to the leap year is the same. Thus, all

dates repeat themselves on the same day of the week in periods of 28 years unless a century year not evenly divisible by 400 intervenes.

QUESTIONS AND EXPERIMENTS

1. Let a series of five letters be written on the board, and find whether all observers agree respecting the order in which they were written.

2. Let a series of five signals be given at intervals of say from 10 to 60 seconds, the intervals being determined by a watch. Let the observers estimate the actual and relative values of the intervals and compare them both with each other and with the actual time.

3. Does the running of a watch depend upon the first law of motion?

4. How long does it take the earth to rotate through one degree? If the diameter of the sun is 32′, how long will it take it to apparently move its diameter? To test it, observe the time it takes the sun to disappear after its lower edge first touches the horizon. Is there anything which would cause this observation to lead to incorrect results?

5. How would the solar and the sidereal day compare in length, if the earth rotated in the opposite direction with the same sidereal period?

6. If a person goes around the earth westward, he loses one solar day; does he also lose a sidereal day? Does he lose any time on the basis of the first law of motion as a definition of time?

7. Develop a rule for leap years whose date numbers are divisible by 1000 so as to improve the Gregorian calendar.

8. February 25, 1905, fell on Saturday and March 25 on Saturday. In what year will the same dates first fall on the same days of the week again?

CHAPTER IX

THE MOON

199. The Moon's Apparent Motion among the Stars. — The moon has an apparent diurnal motion from east to west, like that of the sun and stars, which is caused by the rotation of the earth. But with respect to the stars its motion is eastward, as can be verified by observing its position on any two successive nights. Its eastward motion is also shown by the fact that it crosses the meridian later every night.

In addition to moving eastward, the moon also moves northward and southward. When it is followed during a whole revolution, it is found that its apparent orbit is very nearly a great circle which is inclined to the ecliptic at an angle of nearly 5° 9′. If there were no perturbations, the apparent orbit with respect to the center of the earth would be exactly a great circle.

The point where the moon's orbit crosses the ecliptic, going eastward, from south to north is called the *ascending node;* and the point where it crosses the ecliptic in the other direction is called the *descending node.* The position of the ascending node has an important influence on the moon's apparent motion. When the ascending node is at the vernal equinox,

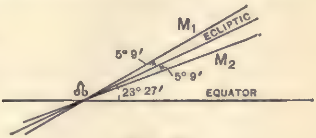


FIG. 93.

the moon's orbit makes an angle of $23° 27' + 5° 9' = 28° 36'$ with the equator, but when the ascending node is at the autumnal equinox it makes an angle of $23° 27' - 5° 9' = 18° 18'$ with the equator.

Suppose an observer is in latitude $+40°$. Then the equator crosses his meridian at an altitude of $50°$. When the ascending node of the moon's orbit is at the vernal equinox, the highest altitude of the moon during the month is $50° + 28° 36' = 78° 36'$, and its lowest meridian altitude is $50° - 28° 36' = 21° 24'$. On the other hand, when the ascending node is at the autumnal equinox the highest and lowest meridian altitudes in a month are respectively $50° + 18° 18' = 68° 18'$, and $50° - 18° 18' = 31° 42'$. When the ascending node is at some other point of the ecliptic, the highest and lowest meridian crossings are between these extremes. The results are similar for an observer in any other latitude. The nodes make a revolution in the retrograde direction in about 18.6 years, and all the possible changes take place during that period.

200. Synodical and Sidereal Periods. — The synodical period is the time it takes the moon to move from any apparent position with respect to the sun, as in conjunction with it, to the same relative position again. It is most easily and accurately determined by comparing ancient and modern eclipses of the sun, for at these times the moon was between the earth and the sun. To fix the ideas, suppose an eclipse of the sun was observed 2000 years ago. The time at which it occurred will not have been accurately observed and recorded, and our knowledge of it will be correspondingly uncertain. Suppose, for example, that the uncertainty is 2 hours, and that an eclipse is now observed with a negligible error in the recorded time. The problem is to find the moon's synodic period. Suppose the whole period expressed in days and fractions of days is T plus or minus the uncertainty of 2 hours. Then the period of one revolution is this whole period divided by the number of revolutions, which can be found from an approximate period based on observations for a few months. There are about 12.4 synodical months in a year, so that in 2000 years the moon will

have made 24,800 synodic revolutions. Hence the period of the synodic month is found to be

$$\frac{T}{24800} \pm \frac{2\text{ hr.}}{24800}.$$

The uncertainty is $\frac{2\text{ hr.}}{24800} = .29$ sec. The actual average synodic period is 29 da. 12 hr. 44 m. 2.86 sec., or 29.53059 days, with an uncertainty of less than one-tenth of a second.

The sidereal period is the time it takes the moon to move from any apparent position with respect to the stars to the same position again. It can be found by direct observations, the chief difficulty being to locate the position of its center. Another method is to compute it from the synodic period and the earth's period of revolution around the sun, which are related by the equation (Art. 154)

$$\frac{1}{S} = \frac{1}{M} - \frac{1}{E},$$

where S is the moon's synodic period, M its sidereal period, and E the length of the sidereal year. The quantities must all be expressed in the same units.

The sidereal period is not so long as the synodic, for the latter is the time it takes the moon to overtake the sun, which is also moving eastward. From the equation above, and also from observations, it is found that the sidereal period is 27 da. 7 hr. 43 m. 11.55 sec., or 27.32166 days. When the period of the moon is referred to in this book, the sidereal period is meant unless otherwise stated.

The periods which have been given are averages, for the perturbations cause them to vary considerably. It follows from the moon's period that its hourly motion is about 33′ on the average. Since one perturbation, the evection (Art. 170), is about 1.27° at its maximum, it follows that the

length of the month may vary by considerably more than 2 hours.

201. Daily Retardation of the Moon's Transits across the Meridian and the Time of Rising. — Since the moon's eastward motion among the stars is faster than the sun's, it follows that it crosses the meridian later every day. It gains a whole revolution on the sun in a synodic period, consequently the average gain in one day is $360° \div 29.5306 = 12°\ 11.4'$. It takes the earth nearly 49 minutes to turn through this angle, but in the meantime the moon has moved forward about $25'$, and it takes the earth nearly 2 minutes more to overtake it.

The period may be computed exactly without difficulty. In a synodic period the moon has gained 24 hours on the sun, and it has crossed the meridian one time less than the sun has. Therefore its gain between two successive transits is 24 hr. $\div\ 28.5306 = 50.5$ m. This period varies considerably because the moon's motion in its orbit is not uniform, and also because its orbit is inclined to the equator. The reasons are the same as those which cause the lengths of the solar days to vary (Art. 187).

The average time of the moon's rising is retarded the same as that of its transit across the meridian, but the variations

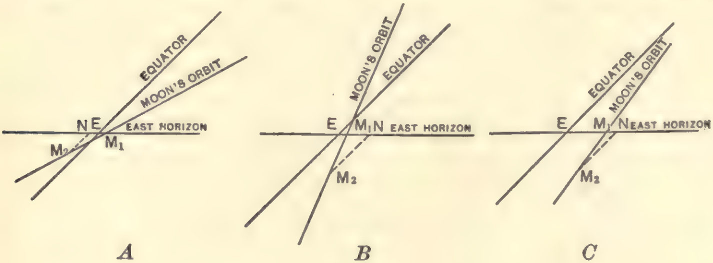


FIG. 94.

are very much greater, depending upon the way its orbit cuts the horizon. Figure 94 shows, in *A*, the situation when

the moon's ascending node is near the east horizon; in *B*, the situation when the descending node is near the east horizon; and in *C* an intermediate case. In each case *E* is the east point, M_1 the position of the moon when it is on the horizon, M_2 its position 24 hours later, and $\overline{NM_2}$ the angle through which the earth must turn in order that the moon may be at the horizon on the second day. It is seen at once that the retardation is least when the angle between the orbit of the moon and the horizon is smallest.

Now the moon moves throughout its whole orbit every month, and in this time it is in the three positions shown in the figure and all the intermediate ones. For this reason the amount of retardation in rising varies greatly during each month, being once very slight and once very great, the amounts depending on the latitude of the observer. For example, if the observer's latitude were great, the angle between the equator and horizon would be small, and the orbit of the moon in the condition shown in *A*, Fig. 94, would nearly, or perhaps exactly, coincide with the horizon. In this case the moon would rise at nearly the same time on successive nights when it was near the ascending node of its orbit. The latitude might even be so great that it actually would rise earlier some night than the preceding one.

It will be shown in the next article that the phenomenon of the slight retardation in the time of the moon's rising occurs when it is full only in the autumn. In September it is known as the *Harvest Moon* from the fact that years ago, when crops were gathered by hand, time was precious and moonlight often useful. In October it is known as the *Hunter's Moon* for reasons which are perfectly obvious.

202. The Moon's Phases. — The moon shines only by reflected light, and its phases as seen from the earth depend upon its position relative to the sun. Figure 95 shows the relation of the earth, sun, and moon at the moon's four prin-

cipal phases, and what portions of the moon are illuminated as seen from the earth. It is seen that the horns of the moon always point away from the sun. The line which separates the illuminated portion from the unilluminated portion is called the *terminator*.

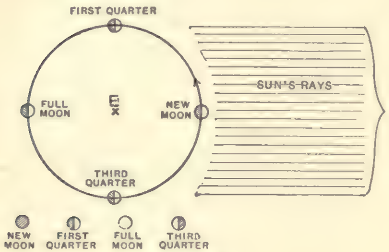


FIG. 95.

When the moon is new it is said to be in *conjunction* with the sun, and when it is full, in *opposition*. In either case, that is when the earth, moon, and sun are in line, the moon is said to be in *syzygy*. Its angular distance from the sun as seen from the earth is its *elongation*, and it is said to be in *quadrature* at the first and third quarters. The whole nomenclature is parallel to that used in case of the superior planets.

It follows from the diagram that the full moon is 180° from the sun. Suppose for the moment that the moon's orbit coincides with the ecliptic, and let us find the time of the year at which the retardation in the rising of the full moon is the least. From Fig. 94, *A*, it is seen that the retardation is the least when the moon is at that part of its orbit which intersects the equator from south to north. That is, it is near the vernal equinox, and since by hypothesis it is full, the sun must be 180° distant, or at the autumnal equinox. The sun is near the autumnal equinox in September and October, and it is, therefore, only at this time of the year that the phenomena of the Harvest and Hunter's moons can be observed.

To illustrate still more fully the relations involved, let it be required to find the time of the year that the retardation of the rising of the moon is least at the first quarter. The retardation is always least when the moon is near the vernal

equinox; consequently it is first quarter when the moon is at the vernal equinox. At this phase the sun is 90° west of the moon, and has, therefore, a right ascension of 18 hours. The sun has this right ascension in December; therefore the retardation in rising at the first quarter is least in December and January.

When the moon is new, the sunlight reflected from the earth partially illuminates its dark side and gives what is known as *earth shine*. This earth shine is probably twenty times as bright as moonlight is on the earth at the time of full moon, and enables us to see the moon's whole outline.

203. Distribution of Sunlight and Moonlight. — The amount of moonlight received by the earth is relatively unimportant except near the time of full moon; hence in the present discussion only this phase will be considered.

It follows from the relations of the sun and moon that the full moon is 180° from the sun, and rises as the sun sets and sets as the sun rises. When the sun is on the part of the ecliptic south of the equator, the full moon is near the part of the ecliptic which is north of the equator, and *vice versa*. Therefore, when the sun's rays strike the earth's surface obliquely as it crosses the meridian, the moon's rays strike it more nearly perpendicularly. When it is perpetual night in the polar regions, the gloom is partially dispelled by the moon always being above the horizon from first quarter to third quarter, and about as far above when it is full as the sun is below.

The inclination of the moon's orbit to the ecliptic and the revolution of its nodes modify these results to some extent, sometimes exaggerating them and at others minimizing them.

QUESTIONS AND EXPERIMENTS

1. Verify by observations the moon's eastward motion among the stars and estimate the distance moved in one day. If it is near a bright star, verify it in a single evening.

2. Verify its northward and southward motions by observing its altitudes as it crosses the meridian in its different phases, or the points of the horizon where it rises.

3. What is the greatest motion in declination the moon can have in a month? The least?

4. What is the position of the line of nodes when the moon's monthly motion in declination is equal to the sun's annual motion in declination?

5. Suppose an eclipse was observed 250 years ago and that its time was determined with an error not exceeding 2 minutes; if it is compared with a recent eclipse whose time is known with an error not exceeding 30 seconds, what is the limit of error in the synodic period found from these data?

6. In comparing different eclipses is it necessary to use the time of the same meridian for both?

7. Compute the length of the sidereal month from the formula expressing it in terms of the synodical month and the sidereal year.

8. If the moon's orbit coincided with the ecliptic, for what latitude would the moon rise on successive nights at the same time when it was near the vernal equinox?

9. Taking the inclination of the moon's orbit to the ecliptic into account, what is the lowest latitude for which the same phenomenon can be observed? How does the position of the nodes of the moon's orbit affect the result?

10. Would the earth as seen from the moon have phases as the moon does as seen from the earth? If so, what are their relations to each other?

11. At what time of the year is the daily retardation of the rising of the new moon the least?

12. Suppose the ascending node of the moon's orbit is at the vernal equinox; at what altitude does the full moon cross the meridian for an observer in latitude 42° in December, March, June, and September?

13. Suppose the ascending node of the moon's orbit is at the autumnal equinox, and make the corresponding discussion.

14. Suppose the ascending node is at one of the solstices, and make the corresponding discussion; does it make any difference at which solstice it is?

15. For what position of the ascending node of the moon's orbit is the distribution of both sunlight and moonlight, considered together, most uniform for the whole year?

16. Suppose the twilight gives sufficient illumination to enable one to make explorations until the sun is 10° below the horizon; suppose the moonlight will answer the same purpose as long as the moon is above the horizon and its phase greater than half moon; how many weeks in a year are polar explorations prevented by darkness?

204. Distance of the Moon. — The method of finding the distance to the moon, which consists in finding the difference in its direction as seen from two points on the earth whose distance apart is known, was explained in Art. 19. This article should be read again in this connection.

The observations show that the mean parallax of the moon is $57' \, 2.3''$; that is, the earth's equatorial radius subtends this angle at the moon's mean distance. From the formula (Art. 20)

$$r = 57.3 \frac{l}{D},$$

where in this case r is the distance to the moon, l the radius of the earth in miles, and D the parallax expressed in degrees, it is found that

$$r = 238{,}840 \text{ miles.}$$

The circumference of the moon's orbit is $2 \pi r = 1{,}500{,}680$ miles. Dividing this by the sidereal period expressed in hours, it is found that the moon's orbital velocity averages 2288.6 miles per hour, or 3357 feet per second.

A body falls at the surface of the earth about 16 feet the first second; at the distance of the moon it would fall about $16 \div 60^2 = 0.0044$ feet, because the earth's attraction varies inversely as the square of the distance from its center. Therefore, in going 3356 feet (nearly two-thirds of a mile), the moon's orbit deviates from a straight line only about one-twentieth of an inch.

205. Moon's Orbit with Respect to the Earth. — The moon's distance varies from about 221,600 miles to 252,970 miles, causing a corresponding variation in its apparent diameter and parallax. The orbit is an ellipse, except for the perturbations explained in Art. 170, with an average eccentricity of 0.05491; the earth is at one of its foci, and the motion is such that the law of areas (Art. 137) is fulfilled (nearly) with respect to the center of the earth as an origin. The

perturbations by the sun cause large deviations from elliptical motion; indeed, no planet or other satellite deviates so much from an elliptical orbit. The point of the moon's orbit nearest the earth is called the *perigee*, and the farthest point, the *apogee*.

206. Moon's Orbit with Respect to the Sun. — The distance from the earth to the sun is about 400 times that from the earth to the moon; consequently the oscillations of the moon back and forth across the earth's orbit are relatively so small that they can hardly be represented in a diagram. Because of this disparity of distances and the relatively long period of the moon's revolution, its orbit is always concave toward the sun.

The reason can be seen from Fig. 96, which is not, however, drawn true to scale. Suppose the earth and moon are at E_1 and M_1 respectively at a certain time. If the moon's orbit is ever convex toward the sun, it will be while its motion is away from the sun. Suppose the earth has arrived at E_2 at the end of half a sidereal month. The moon will have moved

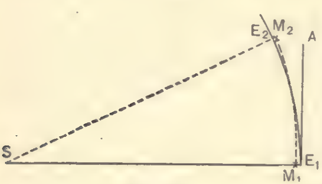


Fig. 96.

toward the line $\overline{E_1A}$ in the interval $2 \times 238{,}840 = 477{,}680$ miles. On the other hand, the appropriate computations show that the earth has moved *away* from $\overline{E_1A}$ 2,790,000 miles. That is, the moon has moved away from the line $2{,}790{,}000 - 477680 = 2{,}312{,}320$ miles, and the moon's orbit has been concave toward the sun at every point.

207. Rotation of the Moon. — The moon always keeps the same side toward the earth, and therefore, as seen from some point other than the earth, it rotates on its axis once in a sidereal month. Its direction of rotation is the same as that of revolution, or from west to east. The plane of its equator is inclined about 1° 32′ to the plane of the ecliptic,

and the two planes always intersect in the moon's line of nodes because of the disturbing effects of the earth's attraction.

The moon's sidereal day is of the same length as its sidereal month. Its solar day is of the same length as its synodic month, as can be seen from the fact that the same face is turned toward both the earth and the sun at every full moon. As compared with the earth, other things being equal, the temperature changes of day and night would be much greater because they are so much longer, and the seasonal changes would be much less because of the small inclination of the plane of the moon's equator to the plane of its orbit.

The most remarkable fact connected with the rotation of the moon, is that it is at precisely such a rate that the same side of it is always toward the earth. It is infinitely improbable that it was started exactly in this way, and the peculiar motion strongly suggests the idea that there have been forces at work which have brought about this unique condition. An explanation has been found in tidal reactions. It has been shown (Art. 177) how the moon raises tides on the earth, and in a precisely similar manner the earth gives rise to even greater tidal forces on the moon. The moon has the appearance of having been at one time in a plastic condition, when there were certainly tides. It will be shown at an appropriate place (Art. 351) that the tides tend to make equal the periods of rotation of the tidally distorted body and of its revolution around the tide-raising body. It is believed that the moon's period of rotation and revolution have become equal from this cause.

208. Librations of the Moon. — The statement that the moon always has the same side toward the earth is not true in the strictest sense. It would be so if the plane of its orbit and of its equator were the same, and if it moved at a perfectly uniform angular velocity in its orbit.

The inclination of the plane of the moon's equator to its orbit is about $5° 9' + 1° 32' = 6° 41'$. The sun shines alternately over the two poles of the earth, because of the inclination of the plane of the equator to the plane of the ecliptic. In a similar manner, if the earth were a luminous body, it would shine $6° 41'$ over the moon's two poles in a month. Instead of shining on them, except by reflected light, the tilting of the moon's axis of rotation enables us to see that distance over the poles. This is the libration in latitude.

The moon rotates at a uniform rate; at least, the departures from uniformity are absolutely insensible. It would take inconceivably great forces to make perceptible short changes in the rotation. On the other hand, the moon revolves around the earth at a non-uniform rate, for it moves in such a way that the law of areas is fulfilled. Consider the motion starting from the perigee. It takes considerably less than one quarter of the period for the moon to revolve through 90°, and the angle of rotation is correspondingly less than 90°. The result is that the part of the moon on the side toward the perigee, that is the western edge, is brought partially into view. On the opposite side of the orbit the other side is brought partially into view.

In addition to this, the moon is not viewed from the earth's center. When it is on the horizon, the line from the observer to the moon makes an angle of about one degree (the parallax of the moon) with that from the earth's center to the moon. This enables the observer to see one degree farther around its side than he could if it were on his meridian.

The result is that there is only 41 per cent of the moon's surface which is never seen, while 41 per cent is always in sight, and 18 per cent is sometimes in sight and sometimes not.

209. The Size of the Moon. — The mean apparent diameter of the moon is $31' 8''$, though the apparent diameter varies

by more than 2′ because of the eccentricity of its orbit and the perturbations of its distance from the earth. From the formula of Art. 20, and the distance of the moon as given above, it follows that its real diameter is 2163 miles. Therefore its diameter is about 27 per cent of that of the earth. Their surfaces are to each other as the squares of their diameters, or as one is to fourteen. Their volumes are to each other as the cubes of their diameters, or as one is to fifty.

FIG. 97. — Relative Size of the Earth and Moon.

When the moon is on an observer's meridian, it is about 4000 miles nearer to him than it is when it is on his horizon. Thus, when the relations are represented as in Fig. 98, the moon is about 4000 miles nearer to A than it is to B. Consequently as seen from A the apparent diameter of the moon is about one-sixtieth greater than it is as seen from B. But every one has noticed that the moon looks larger when it is near the horizon than it does when it is high in the sky. The reason is that intervening objects give us the impression that it is distant, and this influences our estimate of its size.

FIG. 98.

There is no perfectly exact meaning to such a statement as that the moon appears to be, say, ten inches in diameter. But from such a statement we can find how far we unconsciously estimate that it is away. The angular diameter is given and the supposed linear diameter is known; it is required to find the distance. The distance is given by the formula

$$r = 57.3 \cdot \frac{l}{D},$$

where l is the linear diameter, D the angular diameter expressed in degrees, and r the distance. If one says the moon appears to be ten inches in diameter, the formula shows that he unconsciously assumes that its distance is about 90 feet. The assumed distance varies directly as the estimated linear diameter.

If the moon were imagined to be at the distance of most distinct vision, 18 inches, its linear diameter, l, would be estimated to be one-sixth of an inch. Or, expressed differently, an object one-sixth of an inch in diameter held at a distance of 18 inches from the eye would just cover the moon.

210. Mass of the Moon. — Although the moon is comparatively near to the earth, its mass cannot be obtained so easily as that of some other bodies farther away. It will be remembered that the method of determining the masses of bodies is by finding the amounts of their attractions for other bodies. Thus, the mass of the earth was found by comparing its attraction with that of a heavy ball or of a mountain (Arts. 106, 107).

One of the simplest ways, theoretically, of finding the mass of the moon is from its period around the earth. The formula for the period is (Art. 181)

$$P = \frac{2\pi a^{\frac{3}{2}}}{k\sqrt{E+M}},$$

where a is half of the major axis, k a constant depending upon the units employed,[1] and E and M the masses of the earth and moon respectively. Suppose E is known and that P and a are found directly from observations. Then the unknown M can be found. The difficulty of this method lies in the fact that the moon's motion is greatly perturbed by the sun.

[1] When the mass of the sun is taken as unity $k = 0.0172021$.

Instead of the moon's going around the earth, both bodies go around their common center of gravity. Now it is this center of gravity which describes the nearly elliptical orbit around the sun. Sometimes the earth is ahead of this center of gravity and sometimes behind it. The effect is that the sun as seen from the earth is apparently sometimes behind and sometimes ahead of the position it would otherwise occupy. From very delicate observations it is found that the sun is displaced by about $6.4''$. Knowing the distance to the sun, it is found that the center of the earth is 2886 miles distant from the center of gravity of the earth and moon.

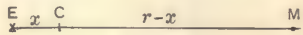


Fig. 99.

Suppose in Fig. 99 that E is the earth, M the moon, and C the center of gravity of the earth and moon. Let the distance $\overline{EC} = x$, $\overline{EM} = r$, and $\overline{CM} = r - x$. The center of gravity is defined by the equation

$$x \cdot E = (r - x)M.$$

Now the observations show that $x = 2886$ miles. Therefore $r - x = 238{,}840 - 2886 = 235{,}954$. Hence the formula gives $2886\,E = 235{,}954\,M$, or

$$E = 81.7\,M.$$

In round numbers, the mass of the earth is eighty times that of the moon.

Since the orbit of the moon is inclined $5°\ 9'$ to the plane of the ecliptic, the earth is sometimes above and sometimes below this plane. This causes an apparent displacement of the sun from the ecliptic in the opposite direction. From the amount of the displacement, determined from observations, the position of the moon's orbit, and the distance of the sun, it is possible to compute, as from the displacement in longitude, the relative mass of the moon.

211. The Density and the Surface Gravity of the Moon.—The volume of the moon is about one-fiftieth that of the earth

and its mass is $\frac{1}{81.7}$ that of the earth. The density of the earth on the water standard is 5.53 (Art. 105). Therefore the density of the moon can be obtained from the equation density × volume (of the earth) = 81.7 density × volume (of the moon); whence the density of the moon

$$d' = \frac{1}{81.7} \cdot \frac{\text{volume of the earth}}{\text{volume of the moon}} \cdot \text{density of the earth};$$

or

$$d' = \frac{1}{81.7} \times 50 \times 5.53 = 3.4.$$

It was shown in Art. 182 that the surface gravity is given by the equation

$$G' = \frac{d'R'}{dR} G,$$

where G is the gravity at the surface of the earth, d and R the density and the radius of the earth, and the accented letters the corresponding quantities for the other body. It follows from the numerical values of these quantities for the moon which have been given that $G' = \frac{1}{6} G$. That is, any body would weigh by a spring balance about one-sixth as much on the moon as it would on the earth. If mountains on the moon were composed of the same material as those on the earth, they could be six times as high without crushing the rock at the bottom. If a body were projected upward, say by volcanic action, it would go six times as high. A force would give a mass the same velocity upward as it would on the surface of the earth, but the moon's attraction would not stop it so quickly as the earth's would.

212. The Atmosphere of the Moon. — The moon has no atmosphere, or, at the most, an excessively rare one. Its absence is proved by the fact that at the time of an eclipse of the sun, the moon's limb is perfectly dark and sharp with no distortion of the sun due to refraction. Similarly, when

a star is occulted by the moon it disappears suddenly, and not somewhat gradually as it would if its light were being more and more extinguished by an atmosphere. Besides this, if the moon had an atmosphere, its refraction would keep a star visible for a little time after it had been occulted, just as the earth's atmosphere keeps the sun visible after it has actually set. In a similar way, the star would become visible a short time before the moon had passed out of line with it. The whole effect would be to make the time of occultation shorter than it would be if there were no atmosphere. This is a very delicate test, and observations show conclusively that the moon has no appreciable atmosphere.

If the moon had an atmosphere, there would be the effects of erosion on its surface, but so far as can be determined it gives no evidence of such action. It consists entirely of a rocky surface, which is perhaps much cracked up because of the extremes of heat and cold to which it is subject, but there is nothing like soil except possibly volcanic ashes.

There can be no water on the moon, for if there were it would be evaporated, especially during the long day, and form an atmosphere. Because of the absence of a sensible atmospheric pressure, the evaporation would be very rapid, just as it is more rapid on the earth in the rare atmospheres of high mountainous regions. Yet W. H. Pickering believes he has some observational evidence of the existence of snow, lakes, and vegetation.

One cannot refrain from asking why the moon has no atmosphere. It may be that it has never had an atmosphere. But the abundant evidence of volcanic action on its surface makes it very probable that vast quantities of vapors have been emitted from its interior. If this is true, they seem to have disappeared. There are two ways in which this could have taken place. One is by the surface vapors uniting chemically with other constituents in forming solids. There

are vast quantities of oxygen in the rocks of the earth's crust which may, possibly, have been largely derived from a great original atmosphere. And, according to the kinetic theory of gases, the moon could also have lost its atmospheric gases by the escape of molecule after molecule from its gravitative control. This would be a rapid process in the case of a body having the low velocity of escape of 1.5 miles per second (Art. 111), especially if its days were so long it became highly heated. It seems probable, therefore, that the moon could not retain an atmosphere if it had one, and that whatever it ever may have acquired from volcanoes or other sources was speedily lost.

213. Light and Heat received by the Earth from the Moon. — The moon is on the average just about as far from the sun as the earth is, and consequently receives about the same amount of light and heat per unit area as the earth receives. Consider the question of the amount of light and heat received by the earth from the moon at the time of full moon. The surface of the moon receives about one-fourteenth as much light from the sun as the earth does, for its surface is only one-fourteenth as great. Suppose all that the moon receives is reflected so as to illuminate uniformly half the sky. The assumption is incorrect, for light is reflected to more than half the sky. If it were not, we could not see the moon at all until the first quarter. But light is not reflected equally over the whole part which is illuminated. The reflection is greatest back toward the sun, and consequently toward the earth at the time of full moon. Thus the two errors in the assumptions, to some extent, balance each other, and the computation will give an idea of what to expect.

The radius of the earth as seen from the moon is nearly 1°. Therefore its apparent surface in degrees is

$$\pi \times 1^2 = 3.1 \text{ square degrees.}$$

The surface of the hemisphere whose radius is R, the distance

from the earth to the moon, is $2\pi R^2$. To express the area in arc measure, R must be replaced by 57.3, from which it follows that

$$2\pi R^2 = 20{,}630 \text{ square degrees.}$$

Consequently, under the hypotheses, the earth would receive about $\frac{3.1}{20{,}630} = \frac{1}{6660}$ of the light which fell upon the moon. Since the moon received one-fourteenth as much as the earth, the moonlight would be about

$$\frac{1}{14} \times \frac{1}{6660} = \frac{1}{93{,}240}$$

that of sunlight.

Now the moon does not reflect all the light received. According to Zöllner, its reflecting power, or, technically, its *albedo*, is only 0.174. Consequently, the amount of light received from the moon would be only about one-sixth of that given above, or $\frac{1}{559{,}440}$ that of sunlight.

It is not easy to compare moonlight with sunlight by direct measurement, and the results obtained by different observers differ considerably. Zöllner's result, which is near the mean and which is commonly accepted, is that the light of the full moon is $\frac{1}{618{,}000}$ that of sunlight. The light received from the moon at any other phase is less in proportion to the illuminated part which is visible, and taking into consideration the whole month, the average amount of light and heat received from the moon is less than one-millionth of that received from the sun. More light and heat are received from the sun in 30 seconds than are received from the moon in a year.

214. Temperature of the Moon. — The temperature of the moon depends upon the amount of heat received, the amount reflected, and the rate of radiation. That which is directly reflected does not heat the moon. This is about 17 per cent, as was stated in the last article; the remaining 83 per cent is absorbed by the moon and raises its temperature. The rate of radiation at a given temperature is not certainly known,

and cannot easily be found. When the moon is eclipsed by the earth, the radiation of heat quickly ceases, which shows that the radiation is rapid. Therefore, the temperature does not become very high, even though the sun shines continuously on a given part of the moon for more than fourteen of the earth's days. On the other hand, Very has concluded from recent observations that during the lunar day the temperature is high, probably more than 200° Fahrenheit. One thing is certain, and that is that during the long lunar night the temperature falls very low, perhaps 200° or 250° below zero Fahrenheit. The lowest temperature ever known in the Arctic regions was about 90° below zero.

These variations in the moon's temperature illustrate in the most striking manner the effects of the earth's atmosphere in retaining heat and equalizing the climate.

215. General Surface Conditions of the Moon. — On the whole the surface of the moon is very rough, showing no effects of weathering by air or water. There are many large, fairly level, and smooth areas, which were called seas ("maria") by the early observers, and the names have been retained, although it is now universally believed that they contain no water. They are given descriptive Latin names, as *Mare Nectaris* and *Mare Serenitatis*. There are a few mountain ranges named after terrestrial ranges, as the *Apennines* and *Alps*. There are a great many craters, which are named after distinguished men, as *Tycho*, *Copernicus*, and *Theophilus*. The map on the next page shows the positions of some of the principal features as seen in an ordinary inverting telescope.

216. Lunar Mountains. — There are ten mountain ranges on the moon. The mountains are extremely jagged and high for their dimensions, reaching up to more than 20,000 feet above the plains on which they stand. If the mountains on the earth were on the same scale, they would be more than fifteen miles high. The height of the lunar mountains is

undoubtedly due, at least partly, to the low surface gravity and the lack of erosion.

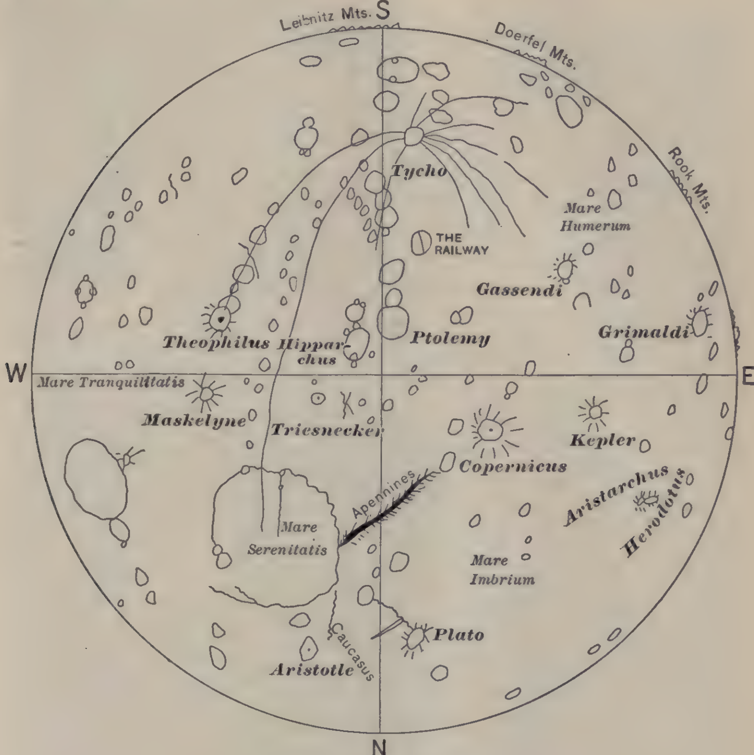


Fig. 100. — Map of Moon.

The height of a mountain on the moon is found from the length of the shadow it casts. Thus, in Fig. 101 (see also peaks in Theophilus, Fig. 104) the mountain M casts a shadow reaching to A. Since the size of the moon is known, the length of the shadow $\overline{AM}$ can be computed in miles. The angular altitude of the sun is equal to the angular

distance from M to the terminator P. Then it is an easy matter to compute the actual height of the mountain from these data. Since there is no atmosphere on the moon, there is no twilight, and the shadows are black and easily observed.

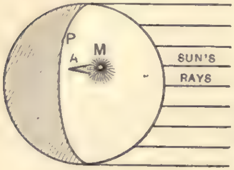


FIG. 101.

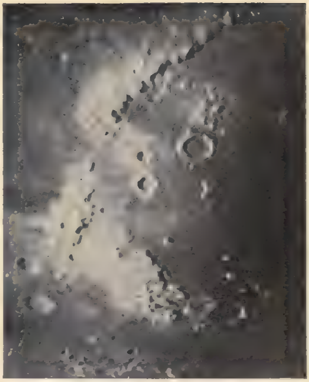


FIG. 102.—Region about the crater Archimedes, showing part of the Apennines and Mare Imbrium. *Photographed at the Lick Observatory.*

217. Lunar Craters. — The most remarkable and conspicuous objects of the lunar topography are the craters, of which more than 30,000 have been mapped. There have been successive stages of activity, for new craters everywhere have broken through and encroached upon the old, like Kilauea on the ruin of Mauna Loa. Often new craters appear within old ones. The newer ones have deeper floors and steeper and higher rims. One of the most interesting things about craters is that

FIG. 103.—Lick Observatory photograph showing how new craters appear around and in old ones.

very many of them have in their centers lofty and spire-like peaks.

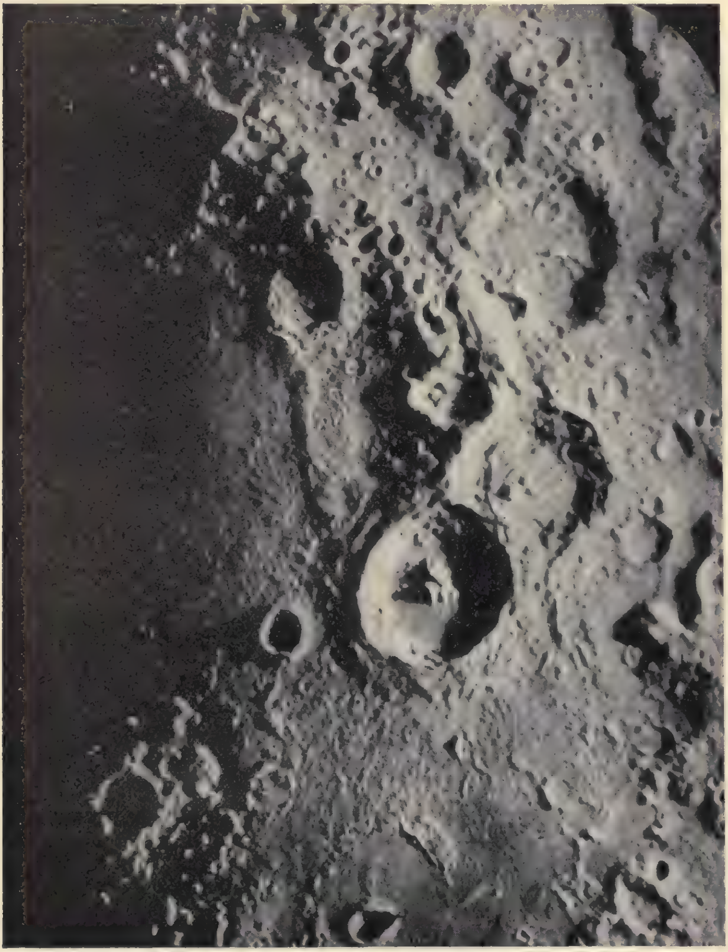


FIG. 104.—Crater Theophilus (diameter 64 miles, depth 19,000 feet). *Photographed by Ritchey with the 40-inch Yerkes Telescope.*

The craters on the moon are of immense size, often being 50 or 60 miles in diameter, and in some cases reaching up even beyond 100 miles. Ptolemy is 115 miles across, while Theophilus is 64 miles in diameter and 19,000 feet deep. The peak in the great crater Copernicus towers 11,300 feet above the floor from which it rises.

FIG. 105. — Crater Copernicus. *Photographed by Ritchey at the Yerkes Observatory.*

The very scale of the lunar craters raises difficulties in explaining their cause. If they are of volcanic origin, the activity which was at one time present there enormously surpassed anything of which the earth now gives any evidence. The cones in the centers of the craters are also peculiarities which have nothing analogous in terrestrial volcanoes. Water is supposed to play a part in volcanic action on the earth, but there is no evidence that there ever has been any on the moon. Gilbert suggested that the lunar craters may have been formed by the impact of meteorites, but it is more generally believed that they are the undisturbed records of the violent volcanic action to which worlds are subject in the early stages of their evolution. There are difficulties in this view, for there is not the distinct evidence of lava flow that one would expect, and in general the whole rim of the crater would not nearly fill the cavity if it were put into it.

Another theory is that the craters were formed by the bursting of huge bubbles which once covered the moon. When the interior gases were relieved, the relatively thin coverings fell back into the cavities.

218. Rays and Rills. — Some of the large craters, particularly Tycho and Copernicus, have long, light streaks called

Fig. 106. — Full Moon, showing rays from Tycho and Copernicus. *Photographed by Wallace with the 12-inch refractor of the Yerkes Observatory.*

rays, radiating from them like spokes from the axle of a wheel. They are not interfered with by hill or valley, and they often extend a distance of several hundred miles. They cast no shadows, which proves that they are of the same level as the adjacent surface, and they are most conspicuous at the time of full moon. It has been supposed by some that they

are lava streams, and by others that they were great cracks in the rocky surface formed at the time when the craters were active, which have since filled up with lighter-colored material from below.

The rills are long cracks in the moon's surface, a mile or so wide, a quarter of a mile deep, and sometimes as much as 150 miles long. They are very numerous, more than a thousand having been so far mapped. The only things like them on the earth are the Grand Cañon of the Colorado and the cut below Niagara Falls, but these gorges are the work of erosion, which has probably been absent from the surface of the moon. The most plausible theory is that they are cracks which have been caused by violent volcanic action or by rapid cooling of the surface.

The rays and rills are very puzzling lunar features which seem to be fundamentally unlike anything in terrestrial topography. Even our nearest neighbor differs very radically from the earth.

219. Changes on the Moon. — There have been no observed changes in the larger features of the lunar topography, although from time to time some minor alterations have been suspected. The most probable change is in the small crater Linné, in the Mare Serenitatis, which was mapped about a century ago, but which was said by Schmidt to be invisible in 1866. It is now visible as originally. It is generally believed that the differences in appearance at the different times have been due to slightly different phases and illumination. Since the moon's orbit is constantly shifting with respect to the ecliptic, and since the month does not contain an integral number of days, it follows that an observer never gets twice exactly the same view of the moon. The differences in appearance due to differences in phase are illustrated in the accompanying photographs.

It is altogether probable that the moon long ago cooled to the point where volcanic action ceases, and since there is

neither air nor water on its surface, the sources of disturbance are the tidal strains and the extremes of temperature between night and day. While it would be too much to say that slight crumbling and disintegration is not still taking place, yet it is certain that, on the whole, the moon is a body whose internal evolution is essentially finished. The seasonal changes are unimportant, but there is alternately the blinding glare of a sunlight never tempered by passing clouds or even an atmosphere, and the blackness and frigidity of the long lunar night. Month succeeds month without any important variation of the phenomena.

220. Seeing the Moon through a Telescope. — When a person looks through a telescope at the moon for the first time, he is astonished that he can see the markings on its surface much plainer without its looking larger. Of course, his idea that it does not look larger is an illusion due to the fact that he has nothing with which to compare it. If the observer looks through the tube with one eye, and along the outside with the other, he will at once convince himself that the reason the telescope shows the surface markings so plainly is because it magnifies.

It is frequently asked how near a particular telescope will apparently bring the moon. This depends upon the magnifying power, which in turn depends upon the eyepiece, as was shown in the chapter on telescopes. A magnifying power of 100 diameter apparently reduces the distance of the moon to $\frac{1}{100}$ of its actual amount, or to 2400 miles. A magnifying power of 1000, which is about as much as is ever used even on the largest telescopes, apparently brings it to within 240 miles of us. These theoretical results are by no means realized in practice. No telescope shows the moon so well as we could see it if it were only 240 miles from us, for the image is subject to a great variety of errors, which are magnified just the same as those things which we wish to observe.

Nearly every one would like to see the moon through a great telescope like that at the Yerkes observatory with the highest-power eyepiece. If his wish were gratified, he would be greatly disappointed, for he would see such an extremely small part of the whole surface that he would not know whether he was looking at the moon or something else. It is easy to see why only a small part of the moon can be seen at once with a high power. The diameter of the moon is a little over half a degree, and if a power of 1000 were used, the whole apparent diameter would be 500°, or nearly a circumference and a half. Evidently this could not all be seen at one time. For an inexperienced observer a telescope of a few inches in diameter is the most satisfactory because of the ease with which it can be manipulated; and a moderately low-power eyepiece should be used so that a considerable part of the moon's surface can be seen at one time. Using a high power is like trying to get an idea of a strange tree through a microscope.

The moon presents a surface to us, and it agrees more nearly with the popular use of terms to call the areal magnification the magnifying power employed, although this is not the usage of astronomers. If a combination of objective and eyepiece magnifies 100 diameters, the areal magnification is $100^2 = 10,000$ times. Thus, when a power of 100 is used, every mountain and crater on the moon looks 10,000 times as large as it does without it.

It is frequently supposed that the best time to view the moon is when it is full, but this is not so. At this phase we see that part which is entirely illuminated, and there are no shadows to bring out the different features. Photographers understand the value of shadows, for they always seat one for a portrait in such a way that the light is strongest from one side. If they should take a front view with the whole face uniformly illuminated, it would look flat. Likewise the full moon looks relatively flat. The most satisfactory view

is obtained when it is near the quarter, for then the mountains near the terminator cast their long, pointed shadows and the walls of the craters shade their floors.

221. Effects of the Moon on the Earth. — The moon reflects sunlight to the earth, and it produces the tides. These are the only influences that can be observed by the ordinary person. It has quite a number of minor effects, such as changing the position of the earth and causing minute deflections of the magnetic needle, but they are all so small that they can be detected only by refined means.

There are a great many ideas popularly entertained, such as that it is more liable to rain at the time of a "change" of the moon, or that crops grow best when planted in certain phases, which have no scientific foundation whatever. It is seen from Art. 213 that the amount of heat received from the moon is insignificant compared to that received from the sun, and it is unreasonable to suppose that it produces any sensible climatic changes. Recorded observations extending over more than 100 years fail to show any certain relation between the weather and the phase of the moon.

The phenomena of storms also show the practical independence of the state of the weather and the phase of the moon. The storm centers move across the country from west to east at the rate of 400 or 500 miles per day, and they can sometimes be followed entirely around the earth. If one passes over a certain locality and causes precipitation at the time the moon "changes," it will pass other places when the moon does not "change." For the first place the theory would be verified and for the others it would not. The moon is popularly supposed to "change" when it is new, and full, and at the quarters. As a matter of fact it changes all the time, and it would be just as reasonable to divide the month into three parts as into four. These superstitions violate the law of cause and effect, and may be taken as illustrating the wonderful vitality certain classes of errors possess.

QUESTIONS

1. How great an error in the computed distance of the moon would an error of 1″ in its parallax make?

2. Draw 60° of the orbit of the earth, and draw along it the orbit of the moon true to scale, supposing that the earth's orbit is a circle, and that the orbit of the moon with respect to the earth is also a circle.

3. Devise a simple experiment to illustrate the rotation of the moon.

4. Describe in detail the apparent motions of the earth as seen from the moon.

5. Does the sun have a diurnal motion as seen from the moon? Does it have an annual motion with respect to the stars?

6. Draw diagrams illustrating the librations of the moon.

7. How large does the moon appear to you? From this compute your unconscious estimate of its distance.

8. Find how far from the eye a coin has to be held to just cover the moon. Measure the diameter of the coin and find what angle it subtends, that is, the angular diameter of the moon.

9. Where is the center of gravity of the earth and the moon with respect to the surface of the earth?

10. If a baseball player could live on the moon, could he throw a ball with any greater speed than he could on the earth? Could he throw it any farther?

11. Find Tycho, Copernicus, Kepler, Herodotus, Theophilus, the Apennines, Mare Serenitatis, and Mare Imbrium in Fig. 106.

12. Can you find more than two generations of craters in any part of Fig. 103?

13. Note the numerous small isolated mountains in Fig. 105.

14. In Fig. 105 find mountains which cast more slender shadows than are cast by the peaks in Theophilus.

CHAPTER X

ECLIPSES

222. Conditions for Eclipses. — The moon is eclipsed when it passes into the earth's shadow so that it does not receive the direct light of the sun. The sun is eclipsed when the moon passes between it and the earth so as to cut off its light from a portion of the earth. Thus, in Fig. 107 (which is

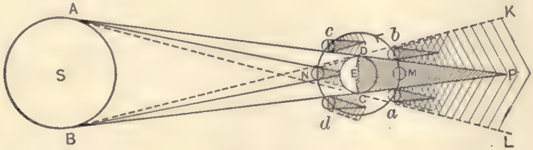


FIG. 107.

not drawn to true proportions), if the moon is anywhere between *a* and *b*, it will be at least partially eclipsed, while if it is anywhere between *c* and *d* it will eclipse the sun for at least some part of the earth.

The shadow *PCD* is the *umbra*, and the parts *PCL* and *PDK* are the *penumbra*. A point anywhere in the umbra receives no light directly from the sun, while a point in the penumbra receives light from only a portion of the sun. The illumination of the penumbra decreases steadily from its outer borders to the complete darkness of the umbra.

Sometimes the moon does not pass entirely into the umbra, and then it is only partially eclipsed. Every eclipse of the moon is visible at one time from a whole hemisphere of the earth, and from the additional part which is rotated into view while it lasts.

The appropriate investigation shows that the angular distance $\overline{ab}$ as seen from the earth is, on the average, 1° 53′. The variations from the average are due to the varying distances of the sun and moon. Since the moon gains about one-half of a degree an hour on the sun, and therefore also on the earth's shadow, it moves straight through the shadow from first contact to last contact in about 3 hr. 45 m. The time of total eclipse is less by the time it takes the moon to move twice its diameter, or nearly 2 hours.

When the moon is at *N*, or anywhere between *c* and *d*, which are 2° 58′ apart, the sun is eclipsed. If the point of the shadow cone reaches beyond the surface of the earth, the eclipse is total for places within the small cone, whose diameter at the surface of the earth never exceeds about 165 miles. If the apex of the moon's shadow cone falls short of the surface of the earth, as it often does, the sun will not be entirely obscured as seen from any point on the earth. For an observer in the axis of the cone the sun will appear to be all covered except a narrow ring around the outside. Such an eclipse is called an *annular* (ring) eclipse. On the average there are seven annular eclipses in eight years.

The apparent diameter of the moon is never much greater than that of the sun. Hence the time of a total eclipse of the sun is short, never exceeding about 7 m. 40 sec. at the earth's equator, and about 6 minutes in latitude 40°.

The moon's shadow moves very rapidly across the earth. The circumference of the moon's orbit divided by its synodic period gives the orbital velocity of its synodical motion, which is about 2120 miles per hour. The shadow moves at nearly the same rate, but since the earth rotates in the same direction at the rate of 1040 miles per hour, the motion of the shadow with respect to the earth's surface at the equator is about $2120 - 1040 = 1080$ miles per hour. Where it strikes the earth obliquely, it is very much greater than this.

Figure 107 is drawn as though the moon's orbit were in

the plane of the ecliptic. If this were so, there would be an eclipse of both the sun and the moon every synodical month. But it was seen in the last chapter that the moon's orbit is inclined to the ecliptic at an angle of 5° 9′; consequently it may pass above or below the shadow cone, so that there will be no eclipse at all. The effects of this inclination must now be considered.

223. Limits for Lunar Eclipses. — Let the plane of the ecliptic and the moon's orbit be represented as in Fig. 108. Let A be the ascending node of the moon's orbit and B its descending node. The earth's shadow travels along the ecliptic at a distance of 180° from the sun. Let $S_1, \cdots, S_4$ be four positions of this shadow. It takes the earth's shadow one year to pass from A around the ecliptic back to A again (neglecting the motion of the node), but the moon moves from A around its orbit back to A again in a sidereal month. If the moon passes the node A while the earth's shadow is anywhere between S_1 and S_2, it will be at least partially eclipsed. The condition at the descending node B is entirely similar.

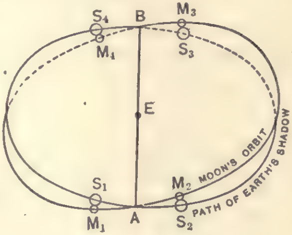


FIG. 108.

Now let us find the distance from S_1 to A. Consider, for simplicity, that S_1AM_1 is a plane triangle; let D represent the angle at A, r the distance $\overline{S_1A}$, and l the distance $\overline{S_1M_1}$. Then the formula (Art. 20) gives

$$r = 57.3 \cdot \frac{l}{D}.$$

In the present case $D = 5° \, 9' = 5.15°$, and l expressed in arc measure is $56.5'$, or half of $\overline{ab}$ (Fig. 107). Substituting these numbers in the formula, it is found that

$$r = 10.5°.$$

Because of the variations in the inclination of the moon's orbit, and in the distances of the sun and moon, this limit oscillates about 1° each way from the value given. In the further discussion the mean value will be used.

224. Number of Lunar Eclipses in a Year. — The distance from S_1 to S_2 (Fig. 108) is on the average about 21°. The sun and the earth's shadow move along the ecliptic at the same rate, or 0.9856° daily. Suppose the moon passes the shadow at S_1 so that it barely misses being partially eclipsed. It will take a synodic month, or 29.53 days, for it to get in conjunction with the shadow again. But in the meantime the shadow will have moved 29.105° from S_1, or beyond S_2, and there will be no eclipse. It follows similarly that if there had been a partial eclipse near S_1 there would have been no eclipse at all near S_2. Therefore we have the following conclusions:

(1) *The earth's shadow may pass one of the nodes of the moon's orbit without there being an eclipse of the moon.*

(2) *There can be but one eclipse of the moon while the earth's shadow is passing one of the nodes of its orbit.*

Suppose that the moon is not eclipsed near S_1, and that it just misses being eclipsed at S_2; let us find whether or not there will be an eclipse while the earth's shadow is passing the node B. In 5 synodic months, or 147.65 days, the earth's shadow and the moon will be in conjunction somewhere near B. In 147.65 days the earth's shadow will move forward 145.5°, and it will be $145.5° + 10.5° = 156.0°$ beyond A. If the moon's nodes were stationary, the conjunction would be $180° - 156.0° = 24.0°$ from B, but they revolve in the retrograde direction once in 18.6 years. Therefore, in 5 months they will have moved backward about 8°, and the conjunction will be $24° - 8° = 16°$ from the node B. It follows that there will be no eclipse near S_3, and by counting forward a synodical month it is seen that there will be none near S_4. By a similar discussion it can be shown that, if there is an eclipse of the moon central at A, there *must* be one near B.

From this discussion it follows that *in a year there may be two eclipses of the moon, or one, or none.* When there are two, they occur six synodic months apart. There is an exceptional case which may happen once in a century or two. There might be an eclipse near the 1st of January, another 6 synodic months, or 177 days, later, and still another 177 days later, which would still be before the end of the year. Only in this way can there be three eclipses of the moon in a year.

225. Character of Succeeding Lunar Eclipses. — Suppose an eclipse of the moon is central at A; in 177.2 days there will be another 174.6° farther along. But in the meantime the node B will have moved back 9.4°, and the eclipse will be $174.6° - (180° - 9.4°) = 4.0°$ beyond it. The moon will pass through the southern part of the shadow.

Consider the following conjunction near A. It will occur 174.6° beyond the point of conjunction near B, or $174.6° + 4.0° = 178.6°$ beyond B. In the interval the node A will have moved back 9.4°; therefore the conjunction will be $178.6° - (180° - 9.4°) = 8.0°$ beyond A, and there will be an eclipse. In this case the moon will pass through the northern part of the earth's shadow.

The conjunction at the end of the next 177.2 days will occur 12° beyond B, and there will be no eclipse; nor will there have been one 29.5 days earlier, for the eclipse limit was missed on the other side. In five synodic months, or 147.65 days, there will be another conjunction $12° + 145.5° = 157.5°$ beyond B. During this time A moves backward 7.8°, so that the conjunction is $(180° - 7.8°) - 157.5° = 14.7°$ before A. This is outside of the eclipse limit, and 29.5 days later the conjunction will be beyond the eclipse limit on the other side. After 177 days more there will be a conjunction 10.7° before B, which is so near the eclipse limit that there may or may not be an eclipse, according as the distances of the sun and the moon, and the position of the moon in its orbit, satisfy the requisite conditions or not.

This process may be continued as far as is desired. It must be remembered, however, that it gives only average results because of the varying distances of the sun and moon and their irregular motions.

226. Solar Eclipse Limits. —Let the ecliptic and the moon's orbit be represented on a plane as in Fig. 109. Let S_1, S_2, S_3, S_4 represent four apparent positions of the sun's center

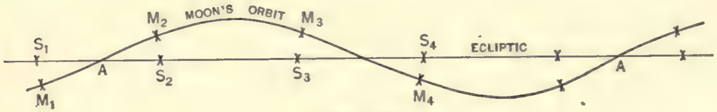


FIG. 109.

on the ecliptic such that, if it is in conjunction with the moon, it will be at least partially eclipsed as seen from some point on the earth.

In Fig. 108 the moon was drawn touching the circle of the earth's shadow at the eclipse limit, but in Fig. 109 it would not be correct to show the moon as apparently touching the sun at an eclipse limit. The reason is, as is evident from Fig. 107, that the sun might be eclipsed as seen from one point on the earth's surface and not as seen from another. When the moon is near c, one part is eclipsed, and when it is near d, the opposite part is eclipsed.

The average angular distance $\overline{c\,d}$, Fig. 107, as seen from the earth, is $2° \, 58'$. Consequently, if S_1, Fig. 109, is at the eclipse limit, $S_1M_1 = 1° \, 29'$. Then from the formula

$$r = 57.3 \frac{l}{D},$$

it is found that, since $D = 5° \, 9'$ and $l = 1° \, 29'$ in arc measure, $r = \overline{S_1A} = 16.5°$. The changing values of the inclination of the moon's orbit to the ecliptic, and of the distances of the sun and the moon from the earth, cause this limit to vary from about $15.3°$ to $18.5°$. The sun and moon must be in

conjunction within this distance of the node of the moon's orbit in order that an eclipse of the sun may occur.

227. Number of Solar Eclipses in a Year. — Suppose the sun and moon are in conjunction near S_1 and that there is an eclipse. In 29.53 days they will be in conjunction again. In the meantime the sun will have moved along the ecliptic 29.1°; but, since the distance from S_1 to S_2 is 33°, it will not yet have passed beyond the eclipse limit, and there will be another eclipse.

In 177.2 days after the eclipse near S_1, the sun and moon will be in conjunction again 174.6° farther along the ecliptic. This will be $174.6° - 16.5° = 158.1°$ from the node A. But the node B will have moved back 9.4°, so that the conjunction will be $(180° - 9.4°) - 158.1° = 12.5°$ before B. Since this is within the eclipse limit, there will be an eclipse at this time. In 29.53 days the sun and moon will be again in conjunction 29.1° farther along, or 16.6° beyond B. This is very near the eclipse limit, and an eclipse may or may not occur, according as the distances of the sun and the moon from the earth and the inclination of the moon's orbit satisfy the requisite conditions or not. From this it follows that there are always at least two eclipses of the sun in a year, and there may be three or four.

228. Character of Succeeding Solar Eclipses. — The determination of the position of the eclipse with respect to the moon's nodes is made just as it was for lunar eclipses in Art. 225. Only results need be given here.

Suppose the sun and moon are in conjunction precisely at A, Fig. 109; then the sun is eclipsed for points on the earth's equator; or rather, since the equator is inclined to the ecliptic by 23° 27′, for points somewhere in the torrid zone. The next eclipse will occur when the sun is 4° beyond B. Since the moon is at this point south of the ecliptic, the eclipse will be visible for points in the southern hemisphere. The next eclipse will occur when the sun is 8° beyond A,

and will be visible for points in the northern hemisphere. The next eclipse will be when the sun is 12° beyond B (in some cases there will be one 29.5 days earlier), and since this is near the eclipse limit, the moon is far south of the ecliptic, and the eclipse will be visible near the earth's south pole. Then there will be eclipses both before the sun passes A and after it passes A. The first will be visible near the earth's south pole, and the other, a synodic month later, near the earth's north pole.

Hence it follows that *when there is but one eclipse at a node passage of the sun, it is visible in the equatorial or temperate zones; and when an eclipse is visible in one hemisphere, the next one will be visible in the opposite hemisphere, if it occurs at the end of a synodical month or of six synodical months, but if it occurs at the end of five synodical months, it will be visible in the same hemisphere again.*

229. Whole Number of Eclipses in a Year. — It is to be noted, first, that the moon is in conjunction with the earth's shadow midway between the times that it is in conjunction with the sun. Suppose there are two solar eclipses a synodic month apart, and that they take place when the sun is near the ascending node A. They will necessarily occur when the sun is near the two eclipse limits, the first being visible near the south pole and the other near the north pole. While the sun is moving from S_1 to S_2, the earth's shadow is 180° from it, and moves from S_3 to S_4. Midway between the solar eclipses the moon and earth's shadow are in conjunction at B, and there is a total eclipse of the moon.

Suppose there are also two eclipses, a synodic month apart, when the sun is near B. Then there will be a total lunar eclipse when the earth's shadow is near A.

Thus there may be six eclipses in a year, four of the sun and two of the moon. The solar eclipses are visible only from high latitudes, while the lunar eclipses are total and visible from a whole hemisphere at a time. Since in this

case the moon's shadow must extend farther in order to strike the earth than when it falls on the equatorial region (see Fig. 107), the eclipse is apt to be annular; or, if the moon is not included between the lines $\overline{AP}$ and $\overline{BP}$, the eclipse will be only partial.

When there are two eclipses of the sun visible in the equatorial regions, there are either no eclipses of the moon, or only partial eclipses.

The conclusion is that *there will be, at the least, only two eclipses in a year, both of the sun ; or, at the most, six eclipses, four of the sun and two of the moon.* There is, besides, the exceptional case where an eclipse occurs near the 1st of January, and, because of the regression of the moon's nodes, a seventh occurs before the end of the year.

230. Regression of Dates of Eclipses. — Since eclipses can occur only when the sun or earth's shadow is near one of the moon's nodes, it follows, from the regression of the moon's nodes, that the eclipses must occur earlier every year. The period of revolution of the line of nodes is 18.6 years. Therefore, the yearly regression in the dates of eclipses must average 365.25 days $\div$ 18.6 = 19.6 days. There are, however, marked variations from this average because of the changes of the number of eclipses in a year.

231. Relative Numbers and Frequencies of Lunar and Solar Eclipses. — There are about four eclipses of the sun to three of the moon, on the average, but the former are seen at any particular place very much less frequently. The reason is very simple. Since an eclipse of the moon is always visible from more than half of the earth, it follows that an observer at any place may, on the average, see a little more than half of the lunar eclipses which occur. On the other hand, an eclipse of the sun is visible from only a very small part of the earth's surface, and an observer does not see it unless its path passes by him. The path of totality is generally less than 100 miles wide and a few thousand miles long. The

eclipse is partial over a region usually four or five thousand miles wide. It is clear that the path of totality will strike a given place only very rarely.

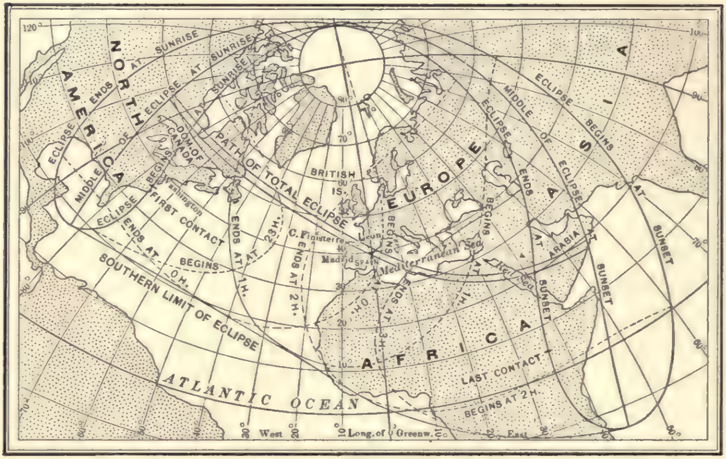


FIG. 110. — Path of Total Solar Eclipse of August 29–30, 1905.

232. The Saros. — The *saros* is the period of time after which the conditions for eclipses very nearly repeat themselves. It was found in very ancient times by the Chaldæans from observations of eclipses extending over a long period, and was used to predict eclipses.

The sun (or earth's shadow) moves forward 360° in a sidereal year of 365.256 days, or 0.9856° in one day. The moon's nodes move backward a revolution, or 360°, in 18.6 years, or 0.0530° in one day. Therefore the sun moves forward with respect to the moon's nodes $0.9856° + 0.0530° = 1.0386°$ daily. The time it takes the sun to move from one of the moon's nodes to the same node again is $360° \div 1.0386 = 346.62$ days. The time it takes the moon to move from conjunction with the sun to conjunction with the sun again is its synodical period, or 29.5306 days.

The eclipse conditions depend upon the position of the sun and moon with respect to the nodes of the moon's orbit. Suppose they are in conjunction precisely at the ascending node, so that there is a total or annular eclipse in the equatorial regions. The same sort of an eclipse will occur next when the sun and moon are again in conjunction at the same node. It is found that 19 periods of 346.62 days equal 6585.78 days, while 223 periods of 29.5306 days equal 6585.32 days. That is, the moon is in conjunction with the sun 0.46 of a day before the sun gets back to the node. In 0.46 of a day the sun moves only 28.6′, from which it follows that the conditions for eclipses are nearly the same

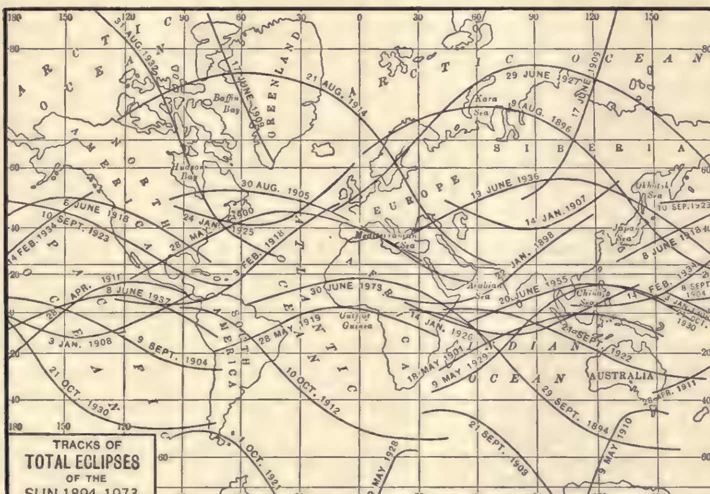


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Fig. 111. — Paths of Solar Eclipses. (From Todd's *Total Eclipses.*)

as they were at the beginning of the period. On the other hand, the earth has turned around 6585.32 times. Consequently the eclipse will be observable about 120° farther west on the earth's surface.

This period of 6585.32 days, which is the *saros*, is 18 years and 11.32 or 10.32 days, according as there are four or five leap years in the interval. In every third period the eclipse occurs at about the same longitude on the earth, but the sun is three times as far from the moon's node, which causes a shift in latitude. Thus, on May 28, 1900, there was a total eclipse visible in the Southern states, across the Atlantic Ocean, and through Spain. On June 8, 1918, another will be visible from Oregon to Florida. This is the next total eclipse visible in this country.

There are two other reasons why eclipses very nearly repeat themselves every saros. The first is that they occur at nearly the same times of the year, from which it follows that the distance of the sun is about the same. The second is that the line of the major axis of the moon's orbit makes a forward revolution in 8.855 years; it follows that it has nearly the same position and that the distance of the moon is nearly the same at the end of a saros as at the beginning. This is very important, for it follows from the relative positions of the sun and moon and their orbits that the perturbations are nearly the same at the two epochs.

233. Phenomena and Uses of Lunar Eclipses. — Lunar eclipses always occur exactly at full moon. When the moon first enters the penumbra of the earth's shadow no change is noticeable, but as it gets up near the umbra its eastern part becomes slightly darkened. When it gets into the umbra the part totally eclipsed is nearly dark and is bounded by a curve which one can see is at least nearly an arc of a circle. It takes the moon about an hour after first contact to become totally eclipsed, about two hours more to begin to emerge from the umbra, and about an hour more to get entirely out. These numbers vary considerably according as the eclipse is central or not, and because of several other factors.

When the moon is in the very center of the earth's shadow,

it is still visible, for the earth's atmosphere acts as a sort of lens and bends some of the rays which pass through it so that they fall on the moon, from which they are reflected back to the earth. Thus, Fig. 112 shows how some of them are bent until they strike the moon. Since those rays which are refracted most are bent out of their paths twice as much as those which come to us from a star on the horizon, or more than a degree, it follows that some of those passing opposite sides of the earth actually cross before they reach the moon. The air absorbs a large part of the violet end of the spectrum and makes the eclipsed moon look copper-colored. The effect is like that on the sun when it is setting, only it is more marked. The appearance of the moon differs greatly at different eclipses, according to the state of the earth's atmosphere in the zone which refracts the light to the moon.

FIG. 112.

Eclipses of the moon are useful in several ways. They afford an opportunity for determining the length of the synodic month, but because of the duration of totality they are much less serviceable in this way than solar eclipses are. It is at the time of a lunar eclipse that observations are most easily made to determine the temperature to which the moon has been raised, for the direct sunlight is quickly cut off at the lunar midday, and we receive only that which is radiated after having been absorbed. The observations of Lord Rosse and many later astronomers show that the moon becomes cold very quickly.

During lunar eclipses the passage of the moon in front of stars can be most easily observed, and such observations are always made, as they give the position of the moon more accurately than it can be found by direct observations.

It is certainly imaginable that the moon may be attended by a small satellite. It could not be seen because of either

the sunlight or moonlight except at the time of an eclipse, but the search for one so far has proved fruitless.

234. Phenomena and Uses of Solar Eclipses. — Solar eclipses occur precisely at new moon. At a position in the path of totality the first sign of the approaching eclipse is that the western edge of the sun becomes obscured by the moon, which is entirely invisible until it apparently encroaches on the sun's disk. The moon moves steadily eastward, cutting off more and more of the light. Before totality the earth is in the penumbra of the moon's shadow. Until the sun's disk is nearly all obscured there is nothing much observable to attract the attention. A cloud cuts off a much greater fraction of the sunlight. When the instant of totality draws near, the light rapidly fails, animals become restless, and everything takes on a strange, weird appearance. Suddenly a shadow rushes across the earth, the sun is covered, the stars flash out, around the apparent edge of the moon are rose-colored prominences of vaporous material forced up from the sun's surface often to a height of 200,000 miles, and all around the sun for half the apparent diameter of the moon extend streamers of pearly light constituting the sun's corona. After a little more than seven minutes at the most the moon passes from the western edge of the sun, daylight suddenly reappears, and the phenomena of a partial eclipse take place in the reverse order.

One of the uses of solar eclipses is to fix the dates in ancient chronology. The ancients used various and much confused systems of reckoning time, and it is now quite impossible to trace back through successive steps the dates of ancient historical events. But suppose an eclipse of the sun was observed and recorded in their method of reckoning time. If it is possible in any way to find, say, in what half century it happened, then it is usually possible to fix the precise date. The reason is that total solar eclipses are at any one place rare phenomena, and especially at a certain

time of the day, which the ancient chroniclers frequently gave. When one date is safely fixed the others can be found by their relations to it. This is, indeed, the very method by which many of the dates in ancient history have been established. For example, Herodotus tells us that during a battle between the Lydians and Medes the day was suddenly turned to night. The combatants, thinking their fighting was offending the immortal gods, ceased from the battle and made a treaty of peace. This eclipse was very probably on the 28th day of May, 584 B.C. It is the one Thales is reputed to have predicted.

The most favorable time for searching for small unknown planets within the orbit of Mercury is at the time of a total eclipse. At every eclipse these so-called intramercurian planets have been looked for, but so far without success. Mention should be made of the observations of Watson and Swift during the eclipse of 1878. Each of these observers saw two small, bright objects which he thought could not be identified with any stars. They did not, however, agree with each other regarding the positions occupied by the objects. Peters has shown that very probably Watson observed two stars in the constellation Cancer, but Swift's observations have never been explained. It seems very probable, though, from the many observations of this and later eclipses, that he was in some way mistaken.

The most important scientific advantages offered by eclipses are the opportunities they give astronomers to study the sun's upper atmosphere and coronal envelope. It is chiefly for these purposes that so much money and time are spent in equipping expeditions to the tracks of totality, even though they be on the opposite side of the earth. Some phenomena can be studied at no other time. These investigations can be treated better in connection with the discussion of the sun.

QUESTIONS

1. Draw a diagram showing the shape of the visible part of the sun as seen from a point in the penumbra of the earth's shadow.

2. Draw a diagram showing the part of the sun visible while an annular eclipse is central.

3. If the inclination of the moon's orbit to the ecliptic were 3°, what would be the lunar eclipse limits? How many eclipses of the moon could there be in a year at the most?

4. Suppose the inclination of the plane of the moon's orbit to the plane of the ecliptic were 90° and that the nodes were fixed; how long after a central total eclipse of the sun before another eclipse of the same sort would take place?

5. If the moon moved in the opposite direction in its orbit and the periods and other things remained the same, would there be any differences in the times of eclipses? Would there be any differences in the eclipses themselves?

6. Suppose there is a partial eclipse of the moon at S_1 (Fig. 108); how many eclipses of the moon will there be all together before there fails to be an eclipse while the earth's shadow passes the node A?

7. If the moon's nodes were stationary, could there be four eclipses of the sun in a year?

8. What period of revolution of the moon's nodes, other things remaining as they are, would bring a central eclipse of the sun every month? Would the moon then be eclipsed?

9. Is the sun totally eclipsed as seen from the moon? If so, is it eclipsed as often as it is as seen from the earth? As seen at a particular place on the moon, is it eclipsed as often as it is as seen from a particular place on the earth?

10. If there are eclipses of the sun as seen from the moon, are they visible, in the course of time, from all parts of its surface?

11. In what respects would an eclipse of the sun as seen from the moon differ from those seen from the earth?

12. How would a cloudy sky around the twilight zone affect a lunar eclipse?

13. Could the moon eclipse (occult) an inferior or a superior planet at the time of a lunar eclipse? At the time of a solar eclipse?

CHAPTER XI

THE SOLAR SYSTEM

235. Members of the Solar System. — The members of the solar system are the sun, the planets and their satellites, the planetoids, the comets, and the meteors, though it may be that many of the comets and meteors are only temporary residents in the solar family. The sun is the one preëminent body, whose gravitative power controls the motions of all the other bodies, and whose rays illuminate and warm them. It is impossible to study the planets without taking into account their relations to the sun. But the constitution and the evolution of the sun are quite independent of the planets. In this chapter the members of the solar system will be treated in their mutual relations, reserving for another the discussion of the peculiarities of the individual planets, and for still another the treatment of comets and meteors. The sun is a great star, and its physical condition and chemical constitution will be considered in a chapter preceding the discussion of the stars.

There are eight known planets. In the order of their distances from the sun they are Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, and Neptune. The first six have been known from time immemorial; Uranus and Neptune were discovered in 1781 and 1846 respectively.

The planetoids (planet-like bodies) are small planets which, with one exception, occupy the zone between Mars and Jupiter. The comets and meteors are wandering members of the system. They pass around the sun in a variety of paths and in all directions; some go off into space perhaps

never to return, while some remain permanently in the system, though their orbits are often radically changed by perturbations.

236. Finding the Scale of the Solar System. — It was shown in Arts. 156 and 157 that the whole solar system can be drawn to scale without knowing any of the actual distances. It follows that if the distance between any two planets can be found, it is possible to compute all the others.

The problem of finding the distances between the members of the solar system is of very great importance, for it is involved in many other problems, such as the determination of the masses of the planets and the distances to the stars. Until after 1700 it was not solved with any considerable approximation, and most of the valuable work on it has been done in the last century. The distances of the planets from the sun are now known with an error not exceeding $\frac{1}{15}$ of 1 per cent.

The direct method of finding the scale of the system is to measure directly the parallax of the sun, just as the parallax of the moon is found (Art. 19). Unfortunately it is of no practical value, for the quantity to be measured is very small, and it is very difficult to use the sun for such measurements. It is not possible to locate its center with the required accuracy, and the heat from it throws instruments of the type that would be used out of perfect adjustment. In using the sun an angle of about $8.8''$ would have to be measured; or rather, this is the difference in direction of the sun as seen from two points on the earth's surface which are at a distance from each other equal to the earth's radius. The difference in direction is about the same as when an object is viewed first with one eye and then with the other at a distance of a mile. The difficulty of measuring so small an angle, using so unfavorable an object as the sun, is easily appreciated.

The direct method is, however, of much value when applied

to the planets and planetoids. Gill, at the Cape of Good Hope, has been particularly successful in using Mars. It is much nearer to us at opposition than the sun ever is, and the parallax is correspondingly larger. The difficulties in using Mars arise from its having a large disk. These are avoided, but at the expense of having a smaller angle to measure, by using the planetoids, which appear as star-like points of light.

In 1898 a new planetoid, called Eros, was discovered by Witt. It is unique in the fact that its orbit lies between that of the earth and Mars; it comes nearer to us than any other large body of the solar system except the moon. It is the best adapted of any body for parallax determinations, and observations and photographs for the purpose have already been made, though they have not been completely reduced.

It has already been mentioned (Art. 140) that, after the velocity of light has been measured and the constant of aberration determined, the velocity of the earth in its orbit, and consequently the size of its orbit, can be found. Newcomb gives this method more weight than any other. The results obtained by it agree very closely with those obtained by observations of the planetoids. It is to be noted that this method does not depend upon the size of the earth.

It was pointed out by Halley, in 1677, that at the time of the transit of Venus its apparent displacement on the sun's disk as seen in different latitudes on the earth could be made to yield the distance from the earth to the sun. The transits are relatively rare, the first after Halley's paper occurring in 1761 and 1769. They were observed with great enthusiasm, and the second with particular success. The results were quite discordant, though they were a great improvement over any previously obtained. The next transits were in 1874 and 1882, and again the results were not all that had been expected, for the atmosphere of Venus introduced some serious difficulties.

The perturbations of the moon by the sun depend upon the distance of the sun. When the theory of these variations from undisturbed motion are worked out, and when the amount of the disturbance due to this cause has been found from observations, it is possible to find the sun's distance. This method gives quite satisfactory results, although it is so complicated that it is not yet of the highest value. Laplace thought it would eventually be the best method of all. Quite similar to this is the earth's perturbation of the nodes of the orbit of Venus, which also gives results fairly accordant with those found by other methods. These methods do not require that the size of the earth shall be known.

The scale of the solar system is always described by giving the parallax of the sun. In 1891 Harkness made a discussion of all of the material bearing on the subject and obtained as a final value of the solar parallax $8.809'' \pm 0.006''$. In 1896 Newcomb obtained from all available material $8.797'' \pm 0.007''$.

From preliminary discussions of the best photographs of Eros secured at Northfield, Bordeaux, and Paris, the resulting solar parallax has been found to be $8.799''$, $8.798''$, and $8.784''$ respectively. The photographs of Eros obtained at nine observatories give, according to the measurements and preliminary discussion made by Hinks at Cambridge, England, a parallax of $8.7966''$. The true value can differ only very slightly from $8.8''$. Consequently the mean distance from the earth to the sun, that is, half the length of the major axis of the earth's orbit, is given by the equation

$$r = \frac{206.265}{8.8} l,$$

where l, the equatorial radius of the earth, is 3963.296 miles. Consequently

$$r = 92{,}897{,}000 \text{ miles}.$$

In celestial mechanics the mean distance from the earth to the sun is ordinarily taken as the unit of length.

237. Size of the Sun and Distances of the Planets. — As seen from the sun the earth's radius subtends an angle of 8.8″, but as seen from the earth the sun's radius subtends an angle of $16' \; 2'' = 962''$. Therefore the radius of the sun is $962 \div 8.8 = 109.32$ times that of the earth, or 433,270 miles. If the sun were represented by as large a circle as could be put on the page, say four inches in diameter, on the same scale the earth would be represented by a circle $\frac{1}{27}$ of an inch in diameter, or little more than a dot. Their surfaces are as the squares of their radii; hence the surface of the sun is 11,950 times that of the earth. Their volumes are as the cubes of their radii; hence the volume of the sun is 1,306,500 times that of the earth. It will assist in getting a conception of the enormous size of the sun compared to that of the earth to note that its radius is nearly twice as great as the distance from the earth to the moon.

The mean distances of the planets from the sun are in round numbers:

Mercury	36,000,000 miles
Venus	67,200,000 miles
Earth	92,900,000 miles
Mars	141,500,000 miles
Jupiter	483,300,000 miles
Saturn	886,000,000 miles
Uranus	1,781,900,000 miles
Neptune	2,791,600,000 miles

Suppose a map of the system is drawn to scale, and that Mercury's orbit is represented by a circle an inch in diameter. On this scale the sun must be represented by a circle $\frac{1}{42}$ of an inch in diameter, and the earth by an invisible dot scarcely more than $\frac{1}{5000}$ of an inch across. Although the radius of the whole solar system will be represented by a line only a little over three feet long, on the same scale the distance to the nearest known fixed star is nearly six miles. The following table gives the diameters of the circles that repre-

sent the dimensions of the various planets' orbits on the chosen scale.

Orbit of Mercury, circle . . .	1.0 inch in diameter
Orbit of Venus, circle	1.9 inches in diameter
Orbit of Earth, circle	2.6 inches in diameter
Orbit of Mars, circle	3.9 inches in diameter
Orbit of Jupiter, circle . . .	13.4 inches in diameter
Orbit of Saturn, circle	24.6 inches in diameter
Orbit of Uranus, circle	49.5 inches in diameter
Orbit of Neptune, circle . . .	77.5 inches in diameter

It follows from the numbers given in the table that if the whole system is to be represented on a page, the diameter of Mercury's orbit will have to be reduced to $\frac{1}{20}$ of an inch. This is so small that it is necessary to use one scale for the first four planets, and another for the last four. Representing Mercury's orbit by a circle half an inch in diameter, we have the following diagram of the orbits of the first four planets. On a scale $\frac{1}{15}$ as great the map of the orbits of Mars and the last four planets is as given in Fig. 114. If this figure is magnified fifteen times and placed around the preceding one, the orbits of all the planets will be represented on the same scale.

Fig. 113.

The thing to be noticed is that the orbits of the outer planets are separated enormous distances from each other. For example, the first six planets are all nearer the sun than the last two are to each other. If a planet is supposed to hold sway in a region extending halfway to the orbit on each

side of it, it follows that the region belonging to each planet is greater than that belonging to all the planets which are interior to it.

The apparent diameters of the sun as seen from the planets are inversely as their distances from it. As seen from Mer-

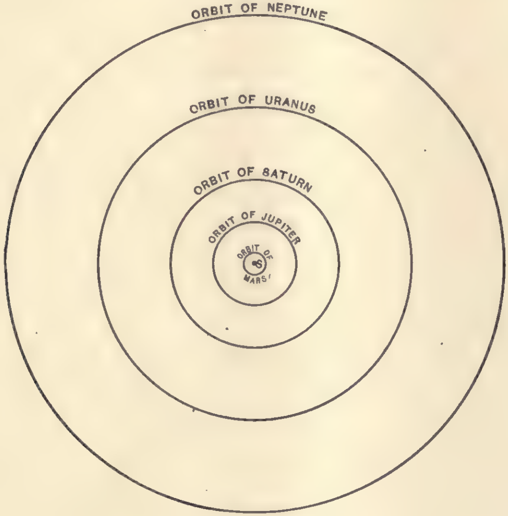


FIG. 114.

cury the apparent diameter of the sun is 83.2′, while as seen from Neptune it is only 1′ 4″. When Venus is nearest to us, its apparent diameter is 1′ 7″.

The light and heat received by the planets per unit area from the sun are directly proportional to the apparent area of the sun as seen from them. The apparent area of the sun varies inversely as the squares of its distances from the several planets. The following lines are proportional to the relative amounts of heat received per unit area by the various planets.

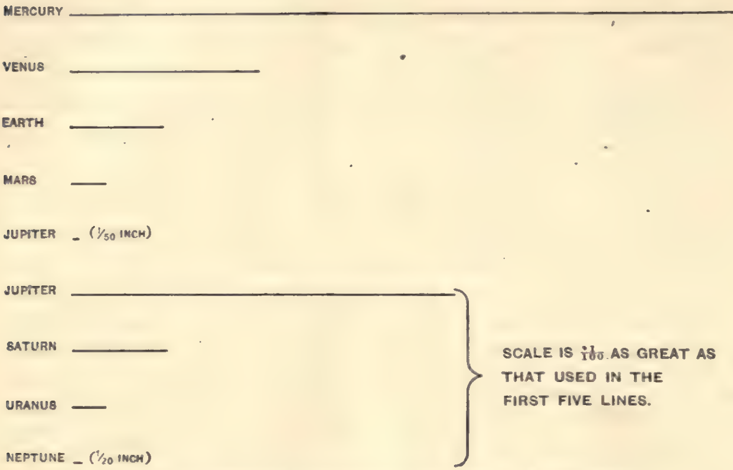


FIG. 115. — Relative Amounts of Heat received by Planets.

If the first five lines are magnified 100 times, the whole figure will be constructed on the same scale.

Taking the amount of light and heat per unit area received on the earth as unity, the following table gives the amounts received per unit area by the eight planets.

PLANET	HEAT RECEIVED	PLANET	HEAT RECEIVED
Mercury	6.8	Jupiter	0.04
Venus	1.9	Saturn	0.01
Earth	1.0	Uranus	0.003
Mars	0.44	Neptune	0.001

Thus it is seen that Mercury receives more than 6000 times as much light and heat per unit area as Neptune does.

238. Observations of the Planets. — When the inferior planets are in inferior conjunction with the sun, they are nearest to the earth, and the superior planets are nearest

when they are in opposition. But when the inferior planets are nearest to the earth, they are in the direction of the sun, and their unilluminated sides are turned toward us. Thus, the advantage of nearness is more than offset by their unfavorable position with respect to the sun. The inferior planets cannot be well observed until their elongation is such that they are about as far from us as the sun is. On the other hand, the superior planets are in the position most favorable for observation when they are in opposition and nearest the earth.

Neglecting the eccentricities of the orbits, Venus comes within 92,900,000 – 48,600,000 = 25,700,000 miles of the earth. Notwithstanding the close approach of Venus, almost nothing is known of its surface because it is an inferior planet. On the other hand the large topographical features of the surface of Mars are very well known because of its favorable position for observation when it is nearest the earth. When Mars is in opposition, its distance from the earth is 141,000,000 – 93,000,000 = 48,000,000 miles. Since this is only about half the distance from the earth to the sun, its parallax is correspondingly greater and more easily measured than that of the sun.

239. The Dimensions of the Planets. — Suppose the angular diameters of the planets are measured when they are nearest the earth; then, since their distances are known, their actual diameters can be computed from the formula

$$l = \frac{D}{206265} r,$$

where D is the angular diameter expressed in seconds, and r the distance expressed in any convenient units, as miles. The following table gives the greatest angular diameters of the planets as seen from the earth, and their actual diameters, according to Barnard's many measures at the Lick Observatory.

PLANET	GREATEST APPARENT ANGULAR DIAMETER	DIAMETER
Mercury	13″	2,765 miles
Venus	67″	7,826 miles
Earth	—	7,913 miles
Mars	25″	4,352 miles (equatorial) 4,312 miles (polar)
Jupiter	50″	90,190 miles (equatorial) 84,570 miles (polar)
Saturn	20″	76,470 miles (equatorial) 69,780 miles (polar)
Uranus	4.1″	34,900 miles
Neptune	2.9″	32,900 miles

The following circles show better than the numbers the relative dimensions of the planets.

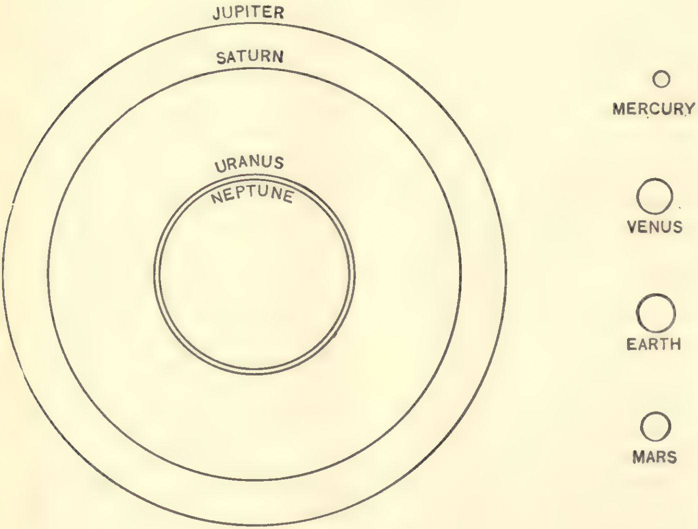


FIG. 116.

The great differences in diameters are much exceeded by those of the surfaces and volumes. The following table gives the comparison, taking as unity the earth's surface and volume respectively.

PLANET	SURFACE	VOLUME	PLANET	SURFACE	VOLUME
Mercury	0.12	0.04	Jupiter	121.9	1346.5
Venus	0.98	0.97	Saturn	85.4	789.2
Earth	1.00	1.00	Uranus	19.5	85.8
Mars	0.30	0.16	Neptune	17.3	71.9

240. The Masses of the Sun and the Planets. — The methods of finding the masses of the planets were explained in Art. 181. There is no difficulty in the case of the sun and those planets which have satellites, but the results given for the others, Mercury and Venus, are still subject to considerable uncertainty. In the following table the masses of the sun and planets are given, first taking the mass of the earth as unity, and then that of the sun as unity, and their mean densities are given on the water standard.

OBJECT	MASS (Earth = 1)	MASS (Sun = 1)	DENSITY (Water=1)
Sun	332,000.0	1	1.41
Mercury	0.033	$\frac{1}{9,647,000}$	3.70
Venus	0.82	$\frac{1}{405,000}$	4.89
Earth	1.0	$\frac{1}{332,000}$	5.53
Mars	0.11	$\frac{1}{3,020,000}$	3.95
Jupiter	317.7	$\frac{100}{104,735}$	1.33
Saturn	94.8	$\frac{1}{3502}$	0.72
Uranus	14.6	$\frac{1}{22,700}$	1.22
Neptune	17.0	$\frac{1}{19,500}$	1.11

The surface gravity of a planet depends upon its mass and radius; the method of computing it was given in Art. 182.

The following table gives the surface gravities of the sun and planets, taking that of the earth as unity.

Object	Surface Gravity	Object	Surface Gravity
Sun	27.7	Jupiter	2.61
Mercury	0.38	Saturn	1.19
Venus	0.86	Uranus	0.88
Earth	1.00	Neptune	0.88
Mars	0.38		

241. Periods of the Planets. — The periods of the planets depend upon their mean distances from the sun and their masses, as was explained in Art. 181. Taking the year as the unit of time, the mean distance of the earth as the unit of length, and the mass of the sun as the unit of mass, the period in years is given by

$$P = a^{\frac{3}{2}}\sqrt{\frac{1+E}{1+m}},$$

where a is the mean distance of the planet, E the mass of the earth, and m the mass of the planet. It can also be found from the synodical period by the formulas given in Arts. 154 and 155, the synodical periods being found from the observations. The following table gives the sidereal and synodical periods of all the planets.

Planet	Sidereal Period	Synodical Period
Mercury	0.24 years	0.32 years
Venus	0.62 years	1.60 years
Earth	1.00 years	———
Mars.	1.88 years	2.14 years
Jupiter	11.86 years	1.09 years
Saturn	29.46 years	1.03 years
Uranus	84.02 years	1.01 years
Neptune	164.78 years	1.006 years

When an inferior planet is at its eastern elongation, it is visible in the southwest after sunset. If the time of greatest eastern elongation is given, the succeeding ones can be found by adding to the date of the given one proper multiples of the synodical period. Superior planets are best seen at the time of opposition, and the times of succeeding oppositions can be found in the same way. The following table gives the dates of greatest eastern elongation or of opposition of all the planets for 1905.

Greatest eastern elongation of Mercury occurred November 27, 1905.
Greatest eastern elongation of Venus occurred February 14, 1905.
Opposition of Mars occurred May 8, 1905.
Opposition of Jupiter occurred November 24, 1905.
Opposition of Saturn occurred August 22, 1905.
Opposition of Uranus occurred June 24, 1905.
Opposition of Neptune occurred January 1. 1905.

QUESTIONS

1. Observe how the relative dimensions of the solar system can be found without knowing any of the actual distances. If every one should fall asleep at the same time, and if, by some miracle, every distance whatever should be divided by two, would it be possible ever to detect the change?

2. Would it be easier or more difficult to measure the sun's parallax if the earth were larger? Explain reason for answer.

3. Light travels at the rate of 186,330 miles per second. How long does it take it to come from the sun to the earth?

4. If the earth stood still and the sun were moving, we would apparently see the sun where it was when the light left it. Does the motion of the earth have a similar effect?

5. Sound travels in the air a mile in about 5 seconds. If it could travel from the sun to us at the same rate, how long would it take in coming?

6. How long would it take a train running at the rate of a mile a minute to go as far as from the earth to the sun?

7. How many times would one have to travel around the earth to go as far as from the earth to the sun?

8. Draw on the blackboard a map of the whole solar system true to scale.

9. If one were to build a model of the solar system, taking for the sun a globe 1 foot in diameter, what would be the distances and sizes of the planets?

10. How does the amount of sunlight received by Neptune compare with the amount of moonlight received by the earth at the time of full moon?

11. Draw to scale the apparent diameter of the sun as seen from the various planets.

12. If the apparent diameter of the sun as seen from Neptune is 1′ 4″, what would be the greatest elongation of the various planets? *Hint:* Suppose their elongations are proportional to the dimensions of their orbits.

13. What is the ratio of the volume of Jupiter to that of all the other planets combined?

14. What is the ratio of the volume of the sun to that of all the planets combined?

15. What is the ratio of the mass of Jupiter to that of all the other planets combined?

16. What is the ratio of the mass of the sun to that of all the planets combined?

17. What is the next date of the greatest eastern elongation of the inferior planets, and of opposition of the superior planets?

242. The Two Natural Groups of Planets. — The preceding data, which give some idea of the solar system as a whole, make it clear that the planets are divided into two sharply distinguished groups. The first consists of Mercury, Venus, Earth, and Mars. They are called *terrestrial planets* because in distance from the sun, period, size, density, and properties not yet discussed they are much like the earth. The second group consists of Jupiter, Saturn, Uranus, and Neptune. They are called the *major planets* because of their relatively great size.

The major planets are on the average 17.6 times as far from the sun as are the terrestrial planets. On the average the terrestrial planets receive per unit area 310 times as much light and heat from the sun as do the major planets. On the average the diameters, surfaces, and volumes of the major planets are respectively 10, 100, and 1000 times greater

than those of the terrestrial planets. The masses of the major planets average 224 times those of the terrestrial planets, while their densities average only 0.22 as much. The periods of revolution of the major planets average 77.6 times those of the terrestrial planets.

243. Similarities among all the Planets. — Notwithstanding the very great diversities between the two groups of planets, they all have a number of properties of motion in common which are of the highest importance, for they furnish the key to the evolution of the planetary system. It is important to note both the differences and the similarities; in short, to study the system relatively.

The planets all move in the same direction around the sun, from west to east. The sun rotates in this direction, and the 500 known planetoids move the same way. This is also the direction of rotation of the earth and of all other planets whose rotations are known. The moon revolves in the same direction around the earth, as do the satellites of all the other planets around their respective primaries, except the ninth satellite of Saturn, possibly the seventh satellite of Jupiter, and the satellites of Uranus and Neptune. This almost universal agreement of direction of motion points unmistakably to a similarity of origin, or to a mode of evolution which leads to this condition.

The planes of the orbits of the planets are not absolutely coincident, but they are inclined only slightly to one another. It is customary to use the plane of the ecliptic for reference, and to give the positions of all the other planes with respect to it. The plane of the ecliptic varies slightly because of the perturbations due to the other planets, but not enough to cause any appreciable difference in the general features of the system. The following table gives the inclinations of the orbits of the planets to the plane of the ecliptic. These inclinations vary somewhat, but never greatly, from the values they now have (Art. 169). When some increase,

others always decrease. At present they average 2° 39′; but if Mercury's orbit be omitted, the average is 1° 56′.

The eccentricity of an elliptic orbit may be anywhere between zero and unity, but the eccentricities of the orbits of the planets are remarkably small. They vary somewhat from their present values, but never greatly, and they always change in such a way that when some increase others decrease. The average of the eccentricities is now 0.060; or, excluding Mercury, 0.039.

TABLE OF INCLINATIONS AND ECCENTRICITIES

PLANET	INCLINATION OF ORBIT TO ECLIPTIC	ECCENTRICITY OF ORBIT
Mercury	7° 0′	0.2056
Venus	3 24	0.0068
Earth	0 0	0.0168
Mars	1 51	0.0933
Jupiter	1 19	0.0482
Saturn	2 30	0.0561
Uranus	0 46	0.0463
Neptune	1 47	0.0090

244. Satellites. — All except two of the planets are known to have satellites revolving around them, just as they revolve around the sun. Mercury and Venus have no known attendants. The earth has the moon, whose mass is greater relatively than that of any other satellite in comparison with the planet around which it revolves. Mars has two little moons only a few miles in diameter. Jupiter has four large satellites, discovered by Galileo, and three very small ones. Saturn has ten satellites, one of which is larger than Mercury, and two of which are very small. Uranus has four satellites and Neptune one. Saturn and its attendants form a sort of miniature of the solar system.

245. The Planetoids. — If the distances of the planets from the sun are examined, it will be found that each one is roughly twice that of the preceding one, with the exception of Jupiter, whose distance is about 3.5 times that of Mars. In 1772 Titius derived a series of numbers by a simple law which gave the distances of the planets (Uranus and Neptune were not known then) with considerable accuracy, except that there was a number for the vacant space between Mars and Jupiter. The law is that if 4 is added to each of the numbers 0, 3, 6, 12, 24, 48, ⋯, the sums thus obtained are nearly proportional to the distances of the planets from the sun. The law rests on no scientific basis, breaks down in the case of Neptune, and would not be mentioned here except for its historical connections. It is commonly known as Bode's law because his writings made it widely known.

The idea that there must be a planet between Mars and Jupiter became strong in the minds of astronomers toward the end of the eighteenth century; it was undoubtedly much enhanced by the number series of Titius. In 1800 Von Zach and five other astronomers met at Lilienthal and organized a plan of search for the suspected planet. Just as they were about to begin observations the announcement reached them that the Italian astronomer Piazzi, at Palermo, had discovered the planet January 1, 1801. He named it Ceres after the tutelary goddess of Italy.

After the discovery had been made, but before the news of it had reached Germany, the philosopher Hegel published a "dissertation" showing by the "most conclusive reasoning" that there were no unknown planets, and remarking on the folly of searching for them.

After six weeks of observations Piazzi was taken ill, and by the time notice of his discovery had reached other observers the planet was nearing superior conjunction with the sun and could not be observed. There was no method of finding from so few observations the elements of its orbit, and

astronomers were fearful of losing it. The great mathematician Gauss, then a young man of twenty-four, came to the rescue with a method of determining an orbit from only three observations of direction of the observed body. His prediction of the position of the planetoid led to its rediscovery on the last day of the year.

On March 2, 1802, Olbers discovered a planetoid, which he called Pallas; on September 2, 1804, Harding found one which he named Juno; and on March 29, 1807, Olbers discovered a fourth, called Vesta. Then no other was discovered until 1845, when Hencke found one, after fifteen long years of search, which he called Astræa. In 1847 three more planetoids were discovered, and in every year since then at least one has been found. Until 1891 they were discovered by comparing the objects visible in certain regions of the sky, especially near the ecliptic, with star charts. If a strange body was found, it was observed on succeeding nights to see whether it was moving or not. The planetoids move just as the planets do, and the character of an unknown body is determined from its motions.

A new epoch began in 1891 when Wolf first discovered a planetoid by photography. The method is very simple. The sensitive plate is moved during an exposure of several hours so that the star images remain fixed on it. In the interval the planetoid will move with respect to the stars, and will leave a short trail on the plate. These trails show where the small planets are. Wolf, at Heidelberg, and Charlois, at Nice, have been very successful in the search for these bodies by this method. All together more than 500 have been discovered so far (1905).

246. Designation of the Planetoids. — Until over 300 planetoids had been discovered, they were named mostly after mythological characters, but their very abundance has become embarrassing. They are now numbered in the order of their discovery, the number being placed in a circle, as

(2) = Pallas. They are also designated by the year of their discovery.

247. Orbits of the Planetoids. — The mean distances of the planetoids vary greatly. Those nearest the sun are about 50,000,000 of miles from the orbit of Mars, and those farthest from the sun are about 80,000,000 of miles within the orbit of Jupiter. The following figure shows the orbits of Mars and Jupiter and the region occupied by the planetoids. They are distributed by no means uniformly over this belt. They are somewhat less numerous on the outer edge than on the inner, and there are some remarkable vacant spaces. Kirkwood first called attention to the fact that these gaps occur at the places where a planet would have a period an exact fraction of Jupiter's period, as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, etc., of it. For such relations between the periods the perturbations are very great, and it may be that Jupiter has forced the planetoids out of these regions. This is not perfectly certain, for there is no known method of treating rigorously the question for the indefinite ages Jupiter may have been acting on these bodies.

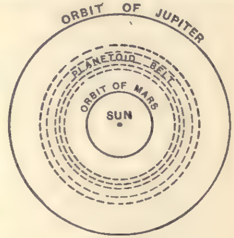


FIG. 117.

The inclinations of the orbits of the planetoids are often very great, ranging even up to 35° in the case of Pallas. The average is about 6°, or nearly equal to the greatest among the planetary orbits.

The eccentricities range from almost zero up to 0.38 in the case of (132) Æthra, the average being about 0.14. The greatest eccentricity among the planetary orbits is that of Mercury's orbit, which is only about one half that of Æthra's orbit. The average eccentricity of the orbits of the planetoids is more than twice that of the orbits of the planets.

248. Dimensions and Masses of the Planetoids. — The planetoids are so small that they are all invisible to the unaided

eye except Vesta, which is barely visible under favorable conditions. Like the satellites of the remote planets, their diameters are estimated from the amount of light they reflect, though Barnard measured the four largest with the great Lick telescope. He found for Ceres, Pallas, Vesta, and Juno diameters of 485, 304, 243, and 118 miles respectively. There are probably a few others whose diameters lie between 100 and 500 miles, but the great majority are undoubtedly much smaller, ranging down to something like 10 miles. It follows from Barnard's measures and Müller's photometric measurements that the albedoes of Ceres, Pallas, Vesta, and Juno are respectively 0.18, 0.24, 0.75, and 0.45. It is difficult to account for these great differences in bodies which we should expect to find much alike.

It is impossible to determine the masses of the individual planetoids, for their disturbing effects on other bodies are absolutely inappreciable. If their combined mass were as great as $\frac{1}{100}$ of that of the earth, they would produce perturbations in the orbit of Mars which observations of this planet show do not exist. Hence the conclusion is that their total mass is below this limit, but it is impossible to say how far below. According to Roszel the aggregate mass of all those so far discovered is less than $\frac{1}{3000}$ that of the earth.

249. Origin of the Planetoids. — When the second planetoid was discovered, Olbers suggested that these bodies are but the fragments of an exploded planet. If this were so, the orbits of all the pieces, except for the perturbations, would cross at the point of the explosion, and in the earlier years of discovery the places where the orbits of Ceres and Pallas cross were carefully searched. Because of the wide range now occupied by them this theory, which was once generally held, has been abandoned. If there were no contradictory observational data, the theory could scarcely be considered probable, for it is impossible to conceive of forces originating within a planet sufficient to tear it asunder after it had

once been aggregated and held together by the mutual attraction of its parts.

Another theory is that when the solar system evolved out of a widely extended nebulous or meteoritic mass, for some reason, perhaps because of the distribution of the original material, or the proximity of the large perturbing body, Jupiter, this part of it failed to aggregate into a planet. The indefiniteness of this theory makes it unassailable, and it probably contains the truth somewhere within its ample domains.

250. The Planetoid Eros. — In 1898 the trail of a planetoid having a very rapid motion was found on a photograph taken by Witt, at Berlin. From observations which were speedily secured, Chandler, of Cambridge, computed an orbit and ephemeris. It was found to be a most remarkable object, in that its orbit lies between that of the earth and Mars, and it at once attracted great attention. On examining photographs taken at the Harvard College observatory during 1893, 1894, and 1896, with the aid of Chandler's ephemeris, the image of the body was several times found, and its positions at those dates were accurately determined by its relations to the fixed stars. This enabled Chandler to get an accurate orbit almost immediately. This is not the only instance in which the Harvard photographic records have furnished most opportune and valuable information.

When it was stated that the orbit of Eros lies between that of the earth and Mars it was meant that its mean distance is between the mean distances of these two planets. Its mean distance is 135,500,000 miles, but its orbit has the high eccentricity of 0.22. When it is at its aphelion its distance from the sun is 165,500,000, which is 24,000,000 miles beyond the mean distance of Mars. There is no danger of its colliding with Mars, for the inclination of its orbit to the ecliptic is nearly 11°, which throws it far out of the plane of the orbit of this planet where it reaches beyond it.

The perihelion distance of Eros is only 105,300,000 miles, or about 12,400,000 miles greater than the mean distance of the earth. But because of the large inclination of the plane of its orbit to the plane of the ecliptic it never gets nearer than about 13,500,000 miles of the earth. These near approaches occur only when it is in opposition at its perihelion, which unfortunately happens rarely. Its sidereal period is 1.76 years, from which it follows that its synodic period is 2.32 years. If an opposition occurs at perihelion, then in 37 years another will occur very nearly at perihelion, for 37 is almost evenly divisible by 1.76 and 2.32. The next most favorable opposition will occur in 1931. As has been mentioned (Art. 236), it furnishes an unexcelled opportunity for obtaining the solar parallax.

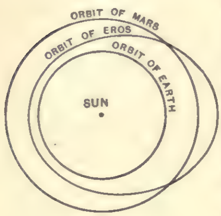


FIG. 118.

251. Variability of Eros. — One of the most peculiar things about this little body, which can scarcely exceed 20 miles in diameter, is that in February and March, 1901, it varied in magnitude both extensively and rapidly. The announcement was first made by von Oppolzer, and the fact was verified by many other observers. The period is 2 hr. 38 m.; or perhaps 5 hr. 16 m. composed of two sub-periods of 2 hr. and 25 m. and 2 hr. 51 m. respectively, but the evidence is not quite conclusive on this point. In February and March the light at its minima was less than one-third that at its maxima, but the variation soon diminished in extent and ceased entirely by the beginning of May.

The variation in the variability shows either that rapid changes are taking place on Eros, or that its light-reflecting power depends upon its relative position with respect to the earth and sun. The first alternative may be safely dismissed. In accord with the second, two theories have been advanced. The first, advanced by André, is that the body is

really a double, the two components revolving around their center of gravity in a period of 5 hr. 16 m. When the earth is in, or near, their plane of revolution, they eclipse each other twice in a period. André accounts for the great variation in their brightness, which, if they were spherical, could not exceed one-half the maximum, by supposing that they are much elongated by tidal forces in the line joining them. The other theory is that the body is very unequally reflective in different parts, and that its brightness varies as the rotation brings these different parts into view. The variability changes as the earth becomes differently situated with respect to the plane of rotation. In certain respects both these theories seem improbable, and we do not know yet which, if either of them, is the true explanation of the very remarkable phenomenon.

252. Scientific Uses of the Planetoids. — The planetoids are subject to very large perturbations by Jupiter because of their proximity to this great body, and it is impossible to keep track of them without computing these perturbations. The work of doing this for so large a number of bodies is enormous, and it has become a serious question whether it is worth while to attempt to keep track of them. Watson left a fund, by his will, to be devoted to the necessary calculations of the orbits of the twenty-two which he had discovered at Ann Arbor. Most of the rest of such work is done under the direction of Bauschinger at Berlin. In response to the question whether this immense labor is profitably bestowed or not, the service that the planetoids have been and are likely to be to astronomy may be reviewed.

In the very beginning the search for planetoids stimulated coöperation and directed the efforts of a large number of observers. It led directly to extensive and accurate maps of the ecliptic constellations. The problem presented by the first discovery attracted to astronomical researches the brilliant Gauss, whose contributions were many and pro-

found. The orbits of the planetoids present many problems in celestial mechanics which do not arise in the theories of the motions of the planets. Their solutions have enriched, and are still enriching, this domain of the science. The vacant spaces where the periods would be commensurable with that of Jupiter have a significance not yet fully understood. An analogous condition exists in Saturn's rings, as will be pointed out in the next chapter. The search for planetoids and their recognition as things worthy of attention led to the discovery of Eros, whose value in parallax determination is very great. In order to find the parallax of the sun over 80 expeditions, equipped at enormous cost, were sent to nearly every part of the earth, especially in high latitudes, to observe the transit of Venus in 1874. It is certain that more valuable results will be obtained from Eros than from all of these expeditions. Besides, the great variability of this body is a new phenomenon in the solar system and may lead to an important discovery. It is true that some of the other planetoids are variable, but to such a slight extent that the phenomena have so far not been of very much interest.

253. The Zodiacal Light and the Gegenschein. — The zodiacal light is a soft, hazy wedge of light stretching up from the horizon along the ecliptic just as the twilight is ending, or as the dawn is beginning. Its base is 20° or 30° wide and it generally can be followed under favorable conditions to 90° from the sun, and sometimes in a narrow faint band 3° or 4° wide entirely around the sky. It is very difficult to decide precisely what its limits are, for it shades very gradually from an illumination perhaps a little brighter than the Milky Way into the dark sky.

The best time to observe the zodiacal light is when the ecliptic is nearly perpendicular to the horizon, for then it is less interfered with by the dense lower air. In the spring the sun is near the vernal equinox, where the ecliptic cuts the

equator from south to north. At this time of the year after sunset the ecliptic comes up from the western horizon north of the equator and makes a large angle with the horizon. Consequently, the spring months are most favorable for observing it in the evening, and for a similar reason the autumn months are most favorable for observing it in the morning. It cannot be seen in full moonlight.

The Gegenschein (German for "counter glow") is a very faint patch of light on the ecliptic precisely opposite to the sun. It appears like an enlargement of the zodiacal band at this point. It is oval in shape, being longest along the ecliptic, and, according to Barnard and Douglass, generally 10° to 20° long and half as wide. It was discovered by Brorsen in 1854, and subsequently it was independently found by both Backhouse and Barnard. Both the zodiacal light and the Gegenschein can be seen only with the unaided eye, for the field of a telescope is so small that it does not enable one to contrast them with the darker sky.

One theory of the zodiacal light, originated by Searle, is that it is due to light reflected from an immense number of meteors circulating around the sun in, or near, the plane of the ecliptic, and extending out somewhat beyond the orbit of the earth. This belt of meteors seems to be densest and thickest near the sun. There is abundant evidence of much meteoric material in this region and the theory seems quite satisfactory. It is furthermore supported by the fact that the spectrum is continuous and the light polarized.

The corresponding and associated theory of the Gegenschein is that the meteors are more numerous precisely opposite to the sun. It was shown independently by Gyldèn and by the author that the attractions of the sun and earth do, indeed, lead to a condensation in this direction at a distance of 930,000 miles from the earth. The meteors occupying this region are constantly changing. As they approach it they are caught in a sort of dynamic whirlpool and after a

few revolutions they escape to go on in their orbits. This theory accounts perfectly for the position and shape of the Gegenschein.

Bigelow has advanced another theory of the zodiacal light. He supposed that it is caused by particles electrically expelled from the magnetic poles of the sun and condensed along the plane of its equator. Now the plane of the sun's equator is inclined more than 7° to the plane of the ecliptic, and in an object less poorly defined it would be a simple matter to decide which it follows. A series of three years' observations by Marchand, published in 1896, seem to show that the zodiacal light is nearly coincident with the plane of the sun's equator. This result needs confirmation before final acceptance, for other observers have not noticed the large deviation of the zodiacal band from the ecliptic. But so far as these results go, they must be taken as being to some extent confirmatory of Bigelow's theory.

Evershed has given a similar explanation of the Gegenschein. He supposes that it is a sort of tail to the earth caused by the escape of molecules of hydrogen and helium away from the earth in a direction opposite to the sun. The tails of comets are something like this, as will be seen in Chapter XIII.

On the whole the meteoric theory of both the zodiacal light and the Gegenschein seems the more probable, though the question cannot be regarded as definitely settled.

254. Possible Undiscovered Planets.— The question of intramercurian planets was mentioned in connection with eclipses. It is probable that no planet of any considerable size exists in this region, though it is probable that it is filled with meteors, or exceedingly minute planetoids.

Planetoids are very frequently discovered in the zone between Mars and Jupiter. There may be thousands of small ones yet which will come within the reach of photographic processes. It is by no means certain that there are not many other planetoids, like Eros, between the earth and Mars. If

there are any between the earth and Venus, they may remain undiscovered because of the difficulty of seeing them in this region; and if there are any farther out than Jupiter, they may escape detection because of their great distance both from the sun and from us. It is scarcely to be expected that any will be found near the planets, which long ago swept up the material near their paths.

The most interesting question is whether there are large planets still more remote from the sun than Neptune. There are two sorts of indications which point to the possible existence of trans-neptunian planets. The first is the grouping of comets' orbits. (See Arts. 295 and 296.) From some such a grouping as is found with respect to Jupiter's orbit, Forbes, of Edinburgh, in 1880, concluded that there are two of these remote members of the solar family, at distances 100 and 300 times that of the earth. They would revolve around the sun in the immense periods of 1000 and 5000 years.

Todd sought to find evidence of an undiscovered planet from certain small residual errors in the theory of the motion of Uranus, those of Neptune not yet having had time to accumulate sufficiently. His conclusion was that there is probably a planet circulating at about 50 times the distance of the earth from the sun, making a revolution in 375 years. The data in both cases were entirely insufficient to establish a conclusive proof, and the question yet remains open.

QUESTIONS AND EXPERIMENTS

1. In what respects are the terrestrial planets similar to each other, but different from the major planets?
2. Is it probable that the satellites have atmospheres?
3. Is it probable that the satellites always keep the same faces toward their respective primaries?
4. The mean distance of Pallas is 257,500,000 miles. What is its sidereal period?

5. The mean distance of Æthra is 242,000,000 miles and the eccentricity of its orbit 0.383. How much does its distance from the sun vary in a revolution?

6. How near is Æthra to Mars at the nearest approach?

7. Look for the zodiacal light.

8. How does the zodiacal light compare in brightness with the Milky Way?

9. How does the zodiacal light compare in width and uniformity of illumination with the Milky Way?

10. Can you see the Gegenschein?

11. At what time of the year is the Gegenschein in most favorable position for observation?

12. At what times of the year is the Gegenschein in the Milky Way?

13. What zodiacal constellations have the fewest bright stars? At what times of the year is the Gegenschein in these constellations?

CHAPTER XII

THE PLANETS

255. The Planets as Individual Bodies. — In the last chapter the planets were studied with especial reference to their relations to the solar system as a whole. In this, those remaining characteristics and peculiarities which have no intimate and direct connections with the other members of the system will be discussed.

In the case of the inferior planets there will be the phases, transits, albedoes, atmospheres, surface markings, rotations, and seasonal changes for discussion. The treatment of the superior planets will involve their satellite systems, albedoes, atmospheres, surface markings, rotations, and seasonal changes.

MERCURY AND VENUS

256. The Phases of Mercury and Venus. — When Mercury and Venus are in inferior conjunction their phase is "new," and they are either in transit across the sun, or the visible illuminated parts are very thin crescents. As their elongations increase their crescents increase, and their disks are just half illuminated at their greatest elongations. As they proceed toward superior conjunction they become gibbous, and finally fully illuminated on the sides toward the earth. These changes of phase, which were first verified observationally by Galileo in 1610, prove the revolution of these bodies around the sun instead of around points between the earth and the sun.

The inferior planets go through phases, like those of the moon, but there are very important changes in their apparent dimensions. When Mercury and Venus are in inferior conjunction, their respective distances from the earth are 56,900,000 and 25,700,000 miles, while their respective distances at superior conjunction are 128,900,000 and 160,100,000 miles. It follows that Mercury at inferior conjunction is less than half as far from us as it is at superior conjunction, and the difference is relatively three times as great in the case of Venus. Therefore the apparent diameter of Mercury when it is a thin crescent is twice as great as when it is full, and the corresponding ratio for Venus is six to one. The following figure shows the relations between the phases and the apparent sizes of these planets.

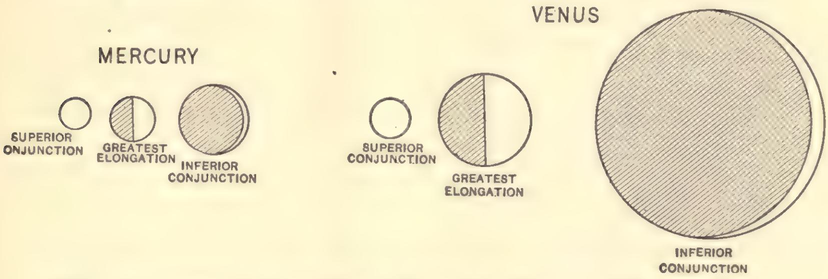


FIG. 119.—Apparent Dimensions of Mercury and Venus at Different Phases.

257. The Transits of Mercury and Venus. — If the inclinations of the orbits of Mercury and Venus to the plane of the ecliptic were zero, they would transit centrally across the sun once every synodical period. But the inclinations prevent these transits except when the planets pass inferior conjunction near one of the nodes of their orbits, just as the inclination of the moon's orbit renders eclipses of the sun rather rare phenomena. The whole matter may be worked out precisely as the conditions for eclipses were, for transits

are geometrically only partial eclipses, but the details will be omitted here.

The earth is near the nodes of Mercury's orbit in May and November, and transits of this planet must occur in these months. But the great eccentricity of Mercury's orbit makes the conditions at the two months quite different; the May transits are only about half as numerous as those in November. The transits occur at intervals of 7, 13, or 46 years, according to circumstances, for these periods are respectively very nearly 22, 41, or 145 synodic revolutions of the planet. The next transits of Mercury, which are, however, of little scientific interest, will occur November 12, 1907, November 6, 1914, May 7, 1924, and November 8, 1927. Mercury is so small that the transits can be observed only with a telescope.

The transits of Venus occur in June and December. They occur in cycles whose intervals are, starting with a June transit, 8, 105.5, 8, and 121.5 years. The last two occurred on December 8, 1874, and on December 6, 1882. The next two will occur June 8, 2004, and June 6, 2012. The chief scientific uses of the transits of Venus are the easy determination of the position of the planet, the investigation of its atmosphere, and the determination of the solar parallax.

258. The Albedoes of Mercury and Venus. — The albedo of a body is the ratio of the light which it reflects to that which it receives. It depends upon whether or not the body is surrounded by a cloud-bearing atmosphere. A body which has no atmosphere and a broken surface, like the moon, has a low albedo, while one covered with an atmosphere, especially if it is filled with partially condensed water vapor, has great reflecting power. The albedo of clouds is about 0.72. Every one is familiar with the intense brightness of thunder heads. The cloud-covered earth would shine as though coated with newly fallen snow.

The observations of Zöllner and Winnecke led them to

the conclusion that the albedo of Mercury is 0.13, while the more recent observations of Müller, made between 1883 and 1893, show that it is 0.17, or just equal to that of the moon. In strong contrast with this low reflecting power, the albedo of Venus is 0.76 according to the observations of Müller.

259. The Atmospheres of Mercury and Venus. — The albedoes given in the last article show what may be reasonably expected. The albedo of Mercury means that it has probably a very rare atmosphere, or none at all. The observations of Müller on the amount of light it reflects at different phases go to show that the reflection is from a solid uneven surface. At the time it transits the sun there is no bright, illuminated, atmospheric ring, nor are there any refractive effects as it crosses the sun's limb. Furthermore, it follows from the kinetic theory of gases (Arts. 110 and 111) that if Mercury had an atmosphere, it would very probably lose it, especially because of the intense heat received from the neighboring sun. Observations of its surface markings, to be mentioned presently, also show that its surface is not obscured by a dense gaseous envelope. Altogether there is abundant justification for the conclusion that Mercury has an exceedingly tenuous atmosphere, if any at all.

Because of precisely contrary evidence it is believed that Venus has a considerable atmospheric envelope, probably as dense as that which surrounds the earth. This conclusion is supported by independent evidence of a most convincing character. First, the brilliance decreases from the center toward the limb where the absorption would be greatest. Then, twilight effects near the terminator have been observed for more than a century. But the most satisfactory evidence is that when the crescent is very thin the atmospheric ring is illuminated beyond the horns, which must be exactly 180° from each other in a sphere. Mädler (1849) found this illumination to extend 240°, and since then many other

astronomers have observed the ring of yellow light entirely around the planet when it has been near conjunction. From observations of the same character made in 1898, Russell concluded that the atmosphere of Venus is considerably less refractive than that of the earth. The conclusion is evidently open to some uncertainty because of the lack of knowledge whether or not the light which came to us passed through the lower strata of its atmosphere, or was reflected from the higher regions. Spectroscopic observations seem to point to the absorption of light by water vapor in the atmosphere of Venus, but the conclusion is at present doubtful.

260. Surface Markings and Rotation of Mercury. — The first astronomer to observe systematically and continuously the surface markings of the moon, sun, and planets was Schröter (1745–1816). He was an observer of rare enthusiasm and great patience, but he seems in some instances to have been led by his lively imagination to erroneous conclusions.

In 1800 Schröter observed that the southern horn of Mercury was slightly rounded, and from the time of the reappearance of the condition he concluded that the planet's period of rotation is 24 hr. 4 m. His work was quite generally accepted until after 1880. About 1880 Schiaparelli took up at Milan the work of making systematic planetary observations. He found that the planet Mercury could be much better seen in the daytime when near the meridian, notwithstanding the illumination of the sky, than at night. He found a rose-tinged disk covered with permanent darker markings. Pursuing his observations hour after hour consecutively, instead of for a few moments at intervals of a day or more, he came to the remarkable conclusion that the planet rotates around an axis sensibly perpendicular to its orbit in a period equal to its period of revolution around the sun. These results have been fully confirmed by Lowell at

Flagstaff and at Mexico. The observations upon which this conclusion is based are difficult, yet it seems that they may be safely accepted.

Figure 120 is Lowell's map of Mercury, and shows an astonishing amount of detail. It must not be supposed, however, that all these streaks were seen at one time. They are the combined results of all the drawings made by six observers with two different telescopes at Flagstaff and at Mexico. The planet is sensibly spherical. The oblateness of the figure is due to the fact that those parts which are brought into view by the libration in longitude are also shown.

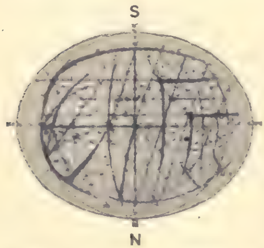


FIG. 120. — Lowell's Map of Mercury.

Figure 121 is a reproduction of three of Lowell's actual drawings showing the slow rotation. From all the observations Lowell drew the following conclusions: The markings on Mercury are very dark, permanent streaks of non-uniform width; the period of rotation of this planet is the same as that of its revolution; the axis of its rotation is sensibly perpendicular to the plane of its orbit; there are on it no clouds, or polar caps, or changes in the markings; and its atmosphere is at most very thin.

FIG. 121. — Drawings of Mercury by Lowell.

The reason Mercury always has its same side toward the sun is very probably that the friction of the tides generated by the sun have worn down whatever excess of rotation above revolution it may have had. At every stage of its development the tides have made its rotation slower than it otherwise would have been.

261. The Mercurian Seasons. — The seasons of Mercury are

due entirely to its varying distance from the sun and to its libration. Because of the large eccentricity of its orbit, which is greater than that of any other planet, it is only two-thirds as far from the sun at perihelion as at aphelion ; consequently the heat and light received at aphelion are only four-ninths of the amount received at perihelion.

The libration, like that of the moon in longitude, is due to the fact that the planet moves at a variable rate in its orbit, while it must rotate uniformly on its axis. Because of the large eccentricity the libration is very great, being about 23.5° each side of the mean. There are thus 133° of longitude on which the sun always shines, an equal amount on which it never shines, and two zones of 47° each on which it alternately shines and does not shine with a period of 0.24 years = 88 days.

On the side of Mercury toward the sun the temperature must be sufficient to dissipate immediately any such liquids as water, while the opposite side must be subject to a frigidity never experienced on the earth. If water were distributed all over the planet, it would speedily be evaporated on the one side and precipitated as snow on the other. If there is any atmosphere, it must be in the most violent circulation in its work of equalizing temperature. Along the temperate zone, which separates that of perpetual sunshine from that of perpetual darkness, there may be a moderate climate so far as temperature is concerned, especially if there is much atmosphere. The seasonal changes due to the varying distances from the sun and the libration must be quite extreme, though the two causes counteract each other somewhat because they have the same period but different phases.

262. Surface Markings and Rotation of Venus. — The history of the study of the rotation of Venus is almost precisely like that of Mercury. From observations of very faint markings by his predecessors, J. J. Cassini, in 1740, concluded that Venus rotates in 23 hr. 20 m. From the observations

of a truncated cusp in 1789–1791 Schröter concluded that the rotation period is 23 hr. 21 m., with an inclination of the plane of its equator to the plane of its orbit of over 53°. These results were the best available until 1880, when Schiaparelli announced that Venus performs a rotation once in a revolution like the moon and Mercury.

The observations of Schiaparelli originally made in 1877 and 1878 were verified by himself in 1895. In 1877, December 15, Holden, at Washington, saw precisely the same markings that were observed by Schiaparelli at Milan eight hours earlier. The conclusions have been still further verified by Perrotin, Tacchini, Mascari, Cerulli, and Lowell since 1890.

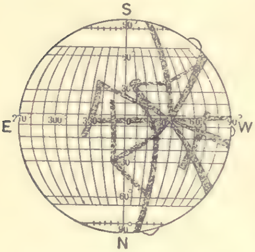


Fig. 122. — Lowell's Map of Venus.

Instead of basing their observations on faint and often obscure dark markings, they followed Schiaparelli in observing the very brilliant white spots whose presence had been known since the Cassinis. These may be snow-covered mountain peaks which extend into the upper atmosphere. There seems to be no libration in latitude, from which it is inferred that the plane of the planet's equator coincides sensibly with that of its orbit.

In 1900 Bélopolsky undertook to determine the rate of rotation by means of the spectroscope. So far as the observations, which were admittedly only preliminary and imperfect, throw light on the question they point to the shorter rotation period. On the other hand, Slipher's work with the spectroscope at the Lowell observatory in 1903 gave no evidence of a short rotation period. Recent direct observations by Niesten, Trouvelot, Williams, and Brenner also point to a period between 23 and 24 hours. But if the bright spots actually belong to the solid surface of the planet, as seems very probable from their permanent character, the

evidence on the whole shows that the period of rotation is 0.613 year = 225 days. The explanation of this condition is the same as for the corresponding conditions in the case of the moon and Mercury.

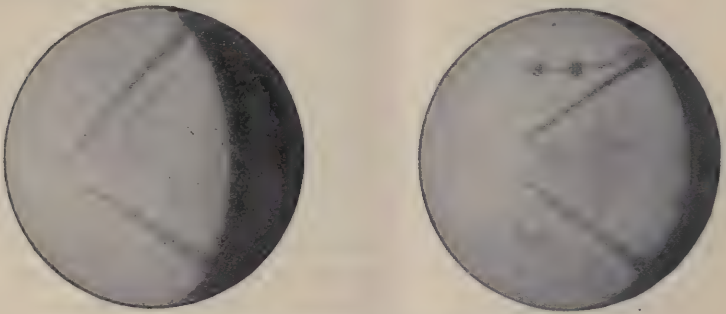


FIG. 123. — Drawings of Venus by Lowell.

263. The Seasons of Venus. — The orbit of Venus is more nearly circular than that of any other planet. The result is that both its change of distance and its librations are very small. Consequently any place on Venus must have a remarkably uniform climate, whether it be the extreme of torridity on the side toward the sun, or of frigidity on the opposite. The changes in the amount of heat received from the sun due to the eccentricity of its orbit are only one-thirtieth as great as they are in the case of Mercury.

MARS

264. The Satellites of Mars. — In August, 1877, Asaph Hall discovered two very small satellites revolving eastward around Mars sensibly in the plane of its equator. They are so minute and so near the bright planet that they can be seen only with a very large telescope, and then Mars must be hidden by a small screen in the focal plane. They are called Phobos and Deimos.

The orbits of the satellites of Mars are very small, Phobos being only 5850 miles from the planet's center, and Deimos 14,650 miles. That is, Phobos is only 3750 miles from the surface of Mars, and could not be seen in latitudes greater than 69° north or south because of the curvature of the planet's surface. The relative dimensions of the planet and the orbits of its satellites are shown in Fig. 124.

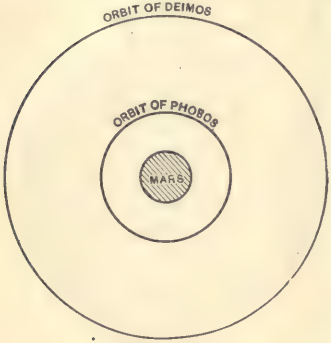


FIG. 124.

On the assumption that the satellites have the same albedo as Mars, E. C. Pickering concluded that the diameters of Phobos and Deimos are seven and six miles respectively. However, they may be considerably larger, as Lowell inferred from his observations.

Because of the small distances of the satellites, their sidereal periods are very short, being 7 hr. 39 m. and 30 hr. 18 m. respectively. Mars rotates on its axis in 24 hr. 37 m. Hence Phobos makes more than three revolutions while the planet turns around once. This leads to the very interesting phenomenon of a satellite rising in the west and setting in the east. The period from meridian around to meridian is 11 hr. 7m. On the other hand, Deimos rises in the east and sets in the west, the period from meridian to meridian being 131 hr. 15 m.

265. The Rotation of Mars. — In 1666 Hooke and Cassini saw dark streaks on the ruddy disk of Mars, and the markings they drew can be recognized at the present day. By comparing those early observations with recent ones the period of rotation is found with a high degree of precision. It is 24 hr. 37 m. 22.7 sec., or a little more than a day.

The inclination of the plane of the equator of Mars to

that of its orbit is about 24°. Therefore, except for its greater distance from the sun, the days and seasons would be very much like those of the earth.

266. The Albedo and Atmosphere of Mars. — According to Müller the albedo of Mars is 0.27, which indicates probably some atmosphere, but not a very dense one.

The surface gravity of Mars is only two-fifths that of the earth, and from the kinetic theory of gases it would not be expected that this planet would retain a very extensive gaseous envelope. The direct observations all point to the same conclusion. In the first place, the surface of Mars can nearly always be seen without any interference from atmospheric phenomena. The meaning of this can be realized by considering the probable appearance of the earth as seen from a distant body. According to Langley, 40 per cent of the vertical rays from the sun are absorbed by the atmosphere before they reach the solid surface. Of the 60 per cent reaching the surface only a small part, say one quarter, are reflected from the whitest rock; and this quarter is largely absorbed before it escapes from the atmosphere. It is probable that not enough rays are reflected from the earth's surface to make its details visible through the dense vapor-laden gases which surround it.[1]

The sharp and sudden occultation of stars by Mars shows that its atmosphere is slight. If the atmosphere of Mars were of the same proportionate mass as that of the earth, it would be rarer at the surface of the planet than that of the earth at the summit of the Himalayas. The rarity of its atmosphere is shown also by the fact that there is no absorption near the planet's limb. Cloudlike formations are sometimes visible, but they are probably due to dust, and curiously, they rise to great heights.

[1] The newspaper talk of communication between the earth and Mars by any imaginable means is utter foolishness. When we see Mars the best, the earth is "new" with respect to Mars, and invisible from that direction.

267. The Topography of Mars. — To the unaided eye Mars appears decidedly red, but with a telescope of even moderate dimensions the general ruddy surface is blotched with darker, greenish patches, while there are often white polar caps. The dark patches are fixed and sensibly permanent, but the polar caps vary with the planet's seasons, as was first noticed by Herschel. In the height of winter for a given hemisphere the white polar cap extends down 25° to 35° from the pole; but as spring comes on it diminishes in size, first gradually contracting around its edges, then breaking into parts, and sometimes it totally disappears. This is more easily observed for the south pole than for the north pole, because the south pole is turned toward us when Mars is in opposition near the perihelion of its orbit.

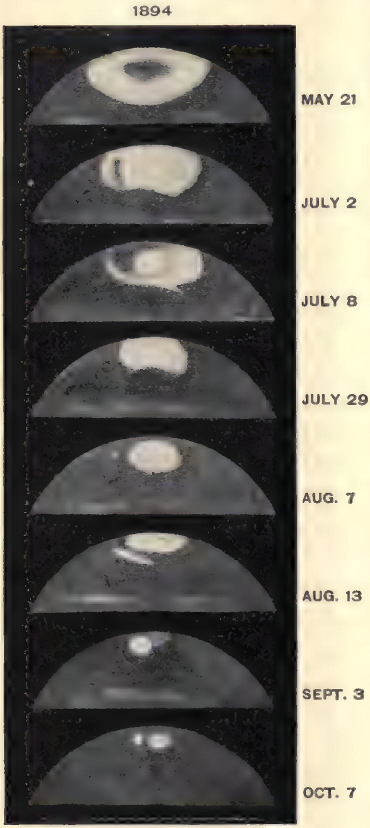


FIG. 125.—Disappearance of Polar Caps of Mars (Barnard).

When the Martian winter comes on in a hemisphere of the planet, the polar cap develops, but by sudden steps instead of gradually. For example, on April 9, 1890, W. H. Pickering photographed a moderate southern polar cap and a large dim surrounding area. On the next day the polar cap had spread all over this dim area, just as though

snow had fallen over a region as large as the United States. These polar caps have every appearance of being caused by snow, and the manner of their appearance and disappearance is equally confirmatory; but it is difficult to account for the warm climate that their vanishing indicates.

If the polar caps of Mars are due to snow, there must be water vapor in its atmosphere. This is a question for spectroscopic investigation. Huggins and Vogel, in 1867 and 1873 respectively, found what they took to be traces of absorption by aqueous vapor, but their conclusions have not been supported by the more recent researches of Keeler and Campbell. Jewell thinks that with present instrumental equipment the presence of water vapor could not be certainly detected unless it is more abundant than it is in the earth's atmosphere. So far as the spectroscope is concerned the question is still open.

The dark areas previously mentioned were supposed to be seas, and the reddish portions the continental regions. The rapid changes of color in low latitudes in spring were ascribed to the flooding of marshy regions from melting snows. If this is the correct division of the surface into land and water, the surface of this planet is only about three-eighths water, whereas that of the earth is about six-eighths water. Like the earth the greater part of the land surface is in the northern hemisphere. But it is very doubtful whether the dark areas on Mars are covered with water, for under good conditions they are seen covered with very complex markings, instead of having the uniformity of actual seas.

268. The Canals of Mars. — In 1877 Schiaparelli made the first of a series of important discoveries respecting the surface markings of Mars. He found that what had been supposed to be continental areas were crossed and recrossed by many dark greenish streaks which always ended in the "seas." They are of great length, extending along the arcs of great

circles from a few hundred miles up to three or four thousand miles. Each streak is very uniform in width, and the different streaks are from 20 to 60 miles across. These streaks were called by Schiaparelli *canali* (channels), which was translated into *canals*, a designation unfortunately too suggestive, for they have no analogy with anything on the earth. The very narrowest of them that can be seen are 15 or 20 miles across. Often many intersect in dark "water areas" called "lakes." For example, seven canals converge in Lacus Phœnicis and six in Lacus Lunæ. According to Lowell the junctions of canals are always provided with lakes, and conversely lakes are nowhere found except at the junctions of canals.

In the winter of 1881–1882 Mars was again in opposition, though not so near the earth as at the previous one. Schiaparelli not only verified his earlier observations, but he also discovered the remarkable fact that a number of the canals are double; that is, that in twenty cases two canals ran perfectly parallel to each other at a distance of from 200 to 400 miles apart. The doubling was found to depend on the seasons and to develop with astonishing rapidity chiefly when the sun was at the Martian equinox. Since his observations were made with a modest telescope of 8.75 inches aperture, and since the results appeared to be so inherently improbable, they were at first accepted with many doubts. But subsequently many skilled observers, working under the most favorable conditions, and with the finest instruments, have verified the observations of the keen-eyed Italian. Among them may be mentioned Perrotin and Thollon at Nice, Williams of England, some of the Lick observers, and the Lowell observers. Lampland has recently partly confirmed these observations by photography at the Lowell observatory. On the other hand, Denning describes them as being somewhat irregular in direction and of varying widths, while Barnard has never been able to see them.

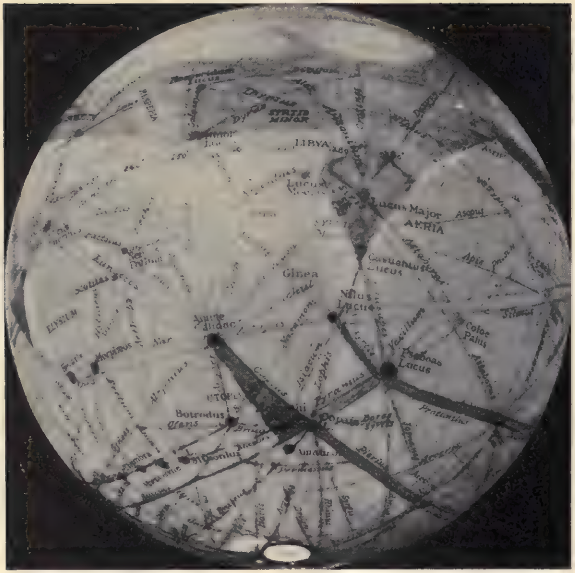
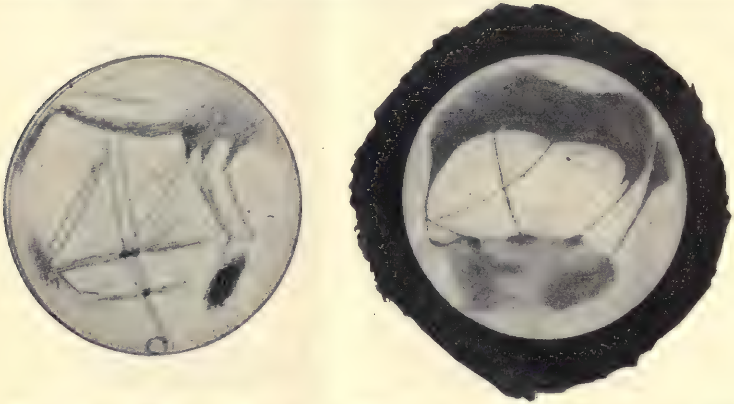


FIG. 126. — One of Lowell's Maps of Mars, showing many Faint Canals, some of which are Double.

269. Explanation of the Canals. The explanation of the canals of Mars can not be given with perfect certainty at present. We may understand at once that the data are not absolutely conclusive, and that every theory is entitled to a respectful consideration. Moreover, it must be accepted that every observer, in stating what he saw, described to the best of his ability the impression which he received.

That canals are *seen* on Mars and that some of them are sometimes seen double is open to no question. It is not quite so certain that they run exactly along the arcs of great circles and that they are of uniform width as Schiaparelli and Lowell describe them. For objects anywhere near the limits of vision these would be difficult questions to settle definitely, and, moreover, some observers reach the opposite

conclusions. However, Lowell is quite positive in the matter, and the idea that he has not in some way deceived himself is supported by the fact that he describes the markings on Mercury and Venus as streaks not having the line-like characteristics of those on Mars. Another important fact is that the canals and the visibility of their double character



Drawing of Mars by Lowell, at Flagstaff. Drawing of Mars by Cogshall, at the University of Indiana.

FIG. 127.

vary with the seasons. In the Martian winter they are invisible, but come out in the spring, sometimes with great rapidity. From an exhaustive study of his very abundant observational material, Lowell showed that the canals first become visible in the high latitudes along the borders of the polar caps, and then appear farther and farther down toward the equator, which they reach in about 50 days.

There are even greater differences in the explanations of the canals than there are in the descriptions of their appearance. Schiaparelli described them without attempting to give any explanation. Yet it is perfectly proper to attempt an explanation, provided we recognize its uncertainty until its truth has been firmly established. The explanation

is the thing of chief interest. It is that which stimulates observers to make great expenditures of time and money, for one would not find otherwise much pleasure in studying the faint markings of a body which always looks smaller than an object an inch in diameter does at the distance of 200 yards.

W. H. Pickering suggested that the canals may be due to vegetation. This idea is to some extent supported by their color and the manner of their changes with the seasons. It does not account for their narrowness, straightness, and frequent doubling. Lowell pushed the theory much further by supposing that in the centers of the canals are actual waterways constructed by intelligent beings ; that the waterways are used for irrigation purposes ; that the canals which are seen are due to the vegetation growing on the irrigated districts along the banks of the waterways ; and that perhaps pumping stations are needed because of the level character of the surface to raise the water so that it will flow over the land. According to this view the "lakes" are sorts of oases where the irrigating waterways cross. The canals become visible first in high latitudes because these regions are first supplied with water from the melting polar caps. A flow of 2.1 miles per hour would lead it to the equator in about 50 days. It will be seen that in this way the phenomena of the canals can be explained very well. Lowell does not pretend to have proved this theory ; he believes that it is one of the possibilities, and that at present it best explains all the observed phenomena. One of the most direct objections to it, the question of temperature, will be treated in the next article.

Proctor suggested that the canals may appear double through some effect of diffraction ; Stanislas, that the phenomena may be caused by reflections from different atmospheric strata ; and Flammarion, that they may be due to mirage-like refractions. These are suggestions for explaining simply the twin-like character of the canals and do not

give a theory of the single canals or why they vary with the seasons.

Recently Evans and Maunder have carried out a series of very interesting experiments for the purpose of determining whether the canals may not after all be some sort of optical illusion. They used a drawing on a circular disk 5 or 6 inches in diameter which represented fairly accurately the general shaded areas which have been seen on Mars. Instead of canals a few faint wavy lines and a larger number of faint dots were inserted promiscuously. This drawing was exhibited to schoolboys who were seated at various distances from it. They were furnished drawing paper on which were circles three inches in diameter, and were told to fill in carefully only that which they could see on the original drawing. They were ignorant of the appearance of Mars and were not told of the purpose of experiment.

It was found that those boys who were at such distances that the faint lines and dots were just beyond the limits of separate visibility drew canals having startling resemblances to those seen by observers of Mars. It is supposed that the eye in some way integrates faint stimuli which are separately imperceptible, and out of irregularly scattered dots constructs fine straight lines. Evans and Maunder drew the conclusion, perhaps somewhat hastily, that therefore the canals seen on Mars have no objective existence on the planet.

Thus it is seen that "the doctors disagree," and the only conclusion we can draw with safety at present is that Mars exhibits a large amount of detail which we can not certainly interpret. The problem of determining directly whether Mars is inhabited, even with the best telescopes and under the finest atmospheric conditions, is as difficult as it would be to find whether this country is inhabited or not from a perfectly accurate relief map of the whole United States, made on such a scale that it would be only 3 inches in diameter and held

at a distance of 3 feet from the eye. On this map the diameter of Ohio would be only $\frac{1}{4}$ of an inch, and the height of the loftiest mountain only $\frac{1}{350}$ of an inch.

270. The Temperature of Mars. — The most direct argument that has been brought against Lowell's theory is that Mars is too cold to support life. In this connection it has been suggested that the polar caps are not snow, but that they are carbon dioxide, which freezes at $-109°$ Fahrenheit into a white solid resembling snow. When the temperature rises above $-109°$ Fahrenheit, the carbon dioxide crystals melt and evaporate. These conclusions are based on the fact that Mars receives only about four-ninths as much heat from the sun as the earth does.

The case is not quite so unfavorable as a superficial examination might lead one to suppose. It by no means follows that the temperature of Mars would be only four-ninths that of the earth. It may be supposed that on the average the planets radiate as much heat during their respective revolutions as they receive. The rate of radiation of a body (a perfectly black body), according to Stefan's law, varies as the fourth power of the absolute temperature. Therefore, if one body is constantly twice as hot as another, it radiates 16 times as much heat, and is supplied with 16 times as much heat. Conversely, if it were constantly supplied with 16 times as much heat, it would be twice as hot on the absolute scale.

On the Fahrenheit scale the absolute zero is about 490° below freezing. If the average temperature of the earth were taken as 60° Fahrenheit, its temperature on the absolute scale would be $490° + (60° - 32°) = 518°$. If we let x represent the absolute temperature of Mars, it follows from Stefan's law that

$$x : 518 = \sqrt[4]{4} : \sqrt[4]{9};$$

whence $$x = .82 \times 518° = 425°.$$

But 425° on the absolute scale is $425° - (490° - 32°) = -33°$ with the Fahrenheit zero. These results are obtained under the hypothesis that the two bodies are both perfect radiators, or, at least, radiate similarly. The difference in the constitution of their atmospheres may cause conditions quite different from those suggested by these figures (Art. 114). At any rate, it is not safe to assume, in the light of our present knowledge, that the temperature on Mars is necessarily so low as these figures suggest it may be.

271. The Major Planets. — It was seen in the last chapter that the major planets have many common characteristics which distinguish them from the members of the terrestrial group. Their detailed study emphasizes the differences in the two groups.

The nearest of the major planets is Jupiter, which, even in opposition, is more than four times as far from us as we are from the sun. Because of this great distance it would not be possible to study fine markings, such as are found on Mars, even if they existed, and the difficulties are greater for Saturn, Uranus, and Neptune. It will be understood that some of the problems are unsolved because of the great distances of these bodies.

JUPITER

272. Jupiter's Satellite System. — The first objects discovered by Galileo when he pointed his telescope to the sky, in 1610, were four of the moons of Jupiter. They are barely beyond the limits of visibility without optical aid, and, indeed, could be seen with the unaided eye if they were not lost in the dazzling rays of Jupiter. No other satellite of this planet was discovered until 1892, when Barnard caught a glimpse of a fifth one very close to the planet. It is so small and so buried in the rays of the planet that it can be seen only by experienced observers through a few of the largest telescopes in the world. Early in 1905 Perrine

found by photography that Jupiter has still two more satellites, which are more remote from the planet than those previously known. They are both about 7,000,000 miles from Jupiter. The eccentricities of their orbits are considerable and their paths actually loop through each other, but their mutual inclination is so high (27°) that they do not often pass very near each other.

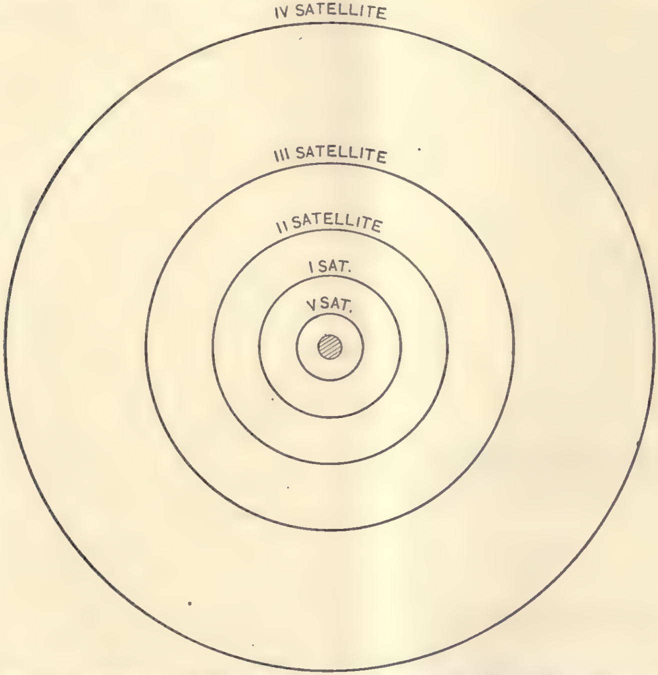


Fig. 128. — Jupiter and Orbits of First Five Satellites to Scale. (On this scale the radii of the orbits of the sixth and seventh satellites would be about 10 inches.)

The satellite discovered by Barnard is called V (the fifth), and the others previously known in the order of their distance from Jupiter are I, II, III, IV. The last two are for the present called the sixth and seventh in the order of their

discovery. The orbits of all the satellites, except the sixth and seventh, are very nearly circular, and all except those of the sixth and seventh lie in the plane of the planet's equator. The size of the fifth satellite can be estimated only from the amount of light it reflects; the four largest ones can be measured. The satellites all revolve around Jupiter from west to east except possibly the seventh, which the photographs so far obtained seem to show, performs a retrograde revolution. The following table gives their distances, periods, and diameters, the last being the results of Barnard's extensive measures.

Satellite	Distance from Center of Jupiter	Period	Diameter
V (unnamed)	112,500 mi.	0 da. 11 hr. 57 m.	About 100 mi.
I (Io)	261,000 mi.	1 da. 18 hr. 28 m.	2452 mi.
II (Europa)	415,000 mi.	3 da. 13 hr. 14 m.	2045 mi.
III (Ganymede)	664,000 mi.	7 da. 3 hr. 43 m.	3558 mi.
IV (Callisto)	1,167,000 mi.	16 da. 16 hr. 32 m.	3345 mi.
VI and VII (unnamed)	About 7,000,000 mi.	About 265 da.	Very small

273. Markings on Jupiter's Satellites. — The great distance of Jupiter renders it difficult to detect any but large and distinctly colored markings on the satellites. In 1890 Barnard found I to be elongated parallel to the equator of Jupiter when transiting the darker portions of the planet, and elongated, or double, in the opposite direction when over the brighter parts. He interpreted this as meaning that the poles of the satellite are dark and that the equatorial belt is white. The accompanying cut, showing the satellite transiting

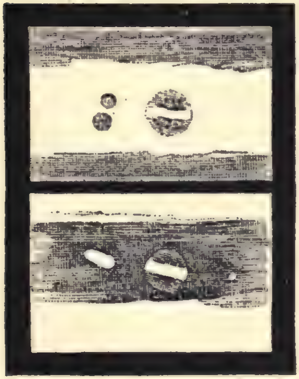


FIG. 129. — Barnard's Drawings of Jupiter's Satellite I.

a light region above and a dark one below, exhibits the observed appearance at the left, and the actual condition at the right. When held at some distance from the eye they look the same.

W. H. Pickering and Douglass observed the satellites III and IV as being of elliptical shape, and with equatorial stripes which had been previously seen by Schaeberle and Campbell. Douglass found many markings on III, much like those on Mars, and from them found the period of rotation to be about 7 days. Barnard with the Lick telescope saw none of these markings, but, instead, bright polar caps and similar ones on IV. They are represented in the accompanying cut. Satellite V presents no sensible disk.

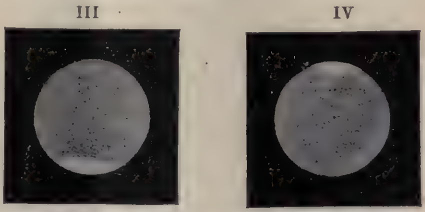


FIG. 130.—Barnard's Drawings of III and IV, Dec. 10, 1893.

274. Discovery of the Velocity of Light. — A very important discovery was made from observations of Jupiter's satellites. Their periods of revolution were found from observations made when the earth and Jupiter were nearest each other. Their periods being known, the times of eclipses could be predicted. Suppose this has been done and that it takes light a measurable time to travel such a distance as from the earth to the sun. Then, when we are nearest Jupiter as at E_1, the eclipses will occur at the predicted times. When the earth has gone round to E_2, they will be late by the time it takes light to travel the distance $J_2E_2 - J_1E_1$. From such observations, in 1675, Römer found that it takes light 499

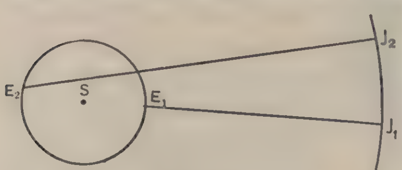


FIG. 131.

seconds to travel the distance from the sun to the earth. When the distance is known the velocity of light can be found.

At the present time the velocity of light can be found more accurately by physical experiments. The work of Fizeau, Michelson, and Newcomb shows that it is very nearly 186,330 miles per second. This velocity and the light equation of 499 seconds being known, the distance to the sun can be computed.

275. The Atmosphere and Albedo of Jupiter. — From its low average density and its great ability to retain an atmosphere, it is to be expected that Jupiter has an extensive gaseous envelope, and such observations as are pertinent support this view. The albedo, according to Zöllner, is 0.62. The light which is reflected to us does not seem to have penetrated far into the Jovian atmosphere, for its spectrum is almost identical with that of sunlight, except for a few absorption lines.

276. Surface Markings and Rotations of Jupiter. — Jupiter is covered with a variety of markings, many of which can be seen with a telescope of moderate dimensions. The principal features are broad parallel belts. The central one is white and at present about 10,000 miles wide; on each side is a belt of reddish brown color of nearly the same width. Several other belts in higher latitudes can be made out, though not with such distinctness, partly at least because we look at them obliquely. The belts vary considerably in width, as the drawings (Fig. 133) by Hough show.

FIG. 132. — Jupiter. *By L. V. Petitdidier.*

A good telescope reveals in the broad belts a very great variety of detail which continually changes as though what we see is cloudlike in structure. Dark spots appear and gradually turn red and finally disappear. The most remarkable and most permanent spot appeared in 1878 beneath the southern red belt as a pale, pinkish oval, extending along the belts 30,000 miles, and about 7000 miles in the other direction. In a year it had changed to a bright red color and was the most conspicuous object on the planet. It has since been known as "the great red spot." It has undergone many changes both of color and brightness and is still generally faintly visible.

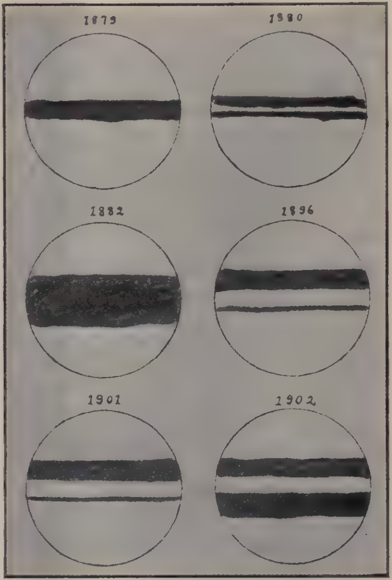


FIG. 133.—Drawings by Hough showing the Varying Width of Jupiter's Equatorial Belts.

From observations of the spots it has been found that, *on the average*, Jupiter rotates on its axis once in about 9 hr. 54 m., which is the shortest known period of rotation of any celestial body. It is necessary to speak of the average, for there are great differences of motion in spots which are very near together. The actual periods vary from about 9 hr. 50 m. to 9 hr. 57 m., or an extreme difference of about $\frac{1}{85}$ of the whole period. The circumference of the planet is nearly 300,000 miles, from which it follows that the rate of rotation at the

equator is about 30,000 miles an hour. Consequently, two spots whose periods differ by 7 minutes drift by each other at the relative rate of $\frac{30,000}{85} = 350$ (about) miles per hour. Compare this with the 70 to 100 miles per hour at which tornadoes sweep along the surface of the earth!

The central white belt is along the equator and rotates the fastest of any portion of the planet except a zone in north latitude about 20° to 24°. This more rapid rotation of the equatorial zone is a remarkable phenomenon which is also observed upon both Saturn and the sun. The rate of rotation does not diminish regularly with increasing distance from the equator, and there is marked dissimilarity in the two hemispheres. The southern hemisphere presents evidences of more rapid changes than the northern; the southern is particularly the home of bright spots, and the northern of dark ones. Another thing not less strange is that the rates of rotation of given spots may undergo rapid and large variations. The period of rotation of the great red spot increased seven seconds in the eight years following its first appearance, but it has been sensibly constant since. The sudden changes in rotation and displacement occur at the times of changes in color and visibility. It is not known why the equatorial zone rotates the fastest, or why the periods of individual spots sometimes vary.

277. Physical Condition of Jupiter. — In considering the physical condition of Jupiter it should be remembered that Jupiter has the low average density of 1.33 on the water standard, that the markings are not permanent, and that there are violent relative motions of the visible parts. These things indicate that Jupiter may be largely gaseous near its surface. The surface gravity is 2.6 times that of the earth, giving great pressures even at shallow depths. These pressures are sustained by the expansive tendencies due to high temperature. It is safe to infer from the

density and gravity pressure of Jupiter that its interior is very hot. It has been supposed, indeed, by some that its surface is very hot and partly self-luminous; but such can not be the case, for the shadows cast by the satellites on the planet are perfectly black, and when a satellite is in the shadow of Jupiter it is invisible. No disturbances distinctly resembling volcanic activity have been observed, but Jupiter is so far away that they could not be seen unless they covered a region several hundred miles across.

In conclusion, we shall probably be near the truth if we suppose that Jupiter is not in an advanced stage of its evolution like the terrestrial planets, but that it contains enormous volumes of gases which are in rapid circulation both along and perpendicular to its surface, and that the energy of its internal fires still gives rise to violent motions.

278. Seasonal Changes on Jupiter. — The inclination of the plane of Jupiter's equator to that of its orbit is only 3° 5′. It follows from this and the small eccentricity of its orbit that the seasonal changes are unimportant. Jupiter gets only $\frac{1}{27}$ as much light and heat per unit area from the sun as the earth does, and when its internal heat becomes exhausted by radiation, so far as we can judge, it will lapse into a condition of perpetual frigidity.

SATURN

279. Saturn's Satellite System. — Saturn has more known satellites than any other planet in the solar system. The largest one was discovered by Huyghens in 1655, then four more were discovered by J. D. Cassini, 1671–1684, two by William Herschel in 1789, one by G. P. Bond and Lassell in 1848, a ninth by W. H. Pickering in 1899, and a tenth by W. H. Pickering in 1905. The planet is so remote that their dimensions are only roughly known from their appar-

ent brightness, and we have no direct knowledge of the mass of any one except Titan, which is about $\frac{1}{4600}$ that of Saturn. The seven which are nearest to the planet revolve sensibly in the plane of its equator, while the orbit of the eighth, Japetus, is inclined to it about 10°, and the ninth about 20°.

When Japetus is on the western side of Saturn, it always appears much brighter than when on the eastern side. This difference in appearance is undoubtedly due to the fact that this satellite always has the same side toward the planet, and that its two sides reflect light very unequally.

The ninth satellite is in some respects the most remarkable satellite in the solar system. It was discovered on photographs taken at the Harvard station at Arequipa in 1898. It is so small and faint that it was not photographed again for several years, but now it has been detected on more than forty plates, and Barnard has seen it with the great Yerkes telescope. It is so far from Saturn that its period of revolution is 1.5 years, or more than six times that of Mercury around the sun. But the most unexpected thing about it is that it revolves around the planet in the retrograde direction — that is, opposite to that of the other satellites, the rotation of the planet, and, in fact, to all other motions of the solar system so far studied.[1]

Satellite	Distance from Saturn	Period	Diameter
I (Mimas)	117,000 mi.	0 da. 22 hr. 37 m.	About 600 mi.
II (Enceladus)	157,000 mi.	1 da. 8 hr. 53 m.	About 800 mi.
III (Tethys)	186,000 mi.	1 da. 21 hr. 18 m.	About 1200 mi.
IV (Dione)	238,000 mi.	2 da. 17 hr. 41 m.	About 1100 mi.
V (Rhea)	332,000 mi.	4 da. 12 hr. 25 m.	About 1500 mi.
VI (Titan)	771,000 mi.	15 da. 22 hr. 41 m.	About 3000 mi.
X (unnamed)	—	— — —	—
VII (Hyperion)	934,000 mi.	21 da. 6 hr. 39 m.	About 500 mi.
VIII (Japetus)	2,225,000 mi.	79 da. 7 hr. 54 m.	About 2000 mi.
IX (Phœbe)	7,996,000 mi.	546 da. 12 hr. —	About 200 mi.

[1] There may be an exception in the case of the seventh satellite of Jupiter.

The preceding table gives the principal data respecting the satellites, and the diagram below illustrates the relative dimensions of their orbits to scale.

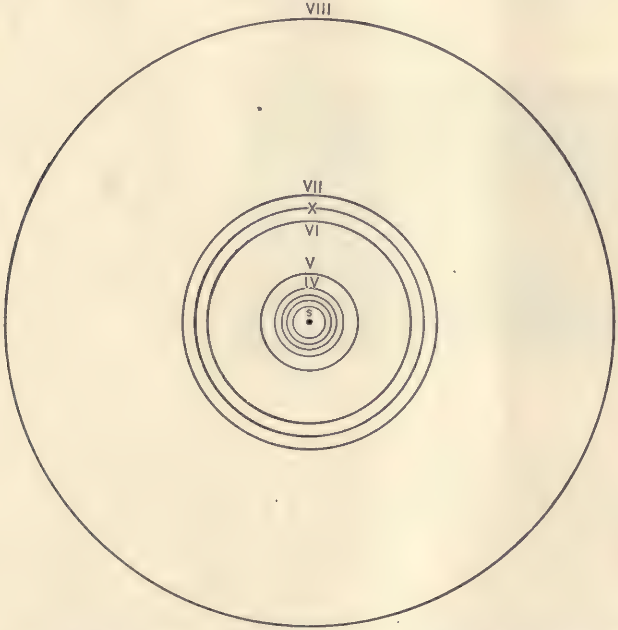


FIG. 134.—The Orbits of Saturn's Satellite System to Scale. (On the same scale, the orbit of IX would be over 11 inches in diameter.)

280. Saturn's Ring System. — Saturn is distinguished from all the other planets by three wide, thin rings which surround it in the plane of its equator. They were seen by Galileo in 1610, but their character was not known until the observations of Huyghens in 1655.

The outer ring has a diameter of 172,610 miles, a width in its plane of about 11,000 miles, and a thickness estimated to be about 50 miles. It is circular around the planet as a

center, and is of nearly the same brightness as the planet. Inside of this ring at a distance of about 2200 miles is another ring whose width is about 18,000 miles. The narrow zone between the rings was discovered by Cassini and is called *Cassini's division.* The outer portion of this inner ring is the brightest part of the whole ring system, and is fully as bright as the planet itself. Inside of this ring is another known as the *crape ring*, which is about 11,000 miles in width. Its inner edge is less than 6000 miles from the surface of the planet. It is very faint on its inner borders and gets gradually brighter until it merges in the bright ring beyond.

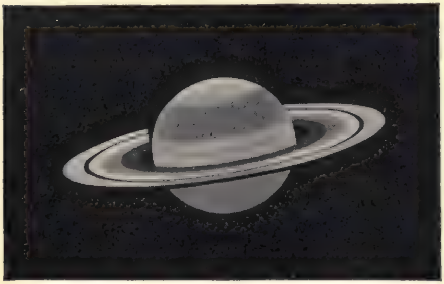


FIG. 135. — Barnard's Drawing of Saturn, July 2, 1894.

The rings are inclined about 27° to the plane of the planet's orbit, and about 28° to the plane of the ecliptic. Consequently we see them at a great variety of angles. When their plane passes through the earth they appear as a very fine line; but when the inclination is high, Saturn and its ring system present, through a good telescope, a sight which can never be forgotten.

The dimensions of Saturn and the ring system are, according to the measures of Barnard:

Equatorial radius of the planet	38,235 miles
From center of planet to inner edge of crape ring	44,100 miles
From center of planet to inner edge of inner bright ring	55,000 miles
From center of planet to outer edge of inner bright ring	73,000 miles
From center of planet to inner edge of outer ring	75,240 miles
From center of planet to outer edge of outer ring	86,300 miles

The following diagram shows the equatorial section of the planet and the rings to scale.

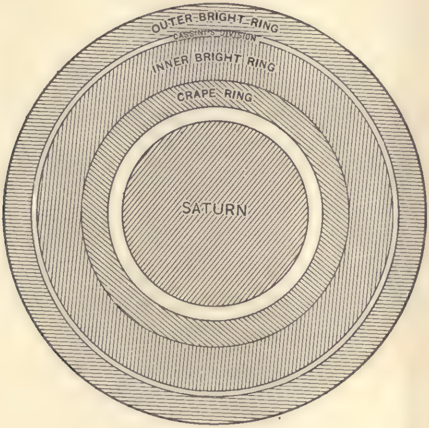


Fig. 136.

On this scale the thickness of the rings would be about $\frac{1}{1600}$ of an inch, while the distance to the ninth satellite would be over eight feet.

281. Constitution of Saturn's Rings. — The bright rings have the same appearance of solidity and continuity as the planet itself. It was generally believed until about one hundred years ago that they were solid or fluid. Yet since 1715, when J. Cassini first mentioned the possibility, it has frequently been suggested that the rings may be simply swarms of meteors, or exceedingly minute satellites, revolving around the planet in the plane of its equator. Such small bodies would exert only negligible gravitational influences upon one another, and their orbits would be sensibly independent of one another except for collisions.

The meteoric theory was first rendered probable by Laplace, who showed that a solid symmetrical ring would be dynamic-

ally unstable. That is, the solid rings would be something like spans of enormous bridges, and they would have to be composed of inconceivably strong material to withstand the strains due to their motion and the gravitative forces to which they are subject. In 1857 Clerk-Maxwell showed from dynamical considerations that the rings could be neither solid nor fluid, and that they were therefore meteoric. Now, if they are meteoric, those parts which are nearest the planet will move fastest, just as those planets which are nearest the sun move fastest, while if they were solid the opposite would be the case. In 1895 Keeler showed by line-of-sight observations with the spectroscope that the inner parts not only move fastest, but that all parts move precisely as though they were made up of totally disconnected parts. (The innermost particles of the crape ring perform a revolution in about five hours.) Hence it may be considered as firmly established that the rings are swarms of meteors.

The rings are strange substitutes for satellites; still a probable explanation of the condition is known. A planet exerts tidal strains upon satellites in its vicinity, and these tendencies to rupture increase very rapidly as the distance of the satellite decreases. In 1848 Roche proved that these tidal forces would break up a fluid satellite of the same density as the planet if its distance were less than 2.44... radii of the planet. The limit would be less for denser satellites, and a little less for solid satellites, but not much less if they were of large dimensions. It is seen from the numbers given above, or Fig. 136, that the rings are within this limit. It is not supposed that they are the pulverized remains of satellites that ever did actually exist, but rather that the material of which they are composed is subjected to such forces that it can never aggregate itself into a satellite under its own gravitative action.

One more interesting thing remains to be mentioned. If a meteor were to revolve in the vacant space between the

rings known as Cassini's division, its period would be nearly commensurable with those of four of the satellites. Kirkwood called attention to this relation, which is entirely analogous to that found in the case of the planetoids, Art. 247.

282. Markings and Rotation of Saturn. — The markings on Saturn are much like those on Jupiter, though of course they are not seen nearly so well because the planet is about twice as far from us. There is a bright equatorial belt, and there are many darker zones in the higher latitudes. The polar regions are darker than any other parts of the planet. It is rather difficult to find spots conspicuous and lasting enough to enable observers to determine the period of Saturn's rotation. From observations made in 1794 Herschel concluded that the period is 10 hr. 16 m.; Hall's observations of a bright equatorial spot in 1876 gave the period of this spot as 10 hr. 14 m. This was generally adopted as the period of Saturn's rotation, particularly after it had been verified by a number of other observers. In 1903 Barnard discovered some bright spots north of its equator. The work of many observers on these spots showed that they passed around Saturn in about 10 hr. 38 m. This difference in period means that there is a relative drift between the equatorial belt and the higher latitudes of 800 to 900 miles per hour.

283. Physical Condition of Saturn. — The density of Saturn is about 0.72 on the water standard; consequently it must be largely in a vaporous condition, for it is so light that it would float on water. Probably no considerable portion of it is purely gaseous. It seems more likely, especially because of its being opaque, that the gases are filled with minute liquid particles just as our air becomes charged with minute globules of water, forming clouds.

The remarkable relative motions in the surface of Saturn show that it must be in a fluid condition. Doubtless it is a world whose evolution is not yet sufficiently advanced to

give it any permanent markings, much less to fit it as a place in any way suitable for organic existence.

284. The Seasons of Saturn. — The plane of Saturn's equator is inclined nearly 27° to that of its orbit, and besides its orbit is rather more eccentric than the orbits of the other large planets. For these reasons the seasonal changes would be marked. However, Saturn is so remote from the sun that it receives only $\frac{1}{90}$ as much light and heat per unit area from this great luminary as the earth does. It follows that its condition depends more upon its internal energies and less on the light and heat received from the sun than is the case with the planets thus far considered.

URANUS AND NEPTUNE

285. Discovery of Uranus and Neptune. — Uranus was discovered by Herschel in 1781, while he was carrying out his program of examining every object in the heavens within reach of his telescope. It is a faint object of the sixth magnitude, and the fact that Herschel recognized that it was not a star by the minute disk it presented is ample evidence of his keenness of sight and of his mental alertness. His discovery of a world, the first in historical times, at once won for him royal favors and made him the most celebrated observer of his time. It was found later that Uranus had been observed several times before, but always as a star.

The discovery of Neptune was a matter of great scientific and popular interest. It was found by 1820 that Uranus was not following exactly its computed path. It must not be supposed that the deviation which gave the astronomers so much unrest was very great. By 1840 the apparent departure of the planet from its theoretical position was less than two-thirds the apparent distance between the two components of Epsilon Lyræ (Art. 47). About 1830 Bessel expressed the opinion that the discrepancy between theory

and observation was due to an unknown planet more remote from the sun than Uranus. Shortly after 1840 Adams, of Cambridge, England, and Leverrier, of Paris, independently took up the problem of finding the position of the unknown body from the perturbations it had produced in the motion of Uranus. The investigation required rare mathematical talents and was of the most laborious character. Adams finished his work first and communicated it to the Astronomer Royal, who, to say the least, did not take a very lively interest in the matter, and allowed the search to be postponed. Leverrier sent his results, which agreed nearly with those of Adams, to the German astronomer Galle, who found the new planet the first evening he looked for it, September 23, 1846. Notwithstanding the fact that both Adams and Leverrier made assumptions respecting the distance of the unknown body which were afterward found to be in error, their work stands as a monument to the perfection of the theory of the motions of the heavenly bodies.

286. The Satellite Systems of Uranus and Neptune. — Uranus has four known satellites, two of which were discovered by William Herschel in 1787, and the other two by Lassell in 1851. Their distances are respectively 120,000, 167,000, 273,000, and 365,000 miles, and their periods are respectively 2.5 da., 4.1 da., 8.7 da., and 13.5 da., and their diameters probably between 500 and 1000 miles. They all move sensibly in the same plane, which is inclined about 98° to the plane of the planet's orbit; that is, if the plane of their orbits is thought of as having been turned up from that of the planet's orbit, the rotation has been continued 8° beyond perpendicularity, and the satellites revolve in the retrograde direction.

Neptune has one known satellite, which was discovered by Lassell in 1846. It revolves at a distance of 221,500 miles in a period of 5 da. 21 hr., and its diameter is probably about 2000 miles. The plane of its orbit is inclined about 145°

to that of the planet's orbit; that is, the inclination between the two planes is about 35° and the satellite revolves in the retrograde direction.

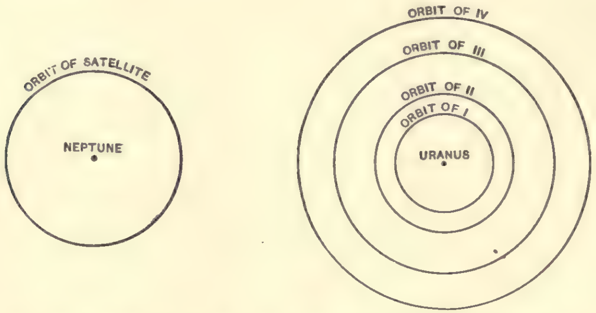


FIG. 137. — Scale Diagram of the Satellite Systems of Uranus and Neptune.

287. Atmospheres and Albedoes of Uranus and Neptune. — Very little is known directly respecting the atmospheres of Uranus and Neptune. From the great gravitative power of such large bodies extensive atmospheres are to be expected. As confirmatory of this general view the spectroscope shows that the light which we receive from them has passed through some extensive absorbing medium besides the sun's atmosphere and that of the earth. The absorbing effects of the element hydrogen are shown in the spectra of both planets, but, according to the recent results obtained by Slipher, more strongly in the case of Neptune than in that of Uranus. Some of the other absorption bands are due to unknown substances, among them one which is found in the spectra of very red stars.

The albedo of Uranus is 0.60 and that of Neptune 0.52.

288. Surface Markings and Rotations of Uranus and Neptune. — Surface markings have been seen on Uranus by many observers, as Buffham, Young, the Henry brothers, Perrotin, Holden, and Keeler, but they are so indefinite and

fleeting that it has not been possible to draw any certain conclusions from them. Nevertheless, so far as they go, they indicate that the period of rotation is 10 or 12 hours, and that the plane of the equator is inclined something like 10° to 30° to the plane of the orbits of the satellites. In 1894 Barnard detected a slight flattening of the disk, with the equatorial diameter inclined 28° to the plane of the orbits of the satellites.

No certain markings have been seen on Neptune and consequently its rate of rotation has not been found by direct means. But by indirect processes both the position of the plane of the equator and the rate of rotation have been found, at least approximately. The dimensions and mass of Neptune are known with considerable accuracy. Now, if the rate of rotation were known, the equatorial bulging could be computed. Suppose the plane of the orbit of the satellite were inclined to that of the planet's equator. Then the equatorial bulge would perturb the motion of the satellite; in particular, it would cause a revolution of its nodes, and the rate could be computed.

The problem is about the converse of that which has just been described. The nodes revolve, and the manner of their motion shows the existence of a certain equatorial bulge inclined about 20° to the plane of the satellite's orbit. The bulging, or ellipticity, is $\frac{1}{85}$, indicating, according to the work of Tisserand and Newcomb, a rather slow rotation as compared to Jupiter and Saturn.

289. Physical Condition of Uranus and Neptune. — We can only infer the physical condition of Uranus and Neptune from that of other planets which are more favorably situated for observation. They are probably much in the state of Jupiter and Saturn, though possibly somewhat further advanced in their evolution because of their smaller dimensions.

One thing to be noticed is that they receive very little heat from the sun. The amounts per unit area are about

$\frac{1}{368}$ and $\frac{1}{906}$ that received by the earth. If their capacity for absorbing and retaining heat were the same as that of the earth, their theoretical temperatures (Art. 270) would be about $-330°$ and $-360°$ Fahrenheit. Nevertheless, it must not be imagined that even Neptune would receive only feeble illumination. Although, as seen from that vast distance, the sun would subtend a smaller angle than Venus does to us when nearest the earth, the noonday illumination would be equal to 700 times our brightest moonlight.

QUESTIONS

1. Does the earth have phases as seen from Mercury and Venus?

2. If the inferior planets revolved around points between the earth and sun, what would be their phase changes as seen from the earth?

3. Do the superior planets have any phase changes at all?

4. Suppose Phobos and Deimos are in conjunction. How many hours before they will be in conjunction again?

5. If the diameter of Phobos is 7 miles, what is its angular diameter as seen from Mars, when it is on the observer's meridian?

6. Is Phobos eclipsed every synodic revolution?

7. As seen from Mars, is the sun ever totally eclipsed by Phobos?

8. Suppose the sun and Deimos rise in the east and Phobos in the west at the same time; describe the phase changes of Phobos and Deimos until they set in the east and west respectively.

9. Which of Jupiter's satellites is most like the moon in mean distance and size? Why is its period so much shorter than that of the moon?

10. Compute the periods of Jupiter's satellites from meridian passage to meridian passage again.

11. Compute the apparent angular diameters of Jupiter's satellites as seen from the planet's equator when they are on the observer's meridian.

12. From Kepler's third law (Art. 138) compute the periods of revolution at the edges of Saturn's rings. Compare the time of revolution of the innermost particle with that of the planet's rotation.

13. Are the rings of Saturn visible from all latitudes on the planet?

14. On the basis of Stefan's law of radiation, and on the assumption that their absorption and radiation of energy received from the sun are like that of the earth, compute the theoretical mean temperatures of all of the planets. [For relative amounts of heat received, see Art. 237.]

FIG. 138. — The Lick Observatory.

CHAPTER XIII

COMETS AND METEORS

290. Comets and Meteors. — The planets are characterized by the invariability of their form and general physical condition, the simplicity of their motions, and their general similarity to one another. In strong contrast to these stable bodies are the comets, whose bizarre appearance, complex motions, and temporary visibility have led astronomers to devote, perhaps, relatively more time to them than they deserve. Most comets are visible only through a telescope, but a few times in a century the sky is visited by one of truly startling appearance. Until the last two centuries they were objects of superstitious terror, and were supposed to portend calamities. With rather elastic limits as to when and where the disasters should come, and history filled with accounts of deeds of violence, it was easy to prove that comets were followed by misfortunes, if not the converse.

Meteors are wandering bits of matter which become visible only when they encounter the earth's atmosphere. As they originate partly, at least, from disintegrated comets, they are appropriately discussed in this chapter.

291. The General Appearance of Comets. — The typical comet is a body composed of a *head*, a brighter *nucleus* within the head, and a *tail*, streaming out in the direction opposite to the sun. The apparent size of the head, or coma, may be anywhere from almost starlike smallness to the angular dimensions of the moon; the nucleus is usually very small and bright, while the tail may stretch away many degrees before it gradually fades out into the darkness of the sky.

The head is the most distinctive part of a comet, being always present and looking much like a circular nebula. Either the nucleus or tail, or both, may be absent from a comet, particularly if it is a small one. Comets are sometimes so bright that they may be observed in full daylight, yet they are so transparent, except in their nuclei, that stars are observed through them without having apparently lost the slightest amount of light.

There are records of about 400 comets having been seen before the invention of the telescope in 1609, and about the same number have been seen since that date. Astronomers now keep such close watch of the sky that from three to ten are discovered yearly. In 1898 ten were found. They are lettered for each year *a*, *b*, *c*, . . . in the order of their discovery. They are also numbered each year I, II, III, . . . in the order that they pass their perihelia; and, besides, they are frequently named after their discoverers. Thus, comet 1904–*a* is the first one discovered in 1904, and 1904–I is the first one to pass its perihelion in the same year.

Fig. 139.
Swift's Comet, April 7, 1892 (Barnard).

292. Dimensions of Comets. — Observations show that comets are usually very far from the earth; that is, at dis-

tances something like that of the sun. In ancient times they were generally supposed to be phenomena of the earth's atmosphere at great altitudes.

When the distances of the comets and their apparent dimensions are known, their actual dimensions can be computed. It is found that the head of a comet may have any diameter from 10,000 miles up to over 1,000,000 miles. The most remarkable thing about the head is that it nearly always contracts as the comet approaches the sun and expands again when it recedes. The variation in volume is very great, the ratio of largest to smallest dimensions sometimes being as great as 100,000 to 1. The only attempted explanation is that of John Herschel, who suggested that the contraction may be only apparent owing to the outer layers becoming transparent. This not only contradicts appearances, but there is nothing definite which supports it.

The nucleus usually is between the smallest that can be seen, say 100 miles, and 5000 miles in diameter. For example, at one time William Herschel observed the great comet of 1811 when its head was more than 500,000 miles in diameter, while its nucleus measured only 428 miles across. The nuclei vary in size during the motion of the comets, but quite irregularly, and no law of variation has been discovered.

The tails of comets are inconceivably voluminous. Their diameters are counted by thousands or tens of thousands of miles at their heads, and by tens of thousands or hundreds of thousands of miles at their visible extremities. Their lengths are from a few millions of miles up to more than 100,000,000 of miles. The strangest thing about them is that they point almost directly away from the sun whichever way the comet may be going. Moreover, the tails develop as the comets approach the sun, and diminish in size and splendor as they recede from it.

293. Masses of Comets. — Comets give visible evidence of remarkable tenuity, but if their densities were even one-

thousandth of that of air at the surface of the earth, their masses would not be insignificant. Their masses are computed by the same principles that are used to find the masses of the planets, that is, from their attractions for other bodies. In the present case it would be more correct to say that their lack of appreciable mass is shown by the fact that they do not produce observable disturbing effects on bodies near which they pass. Many comets have had their orbits entirely changed by planets without their producing any sensible effects in return. Since the action (that is, the product of the mass and velocity generated) is reciprocal, this means that the masses of comets are very small, probably not exceeding one-millionth that of the earth. About the most striking example so far known of the feeble gravitational power of comets was furnished by the one discovered by Brooks in 1889. It passed through Jupiter's satellite system in 1886 without interfering sensibly with the motions of these bodies, while its own orbit was transformed so that its period was reduced from 27 years to 7 years.

294. Orbits of Comets. — Kepler supposed comets moved in straight lines, but Doerfel showed that the comet of 1681 moved in a parabola around the sun as a focus. In 1686 Newton invented a method of computing their orbits from three observations of their apparent position.

The orbits of about 400 comets have been computed, and as nearly as can be determined about 300 are parabolic. Of the remainder about 90 have been found to have elliptical orbits, and 6 or 8 slightly hyperbolic orbits. The parabola (Art. 163) is the curve which separates the family of ellipses from the family of hyperbolas, and it is altogether improbable that any comet moves exactly in a parabola. The question whether an orbit is elliptical or hyperbolic is an important one, for if it is elliptical the comet will ultimately return to the sun, while if it is hyperbolic it will indefinitely recede. About 75 comets have elliptical orbits whose major

axes are short enough to insure their return to the vicinity of the sun. The remainder move in exceedingly elongated orbits, and the character of their motion is less certain. The hyperbolic orbits so far found are all very near the parabolic limit, and it may be that the computed deviations from it in this direction are all due to small errors of observation. The possibility of this suggestion will be accepted when it is remembered that a comet can be observed only while it is about as near the sun as the earth is, and while it traverses an insignificant part of its entire orbit. In addition, the

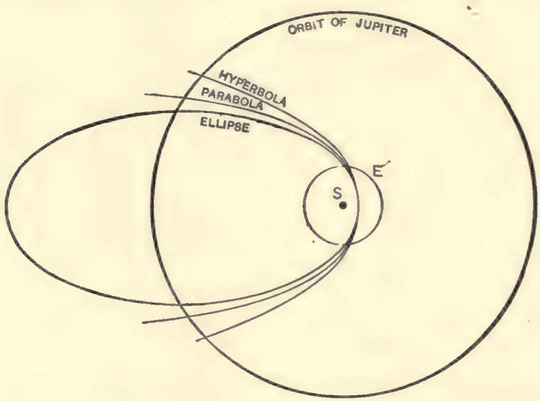


Fig. 140.—Showing the Slight Differences in Elliptic, Parabolic, and Hyperbolic Orbits in the Observed Portions of Cometary Orbits.

generally diffuse character of a comet makes it very difficult to locate exactly the position of its center of gravity. The problem is something like that of attempting to construct accurately, by rule and compass, a circle a foot in diameter having given three dots on it one-hundredth of an inch in diameter and one-tenth of an inch apart.

The orbits of the planets are nearly all in one plane; on the other hand the planes of the orbits of the comets lie in

every possible direction and exhibit no tendency to parallelism. The perihelia are distributed all around the sun, but show a tendency to cluster in the direction in which the sun is moving among the stars, that is, in the direction of Vega. This peculiar distribution of perihelia may have some connection with the sun's motion.

Some comets have perihelion points only a few hundred thousand miles from the sun and actually within its corona. About 25 comets pass within the orbit of Mercury; nearly three quarters of those which have been seen come within the orbit of the earth, and the remainder all far within the orbit of Jupiter. It must be remembered that this distribution refers only to those comets which have been seen. It is precisely what would be expected even if comets were equally numerous with perihelion distances indefinitely great, for they are very inconspicuous, or entirely invisible, from the earth when their distance from the sun is greater than that of Mars. It may well be that a vast majority of those which pass around the sun fail to come near enough to it to be seen from the earth.

295. Families of Comets. — Notwithstanding the great diversities in the orbits of the comets, there are a few groups whose members seem to have some intimate relation to one another, or to the planets. There are two types of these groups, and they are known as *comet families.*

Families of the first type are made up of comets which pursue nearly identical paths. The most celebrated family of this type is composed of the great comets of 1668, 1843, 1880, and 1882. A much smaller one seen in 1887 probably should be added to this list. Their orbits were not only nearly identical, but the comets themselves were very similar in every respect. They came to the sun from the direction of Sirius — that is, from the direction *from* which the sun is moving with respect to the stars — and escaped the notice of observers in the northern hemisphere until they were near

perihelion. They passed half-way around the sun in a few hours at a distance of 100,000 or 200,000 miles, moving at the velocity of nearly 400 miles per second. Their tails extended 100,000,000 miles in dazzling splendor.

One might think that the various members of a comet

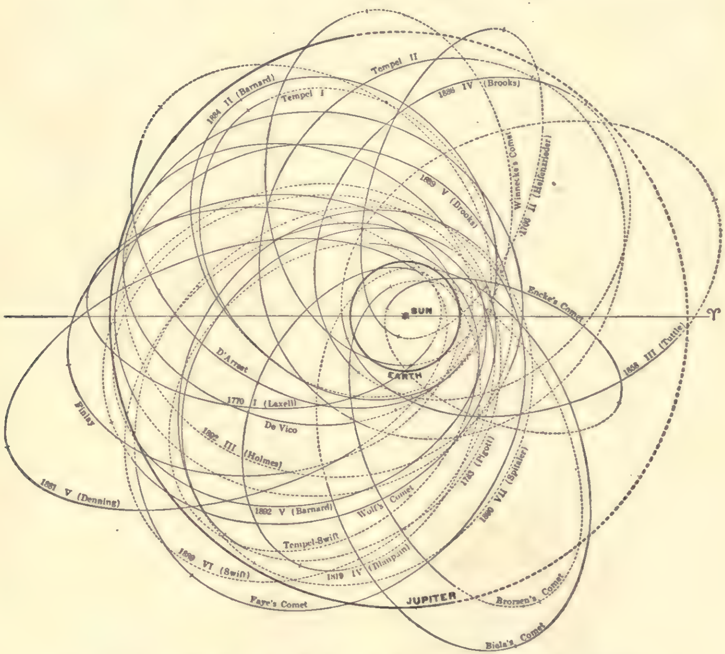


FIG. 141. — Jupiter's Family of Comets. From *Popular Astronomy*.

family are but the successive appearances of the same comet; but such is not the case, for the observations show that, though their orbits may be ellipses, their periods are 600 or 800 years. This means that they recede to something like five times the distance of Neptune from the sun. The most plausible theory seems to be that they are the separate parts of a great comet which in an earlier visit to

the sun was broken up by an encounter with the solar atmosphere, or by tidal disturbances.

Families of the second type are made up of comets whose orbits have their aphelion points and their nodes near the orbits of the planets. About 30 have their aphelia near Jupiter's orbit, and are known as Jupiter's family of comets. Their orbits are, of course, all elliptic, and their periods are from 3 to 8 years. They revolve around the sun in the same direction as the planets do. Half of them have been seen at two or more perihelion passages. These comets are all inconspicuous objects and entirely invisible to us except when they are near the earth.

Saturn has a comet family of 2, Uranus a family of 3, and Neptune a family of 6 members. The terrestrial planets do not possess comet families. There are some comets whose aphelia are about twice the distance of Neptune from the sun, suggesting, possibly, the existence of a planet at this distance (Art. 254).

296. The Capture Theory. — We are confronted with the following facts: A very great majority of comets move in sensibly parabolic orbits. Nearly all of the strongly elliptic orbits are near the plane of the planetary orbits, and have their aphelia near the planetary orbits. These facts suggest that the comets which move in elliptic orbits probably have had their orbits changed from parabolas to ellipses by the disturbing action of the planet with which they are associated. The matter has been worked out mathematically, first by Laplace and more recently by H. A. Newton, and it has been found that if a comet passes closely in front of a planet its motion will be retarded so that it will subsequently move in an elliptic orbit, at least until disturbed in the opposite direction. Then the comet is said to be "captured." It will, in the course of time, pass near the planet again when its orbit may be still further reduced, or it may be driven from the system on a parabola or hyperbola.

It is a generally accepted theory that the members of the comet families of the various planets have been captured. Jupiter has a larger family than any other planet because of its greater mass, and also because if a comet has been captured originally by any planet farther from the sun, it is likely to have its orbit still further reduced by Jupiter. On the other hand when Jupiter has once reduced the orbit of a comet so that it is a member of the Jupiter family, it is removed from danger of important perturbations by planets which are more remote from the sun.

The terrestrial planets have no comet families because their masses are small compared to that of the sun, and because comets cross their orbits with great speed.

The mass of Jupiter is not great enough to reduce a parabolic comet to a member of its own family at one disturbance. The theory has been illustrated by Brooks's comet, 1889–V, whose period, according to the investigations of Chandler, was reduced by Jupiter in 1886 from 27 years to 7 years. Lexell's comet of 1770 furnishes an example of a disturbance of the opposite character. In 1770 it was moving in an elliptical orbit with a period of $5\frac{1}{2}$ years, but in 1779 it approached near to Jupiter, its orbit was enlarged, and it has never been seen again.

297. The Origin of Comets. — The similarities of the motions of the planets point to a common origin, and the direction of the rotation of the sun indicates that they have been associated with it throughout their evolution. Reasoning on this basis the comets do not belong to the planetary family, and they probably have had quite a different origin. Since their orbits are normally sensibly parabolic, it follows that they have come to the sun from regions at least several times as remote as the planet Neptune.

One hypothesis is that comets are merely small wandering bodies which pass from star to star, visiting our sun but once. The intervals of time required for any such excursions are

enormously greater than one would imagine if he had not thought much about the great distances to the stars. For example, suppose the great comet of 1882 actually came from the star Sirius. This is one of the nearest stars, having a parallax of $0.38''$. Suppose the comet moved under the attraction of Sirius until it got half-way through its long journey, and that it then moved sensibly under the attraction of the sun. Let us find how long it would take it to make this second half of the distance. The result will not be much in error if we suppose that the comet came in on an elongated ellipse whose aphelion was half-way to Sirius. From the parallax given above, it follows that the aphelion point would be 270,000 times as far away as the sun is from the earth. From Kepler's harmonic law (Art. 138), it is found that it would take the comet 70,000,000 years to describe this part of its orbit. It would take about twice this time for it to come from Sirius, and eight times this immense interval to come from a star four times as far away; and but few stars are known to be within this distance.

The figures which have been given do not disprove the theory that the comets wander from star to star, but they simply show that if this hypothesis is true comets spend most of their time traveling and but little visiting. The same remarks would apply to another hypothesis, viz., that comets are material ejected by the stars in the violent convulsions which their enormous temperatures induce. It will be seen (Art. 336) that there are certain solar phenomena which lend some support to this explanation.

The strongest objection to these hypotheses is the fact that we are not yet certain that comets ever move in hyperbolic orbits. A comet would not pass around the sun in a parabola unless it had no motion relative to the sun when it was at a very (infinitely) great distance from the sun. Considering the fact that the sun is in motion with respect to the stars, and that they move in every possible direction,

it is entirely unreasonable to suppose that all the bodies coming from the interstellar spaces would start with sensibly zero relative velocities. It should be added, though, that but few of the hyperbolic comets would pass so near the sun that they could be observed from the earth.

A closely related hypothesis is that the parabolic comets have in reality very long elliptic orbits, and that they have all been ejected from the sun. The planetary families of comets are supposed to have been ejected similarly from their respective planets.

Another hypothesis is that comets are the products of the far-remote parts of an original solar nebula. This explains why they have such elongated orbits that they can not be distinguished from parabolas, and why hyperbolic orbits are not certainly found. According to this theory, as well as to that which supposes comets are ejections from the sun or planets, they should be composed of familiar elements. The spectroscope shows nearly always the presence of hydrocarbon compounds, and sometimes sodium and iron when the comet is near the sun. The fact which throws most suspicion on this theory is that the comets show no tendency to move near the plane of the planetary orbits. However, the test is not crucial and does not disprove the theory.

298. Theories of Comets' Tails. — As has been stated, comets' tails usually project directly away from the sun. This suggests a repellent action on the part of the sun. It was first suggested by Olbers as early as 1812 that the repulsion is electrical, due to different electrical conditions of the sun and comets. This theory has been developed in detail by Bredichin of Moscow, and the intensity of the repulsion has been computed in a number of cases from the photographs of the tail at different times. If the comet first repels the particles and then the sun repels them also, it can be shown that they will stream out away from the sun. Because of the motion of the comet the tail will be slightly curved in

the direction from which the comet has come. Now electrical repulsion is a surface action, while gravitation is a mass (volume for constant density) action. Consequently small molecules will be repelled, relatively to their attraction by the sun, much more strongly than large molecules, and the tails which they produce will be more nearly straight. Bredichin supposes the long, straight tails are due to hydrogen gas, the ordinary curved tails to hydrocarbon gases, and the short, stubby, and much curved tails, which are sometimes seen, to the vapor of metals. Sometimes a single comet will have several tails of different types.

If bodies are to repel each other they must be similarly electrified. When a body is subject to ultra-violet light, negatively electrified particles are given off. Moreover, at least the hydrogen in the sun's atmosphere seems to be negatively electrified. So we may propound the following plausible theory. Suppose a comet comes toward the sun from remote space without an electrical charge. The ultraviolet rays from the sun will drive off negatively charged particles which will be repelled by the negative charge on the sun and will form a tail. The repulsion will depend upon the size of the particles and the potential of the sun. The remainder of the comet will be positively charged and consequently will be electrically attracted by the sun. But, since the particles driven off will be only an exceedingly small part of the whole comet, this attraction will not be great enough sensibly to alter the comet's motion.

Another theory, which merits careful attention, has been urged by Arrhenius. According to Maxwell's electromagnetic theory, light exerts a pressure upon bodies upon which it falls proportional to the light energy in a unit of space. For bodies of any considerable magnitude the repulsion is very small, though it has been detected and measured by Nichols and Hull. But for minute bodies, as a ten-thousandth of an inch in diameter, the pressure may greatly

exceed the sun's gravitation. This, of course, depends upon the intensity of the radiation to which it is subject. And smaller bodies are repelled relatively still more until their diameters are as small as a wave length of light, say something like one fifty-thousandth of an inch. Then, as Schwarzschild has shown, the pressure becomes relatively, as well as actually, less.

Arrhenius supposes that the violent motions and collisions in the head of a comet break up the solid parts into minute

FIG. 142. — Photographs of 1893–IV (Brooks), taken by Barnard, November 2 and November 3. The photographs were superposed in making the cut, and show a change of 16° in the direction of the tail, while there was a change of only 1° in the direction of the comet from the sun.

particles which are driven away from the sun by light pressure. It is scarcely doubtful that this action takes place, but at present it is rather generally supposed that electrical repulsion also plays a part in producing the strange phenomena of comets' tails.

There are often sudden and great changes in the character and luminosity of comets' tails which no theory explains. For example, secondary tails sometimes develop making an angle of as much as 45° with the line from the sun. Stranger

still, Barnard found that the tail of Brooks's comet rotated through an angle of 16°, while the direction of the comet from the sun changed but a single degree.

299. Disintegration of Comets. — The particles that leave the head of the comet to form its tail are never gathered up again by the comet. In this way the material of which comets are composed becomes widely scattered.

The sun raises tides in the comets just as it does in the planets, but in comets they are much greater, for the mutual attractions of the various parts of these tenuous bodies are very feeble. The limit of tidal disintegration found by Roche is for a body revolving around its primary in a circle (Art. 281). The conditions are somewhat different for a body moving in a parabola, for the figure of equilibrium changes with the distance, but the results are of the same general character. The limits of tidal disintegration are very great for comets because of their extreme tenuity. Yet it is not to be supposed that comets will ordinarily be torn into widely separated fragments. When the particles have become separated so that they pursue essentially independent paths, they will follow nearly similar orbits, depending upon their initial positions and velocities. After a disintegrated comet has receded to a great distance from the sun, the mutual gravitation of the parts may assemble them again into a body whose density depends upon its constitution. It follows that tidal forces do not immediately widely scatter the material of which a comet is composed.

There is another factor which may be, in particular cases, of importance. The planets exert disrupting forces like that of the sun. These disturbances may conspire with those due to the sun and produce radical changes.

Evidently a comet would be destroyed by striking the sun, a planet, or a large satellite. There seems to be a large quantity of meteoric matter circulating around the sun, and collisions with this would also tend toward the disintegration of

comets. There is evidence that encounters of this sort occur occasionally, if not frequently. At least one comet of Jupiter's family (Encke's) had its motion retarded by some unseen agent at several appearances, after which, moving in a different orbit, it did not suffer so great a disturbance; one

FIG. 143. — Brooks's Comet, showing Broken Tail. *Photographed by Barnard, Oct. 22, 1893.*

comet (Biela's) was broken in two, or separated into two parts, which subsequently traveled independent paths; while more recently photography has shown that the tails of several comets have been suddenly broken up in the most startling manner.

In conclusion we shall be safe in believing that a comet may move around the sun in a parabolic orbit without being totally, or even largely, disintegrated; but that if a comet is captured and becomes a permanent member of the system,

it is only a question of time until the material of which the tail is composed will be entirely driven away, and until the remainder will be so scattered that it will be invisible, except possibly in the faint illumination which is characteristic of the blackest sky. As confirmatory of this view it may be noted that the members of Jupiter's family have small tails, or none at all; that this comet family does not contain many members; and that a number of comets have totally disappeared, presumably by disintegration.

300. Historical. — In this article some of those comets which have exhibited phenomena of unusual interest will be mentioned. The bearing of their peculiarities upon the more general discussions which have preceded will be at once evident.

Comet of 1680. — This comet is worthy of note as being the first one whose orbit was computed on gravitational principles. Newton made the computation and found that its period is about 600 years. It is also one of the family of comets mentioned in Art. 295. At its perihelion it was in the sun's corona and only 140,000 miles from its surface. It flew along this part of its orbit at the rate of 370 miles per second. Its tail was 100,000,000 miles long.

Comet of 1682 (*Halley's Comet*). — The orbit of this comet was computed by Newton's friend Halley. He found that it was almost identical with that of the comets of 1607 and 1531. He came to the conclusion that these various comets were but different appearances of the same one, whose period is about 75 years. The records of comets in 1456, 1301, 1145, and 1066 confirmed this view, for these dates differ from 1682 by nearly even multiples of 75 years. Halley predicted that the comet would next appear March 13, 1759. It came from beyond Neptune's orbit and passed its perihelion within a month of the time predicted. This was the first verification of such a prediction. The comet appeared again in 1835, when it passed within 5,000,000 miles of the earth.

It had at one time a short tail projecting toward the sun, a luminous sector which, according to Bessel, oscillated back and forth in a period of 4.6 days. On the 23d of January, 1760, it was without sensible disk, but the next day its light had increased twenty-fold, it had a disk like Neptune, and the nucleus was inclosed in a nebulous sheath. Its next return will be in 1910.

Comet of 1811. — This comet was visible from March 26, 1811, until August 17, 1812, and was carefully observed by William Herschel. He discovered from the changes in its brightness that it shone partly by its own light, which developed rapidly as it approached the sun. Its tail was at one time 100,000,000 miles long and 15,000,000 miles in diameter. This comet suggested to Olbers the electrical repulsion theory of comets' tails.

Encke's Comet (1819). — This was the first member of Jupiter's family of comets which was discovered, and it has the shortest known period, $3\frac{1}{3}$ years. It is an inconspicuous telescopic object, but is noted for the fact that its period was shortened, presumably by encountering some resistance, about $2\frac{1}{2}$ hours each revolution until 1868; since then the change in the period has been only one-half as great. Its change of volume has been extraordinary. On October 28, 1828, it was 135,000,000 miles from the sun and had a diameter of 312,000 miles; on December 24, its distance was 50,000,000 miles and its diameter was 14,000 miles; while at its perihelion passage on December 17, 1838, at a distance of 32,000,000 miles, its diameter was only 3000 miles.

Biela's Comet (1826). — This comet is also a small member of Jupiter's family and has a period of about 6.6 years. It appeared according to predictions in 1846. On the first of December it presented no unusual appearance, by the 20th it had become quite elongated, and by the first of January it had separated into two parts. They traveled along parallel orbits about 160,000 miles apart. At this time they were

undergoing considerable changes in brightness, usually alternately, and sometimes they were connected by a faint stream of light. At their appearance in 1852 the two components were 1,500,000 miles apart, and they have never been seen again, although searched for most carefully.

De Vico's Comet (1844). — This comet was a small member of Jupiter's family, describing its orbit in 6 years. It is worthy of especial notice only because it never has been seen at subsequent approaches to the sun.

Brorsen's Comet (1846). — This comet has a period of 5.5 years. It was observed at four returns after its discovery, but has since entirely escaped detection.

Donati's Comet (1858). — This was one of the greatest comets of the century. It was visible with the unaided eye 112 days, and through the telescope for more than 9 months. Its tail subtended an angle of 30° as seen from the earth, and was 54,000,000 miles long. Its period of revolution is more than 2000 years, and at its aphelion it is 5.5 times as far from the sun as Neptune is. Its motion is retrograde.

Tebbutt's Comet (1861). — This comet was of great dimensions, but is noteworthy chiefly because the earth passed through its tail. As could have been anticipated from the excessive tenuity of comets' tails, the earth experienced no sensible effects from the encounter.

Great Comets of 1880 *and* 1882. — These comets are two members of the most remarkable family of comets traveling in the same orbits. Both comets, as well as the earlier members of the same family, were remarkable for their vast dimensions, their brilliance, and their close approach to the sun. The comet of 1882 was observed both before and after perihelion passage. Although it swept through several hundred thousands of miles of the sun's corona, its orbit was not sensibly altered. Yet it gave evidence of having been subject to violent disrupting forces. After perihelion passage it

was observed to have as many as five nuclei, while Barnard and other observers saw in the immediate vicinity as many as six or eight small, comet-like masses, apparently broken from the large body, traveling in orbits parallel to it. They all speedily became invisible. After perihelion passage this comet had a projection toward the sun something like a second tail.

From 1882 to the present time no very brilliant comet has appeared. This interval is noteworthy for the number of periodic comets discovered, and for the successful application of photography to the study of their structure and changes. It has been found that comets often vary greatly in dimensions and in light-giving power in short intervals, that the tails generally present a complicated structure of long rays and local condensations which change with great rapidity, and that the breaking off of considerable portions of comets' heads is by no means of unusual occurrence. But in order that our knowledge of the chemical constitution of comets may be much advanced by spectroscopic means, we must await the approach of another great and bright comet.

301. Meteors. — On a clear, moonless night a "shooting star" can be seen darting across the sky every five or ten minutes. It is sometimes erroneously stated that stars have been seen to "shoot," but evidently the observer was mistaken. In 1798 Brandes and Benzenberg, at Göttingen, began to observe meteors at a distance of a few miles from each other. By getting the apparent positions of the beginning and end of the path of a meteor from both stations, and the time of its flight, they were able to compute its absolute positions with respect to the earth, and the velocity of its motion. They, and many later observers, have found that meteors appear at altitudes of 60 to 100 miles, and that they move over paths of 40 or 50 miles at a rate of from 10 to 40 miles per second (Art. 112).

Fig. 144. — Borelly's Comet. Cut on right is from a photograph by Barnard, and one on left from a photograph by Wallace three hours later. The changes in the tail and the motion of the comet with respect to the stars are shown.

The light given out by meteors is due to their being heated by friction with the atmosphere. If a meteor moving at the rate of 25 miles per second should take up half the heat generated by having its velocity destroyed, its temperature would be increased 97,000° Centigrade, provided its specific heat were unity. This explains why meteors are generally consumed before they reach the surface of the earth. The products of their oxidation and pulverization fall in the form of dust.

FIG. 145. — Meteor Trail and Brooks's Comet.
Photographed by Barnard, Nov. 13, 1893.

When the distance, duration of luminosity, and brightness of a meteor are known, the total amount of light radiated can be calculated. If all the energy were transformed into light, this calculation would give the total amount of kinetic energy possessed by the meteor. The kinetic energy is proportional to the mass of the meteor times the square of its velocity. Since the velocity is known, this relation gives a means of finding out something of the mass of a meteor. The uncertainty lies in our incomplete knowledge of the fraction of the whole energy which is transformed into light. But, making what seem to be safe assumptions, it turns out that the masses of meteors are only small fractions of an ounce.

302. Numbers of Meteors. — An observer can see half of

the sky at once, but he can not see half of the meteors which strike into the earth's atmosphere. The reason is that the atmosphere is relatively very thin, and meteors can be seen only when they encounter it near the observer. If the earth were represented by a sphere a foot in diameter, the thickness of the atmosphere on the same scale would be only one-ninth of an inch. Clearly only an exceedingly small part of it is within the range of a single observer.

From very many counts of the number of meteors that can be seen at a single place during a given time, it has been computed that between 10,000,000 and 20,000,000 strike into the earth's atmosphere daily. There are probably several times as many as this which are so small that they escape observation without telescopic aid.

Meteors enter the earth's atmosphere from every direction. Their places and velocities of encounter depend both upon their own velocities and also upon that of the earth around the sun. The side of the earth which is ahead encounters more meteors than any other, for it receives not only all those which it *meets*, but also those which it *overtakes*, while the part behind receives only those which *overtake* the earth. The meridian is on the forward side of the earth in the morning and on the rearward side in the evening. It is found that more meteors are seen in the morning than in the evening, and that their relative velocities are greater.

303. Meteoric Showers. — Occasionally unusual numbers of meteors are seen, and then it is said that there is a meteoric shower. There have been a few instances in which meteors were so numerous that they could not be counted, but usually not more than one or two appear in a minute.

At the time of a meteoric shower the meteors are not only more numerous than usual, but a majority of them move so that when their paths are projected backward, they pass through, or very near, a point in the sky. This point is called the *radiant point* of the shower, for the meteors all

appear to radiate from it. Those whose lines do not pass through the radiant point constitute the strays which are always appearing.

The most conspicuous meteoric showers occur on November 15 and November 24 yearly. The former have their

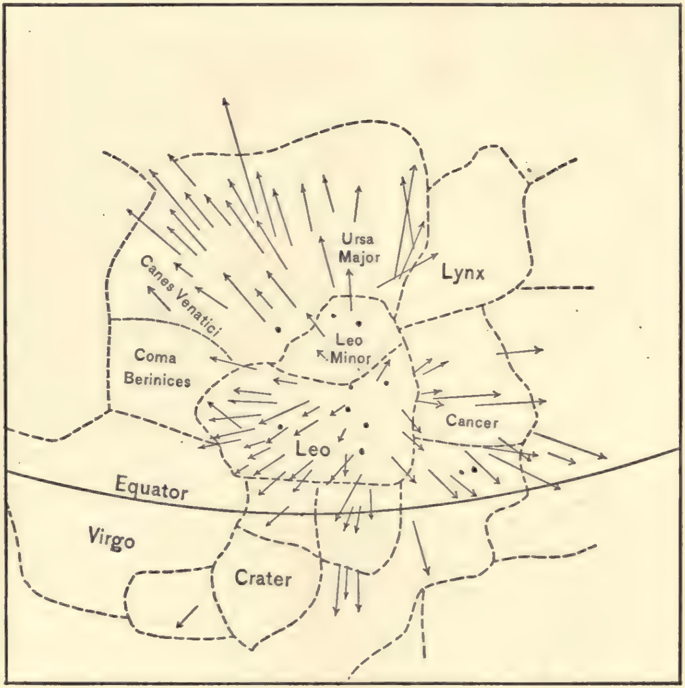


FIG. 146. — Leonids. *Charted by Upton at Providence, R.I., Nov. 15, 1901.*

radiant in Leo within the sickle, and are called the *Leonids.* From the position of this constellation (Art. 41) it follows that they can be seen only in the early morning hours. The latter have their radiant in Andromeda, and are called the *Andromids.* They can be seen only in the early part of the

night. The Leonids and Andromids are not equally numerous every year. Great showers of the Leonids occurred in 1833 and 1866, and less remarkable ones, though greater than the ordinary, in 1898 and 1901. The Andromids appear in unusual numbers every thirteen years.

Besides these meteoric showers, according to Denning, nearly 3000 other less conspicuous ones have been found. The Perseids appear for a week or more near the middle of August, the Lyrids on April 20, the Orionids on October 20, etc.

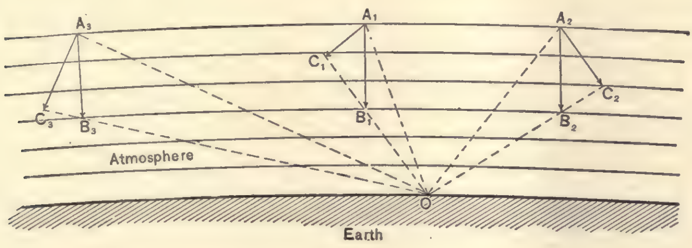


FIG. 147.

304. Explanation of the Radiant Point. — In 1834 Olmsted showed that the apparent radiation of meteors from a point is due to the fact that they move in parallel lines, and that we see only the projection of their motion on the celestial sphere. Thus, in Fig. 147, the actual paths of the meteors are $\overline{AB}$, but their apparent paths as seen by an observer at O are $\overline{AC}$. When these lines are all continued backward, they meet in the point which is in the direction from which the meteors come.

It follows that the meteors which give rise to the meteoric showers are moving in vast swarms along orbits which intersect the orbit of the earth. When the earth passes through the point of intersection, it encounters the meteors and a shower occurs. Thus, the orbit of the Leonids touches the

orbit of the earth at the point which the earth occupies on November 14. In this case the earth meets the meteors, while the Andromids overtake the earth.

305. The Orbits of Meteors. — It was shown in Art. 301 how the velocity of a meteor with respect to the earth's atmosphere may be found. This relative velocity is the resultant of the velocity of the meteor and that of the earth. Until a meteor gets within about 160,000 miles of the earth it is attracted by the sun more than by the earth; after this time the earth's attraction is predominant. The final velocity is the resultant of the initial velocity and the two attractions.

Let us assume provisionally that the meteors are moving around the sun in sensibly parabolic orbits, like the orbits of the comets, and let us find the greatest and least velocities with which they can encounter the earth's atmosphere. If it were not for the earth's attraction they would pass the earth's orbit at the rate of 25 miles per second, the velocity being independent of the angle at which they crossed. The earth's attraction would generate a velocity of nearly 7 miles per second in a body falling from an infinite distance into its atmosphere, whether the sun were attracting it or not. The greatest relative velocity will be when the earth and meteor meet, which is $25 + 7 + 18 = 50$ miles per second. The least will be when the meteor overtakes the earth, which is $25 + 7 - 18 = 14$ miles per second. If the meteors did not come from an infinite distance, these limits would both be lower, for 25 and 7 would both be replaced by smaller numbers. So far as the observations go, although they can not yet be regarded as being conclusive, the limits are so low that it is probable that meteors do not come from the distances of the stars, but that they are permanent members of the solar system.

It is definitely known that the meteors which give rise to the meteoric showers move in elliptic orbits. Their radiant

points give the direction of their motion, while their velocities, the periods from their maximum display to maximum display, and the perturbations their orbits undergo are sufficient to enable astronomers to discover the character of their orbits. The Leonids move in ellipses with a period of $33\frac{1}{4}$ years. Hence at their aphelion they are but a little beyond the orbit of Uranus. Since they are not scattered uniformly along their orbit, the times of maximum display occur every 33 or 34 years. The Andromids move around the sun in about $6\frac{1}{2}$ years.

306. Connection between Comets and Meteors. — In 1866 Schiaparelli showed that the August meteors move in the same orbit as Tuttle's comet of 1862. That is, the comet was but a bright member of the series of bodies traveling in its orbit. In 1867 Leverrier found that the Leonids move in the same orbit as Tempel's comet of 1866, while Weiss, of Vienna, showed that the meteors of April 20 and a comet of 1861 moved in the same orbit, and that the paths of the Andromids and Biela's Comet were likewise the same.

The relations of the orbits of meteors to those of comets point to the theory that a comet is captured by a planet, the material of which the tail is composed is driven off into space, and the remaining material is scattered along the orbit by the disintegrating forces to which it is subject. If the orbit intersects that of the earth, a meteoric shower occurs when the earth and the meteors arrive at the intersection point at the same time. The Leonids were shown by Leverrier to have been captured by Uranus in 126 A.D.

307. Effects of Meteors on the Solar System. — The most obvious effect of the numerous meteors which swarm in the solar system is a resistance both to the rotations and the revolutions of all the bodies. As was stated in Art. 126, the effects of meteors upon the rotation of the earth are at present exceedingly slight, and it is very probable that their influences upon the rotations of the other members of the sys-

tem are also inappreciable. A retardation in the translatory motion of a body causes its orbit to decrease in size. Hence, so far as the meteors affect the planets in this way, they cause them to approach the sun continually.

Another effect of meteors upon the members of the solar system is to make them all grow by the accretion of the matter which may have come originally from far beyond the known limits of the system. As the masses increase, their

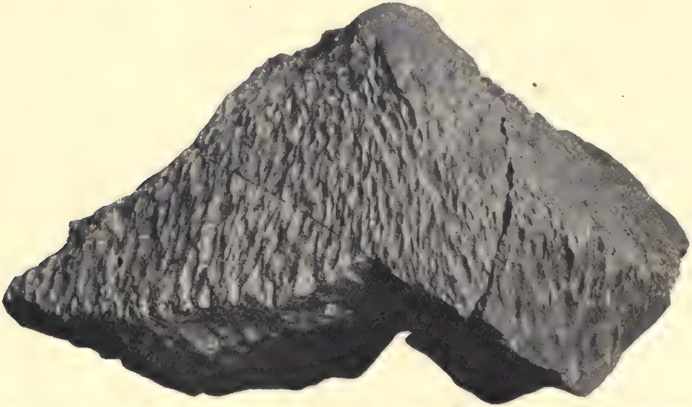


Fig. 148. — Meteorite which fell at Long Island, Kansas; weight 700 pounds. *Photographed by Farrington.*

mutual attractions increase and their orbits become smaller. Looking to the past, we are struck by the possibility that the accretion of meteoric matter may have been more rapid in former times, and that it may have been an important factor in the growth of the planets from much smaller bodies.

308. Meteorites. — Sometimes bodies weighing from a few pounds up to several hundred dash into the earth's atmosphere, glow brilliantly from the heat generated by the friction, roar like a waterfall, produce occasional violent detonations, and end by falling on the earth. Such bodies are called *meteorites*, *siderites*, or *aërolites*.

About two or three meteorites are seen to fall yearly; but, since so large a part of the earth is covered with water or is uninhabited for other reasons, it is probable that at least 100 strike the earth. The outside of a meteorite during its passage through the air is subject to intense and sudden heating, and the rapid expansion of its surface layers often breaks it into many fragments. The surface is fused and on striking cools rapidly. The result is that it has a black, glossy structure, usually with many small pits where the less refractive material has been melted out.

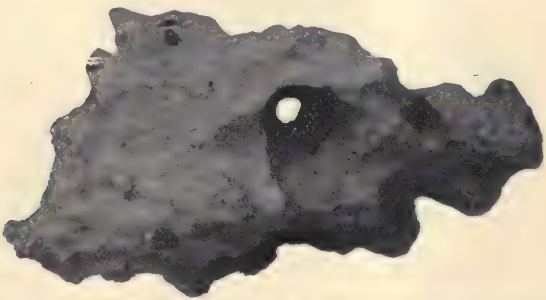


FIG. 149. — Iron Meteorite from Cañon Diablo, Arizona; weight 265 pounds. *Photographed by Farrington.*

Since meteors pass entirely through the atmosphere in a few seconds, only their surfaces give evidence of the extremes of heat and pressure to which they have been subjected in their final flight.

Most meteors are composed of stone, though often mixed with some metallic iron. Even where pure iron is not present some of its compounds are usually found. About three or four out of every hundred are nearly pure iron with a little nickel. All together about 30 elements which occur elsewhere on the earth have been found in meteorites, but no strange ones. Yet their structure is quite different in some respects from that of terrestrial substances. They have

peculiar crystals, they show but little oxidation, and no action of water, and they contain in their interstices relatively large quantities of occluded gases, some of which are combustible. According to Farrington, some give evidence of fragmentation and recementation, others show faulting (fracture and sliding of one surface on another), and others, veins where foreign material has been deposited.

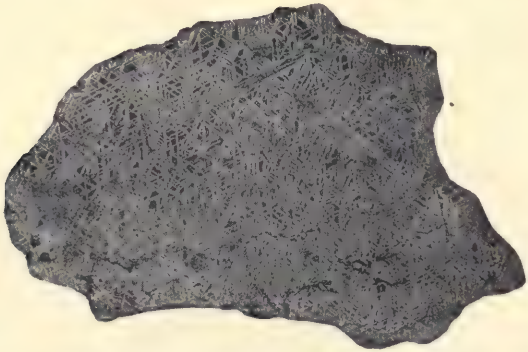


FIG. 150.—Durango, Mexico, Meteorite showing Peculiar Crystallization.
Photographed by Farrington.

309. Theories respecting Origin of Meteorites. — If it were known that meteorites were but meteors which are so large that they reach the earth before they are completely oxidized and pulverized, we might justly conclude that they are probably the remains of disintegrated comets. This would enable us to learn certain things about comets which can not be settled yet. But no meteorite is known certainly to have been a member of any meteoric swarm. However, two meteorites have fallen during the time of meteoric showers, one in France, at the time of the Lyrids in 1905, and the other in Mexico, just before the Andromids in 1885.

The structure of some meteorites is more like that of lava from deep volcanoes than anything else found on the earth.

One theory has been that they have been ejected by volcanic explosions from the moon, planets, or perhaps the sun. This theory would account for many of their characteristics, and would explain why they contain only familiar elements, at least if the other bodies of the solar system contain only those found on the earth. But it does not explain so well the fragmentation, faulting, and veins found by Farrington, for forces great enough to produce ejections would scarcely be found without heat enough to produce at least fusion.

Chamberlin has maintained that they may be the débris of bodies which have been broken up by tidal strains when they have passed within the Roche limit. When suns travel around other suns, it is probable that occasionally they pass so near each other that their planets (if they have any) are broken up. More rarely the suns themselves are disintegrated. Indeed, this may be the origin of all cometary and meteoric matter. Whether it is or not, there is a possibility of disintegration here which must be taken into account in any theory of cosmical evolution.

The present desiderata are more accurate determinations of comets' orbits to find whether any of them are really parabolic, more accurate determinations of the velocities of meteors to find whether they ever come into our system on parabolic or hyperbolic orbits, and finally the settling of the question whether meteors and meteorites are really related.

QUESTIONS

1. What sort of observations would be made to prove that comets are not in the atmosphere of the earth as the ancients supposed they were?

2. If a comet shone entirely by reflected light, how would its luminosity depend upon its distance from the sun and from the earth?

3. If a comet, forming an equilateral triangle with the earth and sun, gave one hundred times as much (reflected) light as a first-magnitude star, how much light would it give when it was in opposition at the distance of Jupiter?

4. Could the earth capture a comet and make it revolve around the earth like a satellite? Hint: At what rate would a comet pass the earth's orbit (Art. 165)? Compare this with the earth's velocity of escape (Art. 111).

5. Can you assign any reason why comets should contract when approaching the sun?

6. Why is not the material of which the tail of a comet is composed repelled so it never approaches the sun at all with the remainder of the comet?

7. On the repulsion theory, should a comet's tail be equally long when it is approaching the sun and when it is receding?

8. Make a list of the fairly well-explained cometary phenomena, and of those for which no satisfactory theory exists.

9. Make observations to determine how many meteors can be seen in an hour.

10. If possible, locate the radiant point of some meteoric shower.

11. Meteors often leave luminous trails which persist for a few minutes. If you are fortunate enough to see one, observe carefully whether the trail drifts with respect to the stars, and from the length of time of the motion and the angular change in direction compute (assuming a height of 50 miles) the actual velocity of the luminous trail. This will be very nearly the velocity of the atmosphere at this height. If the motion of the trail with respect to the stars is observed, will it be necessary to make a correction for the diurnal motion of the stars in getting the motion with respect to the earth's surface?

12. Estimate the intervals of time during which meteors are visible; assume that they are 50 miles away and moving at right angles to the line of sight; from these data compute the velocity of motion.

CHAPTER XIV

THE SUN

310. Problems of the Sun. — The sun must be considered from two points of view. In the first place, it is the dominant member of the solar system, governing the motions of the planets, illuminating and heating them with its bountiful rays, and it is inseparably associated with their evolution. For this reason it is an object of the highest importance in studying the solar system. In the second place, the sun is a star, and the only one of the millions revealed to us by our telescopes which is so near that it can be studied in its separate parts. Consequently, in approaching the broad problem of determining the conditions, constitution, and evolution of the stars, the sun would be the first object studied, and the one likely most amply to repay study. The problems arising in the two ways largely overlap.

Specifically, the problems respecting the sun are its dimensions, its mass, its motions, its radiating power, its temperature, the sources of its heat, its physical condition, its chemical constitution, and its evolution.

The dimensions and mass of the sun were given in the chapter on the solar system (Arts. 237, 240). The motion of the sun with respect to the stars will be treated in the last chapter.

THE SUN'S HEAT

311. Light and Heat received from the Sun. — It is a matter of universal observation that the sun furnishes the earth with an enormous amount of light. The brightest artificial

light ever used for illumination is dull in comparison with it. Even when the sun's light is largely cut off by dense clouds and enters a room only through windows, a lighted lamp compared with it pales into insignificance as a source of illumination.

Physicists have devised methods of measuring the relative amounts of light received from different sources. It is found that the sun sends to the earth 600,000 times as much light as is received from the full moon. Or, expressed in other units, the illumination due to the sun is over 60,000 times that of a standard candle at a distance of a yard.

Light is a wave motion in the ether, and waves of only a limited range of lengths affect the eye as light. Those which are too long to stimulate the optic nerve as light are heat waves, and those which are too short are the chemical rays. Clearly, in a discussion of the radiation of a body like the sun, all rays should be included whether they can be seen or not. The longest waves in the red which affect the ordinary eye as light are about $\frac{1}{39,000}$ of an inch in length, while the shortest ones visible in the violet are about $\frac{1}{62,000}$ of an inch in length. In the phraseology of acoustics, this is less than an octave, while the ear is sensitive to ten octaves of sound.

It is possible to measure quite accurately the amount of radiant energy which reaches the surface of the earth, but the atmosphere absorbs a portion which can be determined only indirectly. Perhaps the best estimates of this atmospheric absorption were made by Langley, who measured simultaneously the intensity of the radiation received from the sun at the summit of Mount Whitney, 14,887 feet above the sea, and at its base. He came to the conclusion that 40 per cent of the rays striking a clear atmosphere perpendicularly are absorbed before they reach the solid surface. Later determinations, recognizing the great absorbing power of the carbon dioxide in the atmosphere, have led to the con-

clusion that a still larger portion is absorbed. The absorption is greatest in the violet end of the spectrum, and least in the red. Langley found that the absorption is inversely as the wave length. The blue rays are largely absorbed when the sun is near the horizon, and this is the reason it looks red when setting, while it is white when near the meridian.

If all the radiant energy which the earth receives from the sun were transformed into mechanical energy, it would amount to *three horse power for every square yard exposed perpendicularly to the sun's rays.* Of course, only a small part of this energy can be used for mechanical purposes. Besides, its amount varies greatly with weather conditions, and always vanishes entirely at night. Consequently, with present appliances, the sun is not a very satisfactory direct source of mechanical energy ; still, in the cloudless skies of southern California, solar engines have been employed in pumping water for irrigation. But we must not forget that all of the mechanical energies which are available have been obtained indirectly from the sun.

In the evolution of organic existence on the earth the sun has been as important a factor as the earth itself. Consequently geologists and biologists have a deep, though indirect, interest in solar theories. The question of most immediate interest is whether the same amount of heat is always received from the sun under the same conditions of distance. It seems reasonable that there may be some slow change, but it has not been supposed until recently that there were any sensible short-period variations. Two years ago (1904) Langley and Abbott announced the startling result that they had found conclusive evidence that the radiant energy received from the sun sometimes varies by as much as 10 per cent in a few days. However, a short change of this amount produces no important climatic changes. If it were to persist indefinitely, the mean temperature of the earth would be changed by only 12° Fahrenheit.

312. The Energy radiated by the Sun. — The earth as seen from the sun subtends as small an angle as Mars does when viewed from the earth. Although this is an exceedingly small part of the whole sphere surrounding the sun, the heat received by the earth is very great; therefore the sun must radiate inconceivable amounts. It can be pictured to our understanding only by stating some of the things it might do.

The energy radiated per square yard from the sun is equivalent to 140,000 horse power. To generate this energy a layer of anthracite coal 25 feet thick would have to be consumed hourly. The heat radiated by the sun would melt a layer of ice 4000 feet thick every hour all over its surface. Only about $\frac{1}{100,000,000}$ of the vast flood of energy which the sun pours forth is intercepted by the planets, the rest traveling on through the ether to the regions of the stars at the rate of 186,000 miles per second.

313. The Temperature of the Sun. — Although the amount of radiant energy emitted by the sun is fairly accurately known, yet it is a difficult problem to find its temperature. The first difficulty arises from the fact that there is no single surface which alone radiates. There is an enormous highly heated atmosphere around the sun which gives out radiant energy and absorbs part of that which comes from a more highly heated interior. This absorption is strikingly shown by the fact that the sun's disk is brightest at the center, gradually darkening toward the edge, until at the limb it is only one-third as bright as it is in the center of the disk (see Fig. 153). Langley believed that if the sun's absorbing atmosphere were removed, it would radiate three or four times as much light and heat as at present, and that it would be decidedly blue.

Another difficulty arises from the fact that it is not known how radiation varies with the temperature, particularly where the temperatures are excessive, as they are in the sun.

Newton assumed that a body radiates directly proportional to its temperature. If this were so it would be an easy matter to find the temperature of a body whose rate of radiation is known. On the basis of this law it would follow that the sun's temperature is something like 4,000,000° Fahrenheit. But it is known, both from experiment and from theoretical considerations, that Stefan's law (Art. 270), which asserts that the radiation varies as the fourth power of the absolute temperature, represents quite accurately the truth for moderate temperatures. It follows from this law as a basis for computation that the temperature of the sun is about 10,000° Fahrenheit. Or, it would be more accurate to say that a radiating surface having this temperature would radiate at about the same rate the sun does. This temperature, which is several thousand degrees higher than has been obtained so far in the most powerful electrical furnace, is now generally regarded as being somewhere near the truth for those portions of the sun which are near enough the surface so that their radiations escape directly into space. Undoubtedly the interior is vastly hotter.

Another method, used first by Zöllner in 1870, and later by Hirn in 1884, has some good features, though on the whole involving many uncertain factors. It consists in inferring the differences of temperature from the violent observed motions which they produce. This method has led to exceedingly high temperatures, ranging from 50,000° to 180,000° Fahrenheit.

314. Rate of Cooling of the Sun. — A given amount of heat will not in general raise the temperature of equal masses of different substances the same amount. For example, it takes about five times as much heat to raise the temperature of a pound of water one degree as it does a pound of stone. And conversely, a pound of water of a given temperature contains about five times as much heat as a pound of stone at the same temperature.

If the sun had the same heat capacity as water (hydrogen is the only known substance which has a greater capacity), the heat which it radiates annually would lower its temperature about 4° Fahrenheit if it were not in some way replenished. Consequently, if the sun were simply a hot body cooling off, it could not maintain its present radiation more than about 3000 years. Direct historical evidence, to say nothing of the indirect inferences from geology and biology, shows that the sun has been radiating sensibly at its present rate for a much longer period than this.

315. The Combustion Theory of the Sun's Heat. — Experiments show how much heat is developed by the combustion of a given amount of coal. On the basis of these experiments, it follows that if the sun were pure coal and oxygen in such proportions that after their chemical union there would be no residue of either, the amount of heat generated would be sufficient to maintain the present radiation only about 1000 years.

No process of successive dissociation and subsequent burning would explain the enormous and long-continued radiation, for the same amount of heat would be used in separating the carbon and oxygen that would be given up when they united again. Consequently the combustion theory is entirely inadequate.

316. The Principle of the Conservation of Energy. — One of the cardinal principles of physical science for a long time has been that matter can not be destroyed. If the ashes, smoke, and gases from a body which had been burned were all gathered up, they would weigh as much as the original body plus the oxygen used in the combustion. But, until about 1840, it was generally supposed that energy might be destroyed as well as be transformed. For example, it was supposed that the energy lost by friction ceased to exist. However, it had long been known that friction generates heat, and at this time it was recognized that the heat pro-

duced might be equivalent to the mechanical energy lost. Many elaborate experiments, particularly by Joule, showed the correctness of this view and led to the generalization that *the total amount of energy in the universe is always the same.* Energy in this connection means the "potential energy" of position as well as the "kinetic energy" of motion. This is one of the most far-reaching principles of natural science, and, like the law of gravitation, it is concerned in every phenomenon involving the motion of matter.

317. The Meteoric Theory of the Sun's Heat. — Following the development of the theory of the conservation of energy, it was suggested, and for a few years believed, that the sun's heat is maintained by the energy contributed by the impact of the vast number of meteors which doubtless fall into the sun. This theory, while qualitatively correct, has been shown to be quantitatively unsatisfactory, for so much meteoric matter would be demanded, notwithstanding the enormous velocity of its impact, that it would produce perturbations in the motions of the planets, which observations show do not exist. And it can be proved that, under the hypothesis that the meteoric matter comes to the sun equally from all directions, the amount of heat received by the earth directly from the impact of meteors in its atmosphere would be $\frac{1}{231}$ of that received from the sun. Since this is millions of times as much heat as is received from the meteors, it follows that the theory must be abandoned as being entirely insufficient.

318. Helmholtz's Contraction Theory of the Sun's Heat. — Suppose the sun were to contract under the mutual gravitation of its parts. This would be equivalent to a slight fall toward its center of every one of its particles, and every one of the impacts of these particles would generate heat. The fall of a particle would take place continually, but it can be proved that the total amount of heat generated would be the same as if it fell the whole distance and gave up all its

energy of motion at one time. Therefore, heat would be continually generated by a continually contracting mass. Every one is familiar with the fact that a bicycle pump gets hot when it is used to force air into a tire. If the sun should contract it would get hot for the same reason, only in the case of the bicycle pump the energy comes from without instead of from the mutual gravitation of the masses involved. It is clearly not the friction of the piston with the cylinder which produces the heat, for if it is not used to force air into the tire no appreciable heat is generated.

In 1854 Helmholtz computed the heat generated by a contracting body in an attempt to explain the source of the sun's supply of heat. Under the assumption that the sun contracts so that it is always homogeneous, he found the astonishing result that a contraction of about 100 feet annually in the radius would account for all the heat radiated. More recent figures showing a greater rate of solar radiation demand a contraction annually of about 180 feet in the sun's radius. This amount is so small in comparison with the vast dimensions of the sun, that its accumulated effects could not be detected by present instrumental means until the lapse of more than 6000 years. The assumption of homogeneity is certainly false, but under the hypothesis of greater density at the center the contraction would be less for the generation of a given amount of heat. Therefore the shrinkage requisite for maintaining the present radiation of heat would not be so great as that given above.

In 1870, Lane, of Washington, proved that a body whose constitution is such that it obeys the laws of gases will actually contract under its own gravitation as it loses heat by radiation. Moreover, as the volume diminishes so that the gravitational forces become greater, the ability to withstand the expanding tendencies of high temperatures correspondingly increases, and the temperature will be greater than before. This is Lane's celebrated paradox, viz., that

a gaseous body, in a state of momentary equilibrium under its internal heat and gravitation, will grow hotter the more heat it radiates. But if the laws of gases should ever fail as a consequence of the interior parts becoming liquid or solid, Lane's law would no longer hold, and the temperature would fall. Lane's investigations have been improved and extended by Ritter, G. H. Darwin, and Hill.

319. The Past and Future of the Sun on the Basis of the Contraction Theory. — It is clear from the preceding discussion that the contraction theory is a satisfactory explanation of the maintenance of the sun's heat. The principal question remaining is whether there are not other important sources of heat besides this. If there were not, and if we knew the rate of radiation in past time, for example if it had always been the same as at present, it would be possible to compute a limit to the time the sun and planets could have existed with their present relations.

Computation shows that if the sun had contracted from infinite expansion, less than 20,000,000 times as much heat would have been generated as is now radiated annually. Consequently, if the contraction theory is the true explanation of the greater part of the sun's heat, the sun can not have radiated sensibly at its present rate for more than 20,000,000 years. But geologists and zoölogists believe they have evidence of a much longer evolution on the earth under conditions of not greater frigidity than exist at present. Some of these estimates demand 100,000,000 or 200,000,000 years, but the data are meager and difficult to use quantitatively.

According to the contraction theory the sun will continue to contract until it becomes so dense in its interior, if indeed it has not already reached this stage near its center, that it will cease to obey even approximately the laws of gases. Then its temperature will begin to fall, and it will finally end by becoming dark and cold like the moon. It is not

certainly known how long it will take for this evolution, but it is generally supposed that in 10,000,000 years not enough heat will be received from the sun to support life on the earth. The theory provides for no escape from this condition of perpetual frigidity except by the possible collision of the sun with some other body of large mass. The heat generated by the impact of two such bodies as the sun rushing together under the impulsion of their mutual gravitation would be sufficient to vaporize both of them.

Many positive statements about the possible past and future duration of the sun have been made, but recent discoveries, while not modifying the contraction theory, show that there are other sources of energy which are probably very important in the consideration of the question. The contraction theory takes into account only the energy developed by molecular motions in the process of contraction. The atomic motions involved in chemical reactions, as combustion, can not add a relatively important amount of energy. But in the last four or five years it has been found in connection with the study of the *cathode rays* and *radioactive* substances, that there are units of matter smaller than any atom, in fact, about one-thousandth the mass of the hydrogen atom, which is the smallest atom known. The internal energies of atoms are found to be incomparably greater than they possess in any motions with which they have ever been known to be endowed. Now it is possible that in some way this internal energy, especially in the dense interior of the sun where the atoms are closely crowded together, may be partially or wholly transformed into molecular motion which is manifested as heat. For example, under all laboratory conditions radium continually sends off these small portions of atoms, called *corpuscles*, and in this process of disintegration at least a million times as much energy is given up as by the combustion of any known substances of the same weight.

The conclusion is that the contraction theory gives a true explanation of the origin of a vast amount of solar heat. But it is possible, and indeed probable, that there are other sources of great importance. At present no positive statements can be made respecting the age of the sun, or the time during which it will continue to illuminate the planets with its beneficent beams.

QUESTIONS

1. How large is a body whose volume is to that of the earth as that of the earth is to that of the sun?

2. The illumination of an object by the sun is proportional directly to the sun's surface, its brightness, and inversely as the square of its distance. The same is true of illumination by any other source of light. Place an opaque body in sunlight and bring some artificial source so near that its shadow is illuminated by the artificial light as fully as where the full sunlight falls. From the known distances and dimensions of the sun and the artificial source compare their brightness.

3. Are the light and heat absorbed by the atmosphere effective in raising the temperature of the earth's surface?

4. Explain in detail how the various mechanical energies we use are derived indirectly from the sun; for example, wind power, water power, energy from combustion, and energy of animals.

5. What trouble is there with the theory that the sun's heat is maintained by the friction on each other of different currents which may traverse it?

6. The law of the conservation of energy is concerned in every phenomenon involving the motion of matter; are there any phenomena which do not involve the motion of matter?

7. What is the shrinkage in the sun's apparent angular diameter in 1000 years, if it shrinks 360 feet yearly?

8. If the sun's radius decreases 180 feet yearly, how much will the density of the sun increase in 1,000,000 years?

SPECTRUM ANALYSIS

320. Problems of Spectrum Analysis. — The pitch and quality of a sound depend upon the object which produces it. The tones of different pitch produced on a piano, for example, depend upon the length, tension, and weight of

the string which has been struck. The loudness of the tone depends upon the violence of the blow which produces it. When a trained ear hears a musical tone it can tell from its character on what instrument it is produced, from the pitch of the tone the nature of the part of the instrument producing it, and from its loudness the acuteness of the disturbance which has put the instrument in motion.

Light is a wave motion in the ether somewhat as sound is in the air. Spectrum analysis consists of a study of light waves with the purpose of finding the constitution, density, and the temperature of the body producing them. Considerable progress has been made in the solution of these problems, and the results which have been obtained are very important, for they pertain to bodies which, because of their remoteness, can not be investigated in these respects in any other way.

The pitch of a locomotive whistle is perceptibly higher when a train is approaching us than it is when the train is receding. The reason is that when the train is approaching the sound waves are slightly crowded together and shortened, and when it is receding they are slightly lengthened. If the true pitch of a locomotive whistle were known, and if the pitch as modified by the train's motion were accurately measured, it would be possible to compute the velocity of the train toward or from us. The results would be almost exactly the same if the whistle were stationary and the hearer were riding on a train. Similarly, motion of the source or recipient of light affects its apparent wave length, and if the change is known the relative velocity in the line joining them can be determined. Measurements of this sort now constitute an important part of spectrum analysis.

321. Nature of Light. — There have been two principal theories respecting the nature of light. According to the first, which was developed by Newton, light consists of minute corpuscles shot out in straight lines by the luminous

body. According to the second, which was developed by Thomas Young and many later physicists, light consists of a wave motion in an all-pervading substance called the *ether*. The wave theory has entirely superseded the corpuscular theory, for it explains several phenomena that the latter does not. For example, when two similar rays of light meet, they destroy each other where the phases of the waves are different. This phenomenon is inexplicable under the corpuscular theory.

Experiments show that the waves in the ether are at right angles to the line of their propagation, like the up and down waves which travel along a steel bar when it is struck, or the torsional waves when one of its ends is suddenly twisted. The waves in the ether are like those in a solid. The velocity of a wave in ordinary matter is proportional to the square root of its elasticity divided by its density. It follows from the high velocity of light that, if we may speak of the ether as having an elasticity and a density, the former must be great and the latter small. However, we are apt to build up false notions in applying to it the terms which have grown up in studying the physics of ponderable matter.

322. The Production of Light. — We are not perfectly certain just how light is produced, but the following theory gives a fair picture of matter and the way these ether waves may be produced. In the first place, matter is made up of molecules which are the smallest masses having the properties of physical bodies. In all such physical changes as subdividing, melting, evaporating, etc., the molecules are not broken up. But the molecules are made up of atoms which are the smallest units involved in chemical changes, and in chemical changes the molecules break up and re-combine into other kinds of molecules.

It has been mentioned that quite recently particles smaller than atoms, called *corpuscles* or *electrons*, have been shown to exist, and it has been found that they have about one thou-

sandth the inertia of a hydrogen atom. The evidence so far goes to show that the atoms of many, if not all, elements are made up of the same kinds of corpuscles. These corpuscles carry (or perhaps are) negative charges of electricity. According to present ideas, developed and elaborated particularly by J. J. Thomson, the corpuscles of which an atom is composed are in extremely rapid revolution. Their revolution and mutual repulsion (being electrified alike) tend to scatter them, but it is supposed that they are in the midst of a more powerful positive charge of electricity which attracts them. The corpuscles are something like the planets in the solar system with the sun representing the central positive charge, though the analogy is far from perfect. These corpuscles are in a condition of equilibrium whose stability is increased by their rapid revolutions, something as the equilibrium of a rider balanced on a bicycle is increased by his moving swiftly.

When an atom is not disturbed by others it gives no light. But if it strikes another atom the little corpuscles are slightly displaced from their standard orbits, and they oscillate rapidly around their regular positions until these motions are destroyed by friction with the ether. These small oscillations, which have definite periods depending uniquely on the structure of the atom, produce the light waves. In a rough way it is like a bell rotating around its axis. If it were not struck it would produce no sound, but if it were given a slight blow the particles of which it is composed would oscillate rapidly around their undisturbed positions and produce a sound. The pitch would depend upon the relations of the parts of the bell to one another, and the vibrations would continue until they were destroyed by friction with the atmosphere.

323. Light from an Incandescent Solid or Liquid. — In a solid body the molecules are so near together that their motions are restrained by their neighbors, and they always keep

the same positions with respect to one another. In a liquid body the molecules are also very close together and constantly interfere with the movements of one another, but they can move slowly around among one another. When the molecules jostle against one another the impacts produce waves, or pulses, in the ether, which constitute light. But the impacts also set the little corpuscles in rapid oscillation around their positions of equilibrium, and these vibrating corpuscles also produce light waves in the ether. To take the rough analogy of the bells, the solid or liquid body corresponds to a very great number of bells, in general, of different types, very near together. The heat of the solid or liquid body corresponds to rapid vibrations of the bells as wholes. Sound waves are produced in the air both by the mutual impacts of the bells, and also by the vibrations in the bells which the collisions set up.

The problem here is to find the character of the light emitted by a solid or liquid body, and it will be well to consider first the case of the bells. Suppose they are related to one another so that when they move they oscillate and collide many times a second, but quite irregularly as to frequency of impact. The result will be that the vibrations of the parts of bells, which, when undisturbed, have definite periods and hence produce tones of definite pitch, will be constantly subject to interruptions. Consequently similar sound waves succeed each other after all sorts of intervals, and the effect is equivalent to the simultaneous production of tones of every possible pitch. Similarly, in a solid or liquid body the collisions of the molecules are of the same order of frequency as the oscillations of their corpuscles around their positions of equilibrium. The result is that light waves succeed each other after all possible intervals, and all colors are produced. That is, a solid or liquid body emits all colors.

It is not true that all the colors are emitted equally. Suppose the body is heated only a little; the impacts of the molecules will not be violent enough to give the corpuscles

oscillations of much amplitude. The result is that they do not give out light waves of great enough intensity to be perceived by the eye. But if the body is heated more, the impacts will become more frequent and violent, and the body will emit a dull red glow. With increasing temperature the collisions will be still more frequent and violent, and the color of the light will be higher in the spectrum. The body will continue to emit waves of all frequencies, but the ones which occur in greatest number will be related to the temperature of the source. Consequently the character of the light which a solid or liquid body emits gives some information respecting its temperature. The hotter a body is, the farther toward the blue will be its maximum radiation.

324. Light from an Incandescent Gas. — In a gas the molecules do not sensibly influence one another except at the instants of collision. Although under conditions of atmospheric pressure and 32° Fahrenheit temperature, the molecules of oxygen collide on the average 5×10^9 times in a second, yet the time during which they are interfering with the motions of one another is very short compared to the whole time. Since light travels at the rate of 186,000 miles per second, while the light waves are about $\frac{1}{50,000}$ of an inch apart, it follows that a luminous body emits in round numbers 6×10^{14} light waves in a second. Consequently something like 10^5 light waves are emitted between collisions in a gas at atmospheric pressure and at the temperature of freezing water.

The character of the light given by a gas is quite different from that emitted by a solid or a liquid. The waves, except at the times of the collisions, are produced by the little corpuscles at regular intervals depending upon the structure of the atom. The frequency of the waves of a given type determines the color of the light. Now the corpuscles may be oscillating in several different ways in the same atom, and in this case the gas will emit several colors simultaneously.

Different kinds of atoms give forth light of different colors, but no two different kinds of atoms give the same color, just as no two different keys on a piano give tones of the same pitch. (It must be remembered that there is an infinite number of colors in the sense that the word "color" is used here, for there is an infinite number of wave lengths.) When the light given out by a gaseous mass is separated into its colors, the substances which emit it can be determined, just as the piano wires which have been struck can be determined from the pitch of the tones which have been produced.

A gas may be pictured in a rough way by a system of bells filling so large a space that they are far apart compared with their dimensions. The bells are moving rapidly and collide at intervals with such violence that they are set vibrating so as to give out sounds. The sounds produced by the impacts are relatively so infrequent as not to affect sensibly the general result. Then there are a number of tones of distinct pitch produced depending upon the number of kinds of bells.

325. Absorption of Light by a Gas. — Consider first a heavy pendulum of such length that it naturally makes complete oscillations in seconds. Suppose it is at rest and that it is struck on one side a very slight blow every *half second*. The first blow gives it a slight motion, and the second meets it on its return swing and totally destroys its motion. This process is repeated indefinitely and the pendulum is never made to vibrate sensibly. The results are similar if the blows occur at any other fractions of a second.

Now suppose the slight blows are struck once *every second*. The first gives the pendulum a very slight motion. At the end of the second the pendulum is again moving in the same direction and the second blow adds to its motion. This continues until the oscillation of the pendulum is so great that it no longer takes place exactly in seconds.

The same principles are illustrated in many ways. For example, rather small waves will make a ship roll badly if

their period is the same as that of the natural roll of the ship. Better still, if the key of a piano is held down and the corresponding one on another piano is struck and held down a few seconds, it will generate air waves which will give rise to a vibration of the wires in the first piano. When the key of the second piano is released so that the felt deadens its tone, the induced tone of the first piano can be heard. But this can not be done unless keys having the same pitch are used. Suppose some instrument produces simultaneously tones of every pitch, and that one key of a piano is held down. Its wire will be set in vibration by those waves having the frequency in which it naturally vibrates, and a large part of the energy of the wave will be used in overcoming the inertia of the wire and giving it motion. Therefore beyond the piano wire this particular tone is feebler than it was before. With a whole screen of similar wires the tone of the corresponding pitch would be largely absorbed.

Suppose white light (*i.e.* light made up of all colors) shines through a gas which is cooler than its source. The waves having the periods in which the corpuscles of the atoms naturally vibrate will give up their energy to these corpuscles, just as air waves give up their energy to piano wires whose natural period of vibration is the same as their period. The result will be that the white light will now lack certain colors, and the important thing to notice is that *they are precisely the ones which the gas would emit if it were luminous.*

326. The Spectroscope. — The spectroscope is an instrument for analyzing and studying the character of the radiant energy emitted by any source. Its essential parts are shown in outline in Fig. 151. A is the source of light, L_1 is a collecting lens used to bring the rays on the narrow slit o in the opaque screen S, and P is a prism on which the wedge of light passing through o falls. The colors are spread out by P, the violet being represented by full lines, and the red by broken

lines. L_2 is a lens used to bring the rays of different colors to respective foci on the line M, where they may be photographed or viewed with an eyepiece.

In another type of spectroscope, perfected by Rowland, a "grating" is used instead of a prism. A grating is a slightly concave piece of speculum metal ruled with from 12,000 to 20,000 parallel equidistant lines to the inch. When parallel rays fall upon a grating they are reflected back from the spaces between the lines, and by interference form a spectrum.

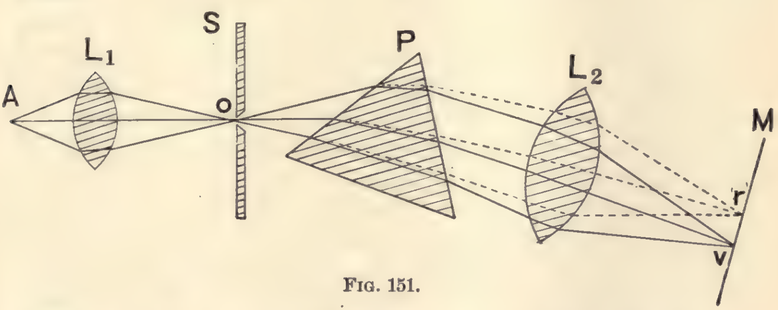


FIG. 151.

The faults of the prism spectroscope are that the colors are not refracted strictly in the inverse proportion to their wave lengths, and that many prisms must be used to get high dispersion. The grating spectroscope produces a spectrum through the interference of the light waves from the surface between the lines, and possesses neither of the faults of the prism spectroscope. Unfortunately the grating can make use of only a small part of the incident light. Consequently the prism spectroscope must be used for feeble sources of light such as the stars, nebulas, and comets, while the grating spectroscope may be used for the sun and laboratory sources. A third kind of spectroscope has been invented by Michelson, which gives the highly dispersed spectrum of the grating without the great loss of light. It consists of a pile of equally thick, accurately plane pieces of glass piled up like a stairway.

327. The Bolometer. — Those parts of the solar spectrum to which the variously prepared photographic plates are sensitive are mapped by photography. But these processes have a limited range of application, particularly in the infra red. In order to investigate this part of the spectrum, Langley invented the *bolometer*, which shows quickly excessively minute changes of temperature. The infra red spectrum is allowed to fall on a very narrow and thin piece of metal through which an electric current is passing. The resistance of the metal to the current depends upon the amount of heat which it receives, and the changes in the current are indicated and automatically recorded by a galvanometer.

Langley has used the bolometer, of course in connection with a spectroscope, with rare skill and perseverance. He has mapped below the red a spectrum twenty times as broad as the visible part.

328. The Principles of Spectrum Analysis. — The first theoretical discussion of the principles of spectrum analysis approximating to the truth was made by Ångström in 1853. The principles were given, substantially in their present form, by Kirchhoff in 1859. He established, on the basis of certain assumptions, the law that the ratio of the absorptive power to the emissive power of all bodies is the same for each kind of rays under the same conditions of temperature. Instead of following him in deriving the principles of spectrum analysis from this law, it will be simpler for us to attach them to the theories respecting the constitution of matter and the nature of light which have been described (Arts. 321–325).

Suppose the source of light is a solid or liquid; then, according to the principles which have been explained, light waves of all lengths will be given out. After the light passes through the slit and prism (considering the prism spectroscope) it will be refracted, depending upon its wave length. Of the visible light, the violet will be refracted the most and the red the least. Consequently there will be a band of colors

extending along the screen S_2 at right angles to the line of the slit. The maximum intensity of illumination depends upon the temperature of the source, being farther toward the violet the hotter the radiating body.

The use of the slit is now clear. For, suppose there were another one a little above the one represented in Fig. 151. It would admit to the screen S_2 another band of colors a little above the first one, so that the colors from one slit would fall on different colors from the other, and confusion would result. A third slit would make the confusion of colors worse, and so on. But a wide opening is equivalent to many slits actually touching, and if one were used the colors would be so mixed up that the light would be white except at the least and most refracted ends, where there would be respectively a red and a violet fringe of light.

If the source of light is a gas and gives out rays of but a few distinct wave lengths, the whole screen S_2 will be unilluminated except where these few colors fall. That is, the spectrum will consist of a series of bright lines. The number and position of these bright lines will depend upon the nature of the gas which emits them.

If the gas is subjected to continually increasing pressure, it will become constantly denser, and the light waves produced directly by the impacts of the molecules will become relatively more numerous. The periods of vibration of the little corpuscles will be changed more and more as the gas approaches the liquid state. The result on the spectrum will be that the lines will become broader and broader, and they will finally form a continuous spectrum.

Suppose, finally, that the light issues from an incandescent solid or liquid, and passes through a cooler gas before reaching the spectroscope. If it were not for the cooler gas interposed, the spectrum would be continuous, but this gas absorbs the light waves which have the same periods as the natural periods of its corpuscles. The result is that the spectrum

will be crossed by relatively dark lines, and these dark lines will be precisely where bright lines would appear if light were received only from the gas. In fact, the gas itself may be incandescent and give a bright line spectrum; but if the solid beyond it is much hotter, the gas will absorb so much light energy that the lines will appear dark compared to the intensity of the bright background.

For convenience the principles of spectrum analysis may be stated all together.

(1) *An incandescent solid or liquid (or a gas under very great pressure) gives a continuous spectrum whose position of maximum intensity is higher in the spectrum the greater the temperature of the source; and conversely, a continuous spectrum shows that the source of light is a solid or a liquid (or a gas under very great pressure).*

(2) *An incandescent gas under low pressure gives a bright line spectrum, the positions of whose lines depend upon the nature of the gas (and in some cases to some extent upon its temperature, density, and electrical condition); and conversely, a bright line spectrum shows that the source is an incandescent gas (or gases), and the positions of the lines show of what gas (or gases) it is composed.*

(3) *Light from an incandescent solid or liquid shining through a cooler gas (or gases) gives a dark line spectrum, the positions of whose lines depend upon the nature of the gas; and conversely, a dark line spectrum shows that the light has come from an incandescent solid or liquid through a cooler gas (or gases), and the positions of the lines determine the nature of the gas (or gases).*

329. The Doppler-Fizeau Principle. — In 1843 Doppler discussed the effects on the colors of double stars of the components of their motions in the line to the earth, and in 1848 Fizeau considered the effects of such motions on the positions of the spectral lines. The problems are essentially one, and the names of both the German and the French physicist are now attached to the principle.

Suppose, first, that the receiver of the light is at rest with respect to the ether and that the source is receding. At a certain instant its corpuscles give forth light waves toward the receiver, and a little later more similar waves. If the source were at rest, the waves would have a certain distance apart, depending upon their frequency and the velocity of light; but if the source is receding, the distances apart of those waves which go back to the receiver is increased. That is, the waves reach the receiver less frequently, and this results in a slight change of color toward the red end of the spectrum, or in a slight shifting of the positions of the spectral lines in the same direction. If the source were approaching the observer, the shift would be in the opposite direction.

If the source is stationary in the ether and the receiver is moving from or toward it, the results are very nearly respectively the same as when the source moves. When the receiver recedes from the source, the spectral lines are shifted toward the red end of the spectrum, and when it approaches, toward the violet end.

Let V represent the velocity of light, v_s the velocity of the source away from the receiver, v_r the velocity of the receiver away from the source, λ the original wave length, and λ' the observed wave length. Then the formula relating these quantities is[1]

$$\lambda' - \lambda = \lambda \frac{v_s + v_r}{V - v_r}.$$

The velocity of the receiver is small compared to the enormous velocity of light; consequently $V - v_r$ may be replaced by V without appreciable error. Let $v = v_s + v_r$, the relative rates at which the source and receiver recede from each other. This quantity v is called the *radial velocity*, and it follows from the formula above that is given by the equation

$$v = \frac{(\lambda' - \lambda)}{\lambda} V.$$

[1] Frost's translation of Scheiner's *Astronomical Spectroscopy*, p. 138.

This method can be actually applied to measuring the radial velocities of the stars. Certain lines, which are known from their general positions and relations, are selected. Their positions, unmodified by radial motion, are known from laboratory experiments; or better, a comparison spectrum can be obtained beside the star spectrum by sending the light from some known gas through one end of the slit at the same time. When the difference $\lambda' - \lambda$ is measured, v can be computed, for V and λ are known. If v comes out positive, the source and receiver are receding from each other; and if negative, they are approaching each other.

QUESTIONS

1. What problems are solved by spectrum analysis that can not be solved in any other known way?
2. Outline the theory of the nature of light and the manner of its production.
3. Why do incandescent solids and liquids give white light?
4. Why do incandescent gases give only particular colors?
5. Explain the absorption of light by a gas.
6. Draw a diagram of a spectroscope with two slits and show the confusion in the images which would result.
7. Trace out the changes in the spectrum of a gas as it is subjected to greater and greater pressure.
8. Draw a diagram illustrating the Doppler-Fizeau principle.

THE CONSTITUTION OF THE SUN

330. The Different Parts of the Sun. — The sun is an enormous body in a state of temperature and of gravitational pressure quite different from that of any material on the earth. The problems respecting its physical and chemical constitution and dynamics are of great difficulty, and can be investigated directly only in those parts which are accessible to observations.

The apparent surface of the sun is called the *photosphere* (light sphere). It is the part that gives forth most of the

light and heat which the sun radiates. It is the surface of the opaque part, and the dimensions of the sun refer to it as the boundary. The rotation of the photosphere gives us what is called the rotation of the sun.

Above the photosphere lies a sheet of gas, probably from 500 to 1000 miles thick, called the *reversing layer*. It contains many terrestrial elements, such as iron and calcium, in a vaporous state; still it is cooler than the underlying photosphere.

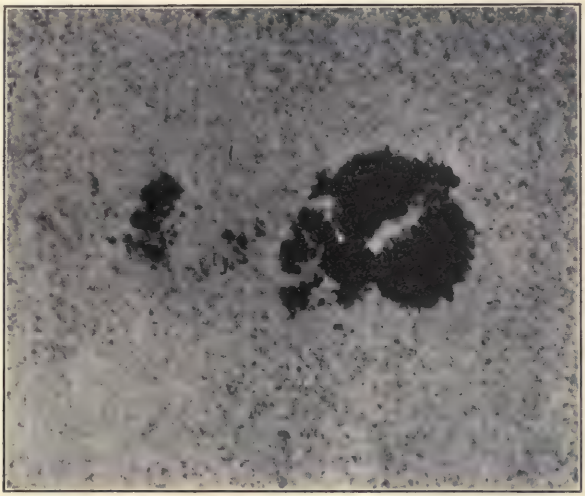


FIG. 152.—Great Spot of July 17, 1905, and Surrounding Region, showing the Structure of the Photosphere. *Photographed by Fox with the 40-inch refractor of the Yerkes Observatory.*

Outside of the photosphere is another layer of gas, from 5000 to 10,000 miles deep, called the *chromosphere* (color sphere). At the time of a total eclipse of the sun it is seen as a brilliant scarlet fringe whose outer surface seems to be covered with vast tongues of flame.

The outermost portion of the sun is the *corona* (crown). It is a halo of pearly light surrounding the sun, but it can not be seen, owing to the illumination of the earth's atmosphere, except at the time of a total eclipse. It is of irregular form and gradually fades out into the blackness of space at the distance of from 1,000,000 to 3,000,000 miles.

331. The Photosphere.—When the sun is examined through a good telescope it is seen to present a finely mottled appearance instead of the uniform luster which might be expected. The brighter parts are intensely luminous nodules, somewhat irregular in form, 500 or 600 miles across. These "rice grains," as they have been called, have been resolved into smaller elements having a diameter of not over 100 miles, and these small granules which all together do not constitute over one-fifth of the sun's surface radiate, according to Langley's estimates, about three-fourths of the light.

The photosphere of the sun gives a continuous spectrum and is, therefore, a solid or liquid, or more probably a gas filled with liquid particles. It can not be a solid or liquid such as we know on the earth, because of its enormous temperature. Besides this, there is direct telescopic evidence of continuous and rapid changes. The cloudlike nodules which produce the mottled surface appear and disappear with relative velocities often as great as 1000 miles an hour. The pressure can be inferred from the fact that the photosphere is buried under an immense atmosphere which is held down by 27 times the gravitation to which our atmosphere is subject. Still, it is probable that the photosphere is not simply a gas under great pressure, for it does not grade by insensible changes into the overlying gases. It has generally been supposed that it is the partially condensed vapors of the refractory metals, or carbon, somewhat as the clouds in the air are composed of minute drops of water. The chief objection to this theory is that all the evidence

available points to a temperature for the photosphere higher than that required to vaporize any known substance.

It is certain, however, that the photosphere is the surface (or region) which separates the intensely heated interior from the relatively cooler exterior; it is the place where the

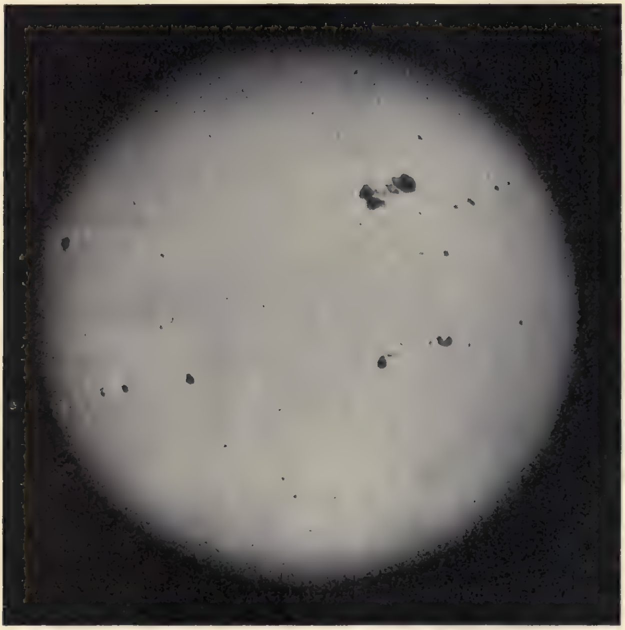


FIG. 153. — Photograph of the Sun taken at the Lick Observatory, showing how much brighter it is in the Center of the Disk than near the Margins.

sun loses heat by radiation. It must be a region of violent convective currents, for heat could not be *conducted* to the surface anywhere nearly so rapidly as it is radiated away. It is probable that the bright nodules are the summits of the ascending currents which, by expansion and cooling, get into

the (solid or liquid) state where they radiate most rapidly, while the darker spaces between are where the cooler currents descend.

332. The Dusky Veil. — The sun is much less luminous near its margins than in its center, as the telescope readily shows, and as comes out remarkably in photographs. Since the sun presents all sides to us as a consequence of its rotation, it follows that its light is absorbed by some medium. The absorption of light, which is estimated from the way in which the luster changes from the center to the limb, is very great, amounting, according to Vogel, to one-half of that which the photosphere radiates.

The dusky veil is a shallow absorbing layer, for the extinction of light increases very rapidly near the sun's edge, and elevations of the photosphere reach nearly through it and shine with intense brightness almost to the very limb. It is not merely a gas, for if it were it would absorb only particular rays depending upon its constitution, whereas it absorbs all rays, though the blue end of the spectrum the most. The result is that it renders the sun decidedly less blue in color than it would otherwise be.

It has been suggested by Hastings that the absorption is due to solid particles, that is, a sort of smoke floating in the atmosphere above the photosphere. But again the question of temperature is one of difficulty.

333. Sun-Spots. — The most conspicuous objects ever seen on the sun are relative dark spots which frequently appear in the photosphere and last from a few days up to several months. The average duration of a spot is a month or two. The typical sun spot consists of a round, relatively black nucleus called the *umbra*, and a surrounding less dark belt called the *penumbra*. The penumbra is made up of converging filaments, or "willow leaves," of brighter material, as though the intensely luminous photospheric columns were tipped over so that their sides could be seen. The umbra and

penumbra do not gradually merge into each other, and likewise the penumbra and surrounding photosphere have a fairly definite line of separation. The umbra has every appearance of being a deep, dark hole in the photosphere, and the penumbra seems to hang out over it like the untrimmed ends of a straw thatch. On the other hand, Frost's observations,

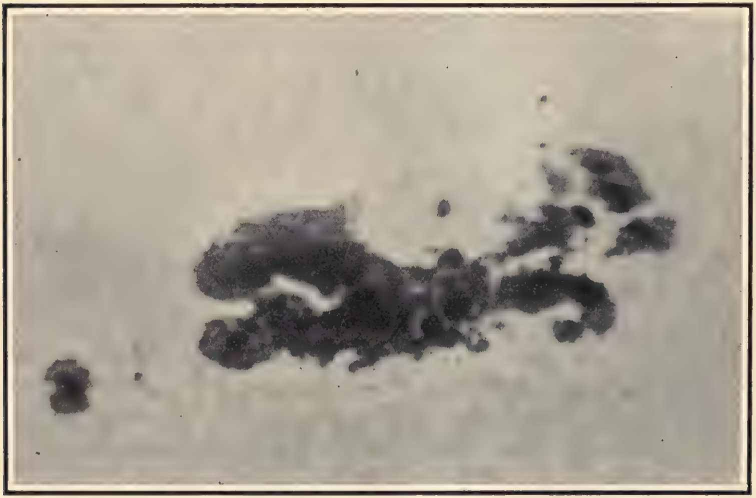


FIG. 154. — Large Sun-spot showing Umbra and Penumbra. *Photographed at the Yerkes Observatory.*

which show that the loss of light from spots as they approach the sun's limb is less than that of the ordinary photosphere, indicate that they are regions of unusual elevation. Probably spots are in general on plateau-like portions of the sun, and have relative depressions at their centers.

The umbra of a sun-spot may be anywhere from 500 to 50,000 miles across; the diameter of the penumbra may reach up to 200,000 miles. Often a single penumbra contains many umbras in its interior. The appearance of a spot

is usually preceded by brilliant points with intervening small dark points. The dark points increase in size, unite, and form a spot after a few hours, or perhaps days. The disturbed area generally becomes elongated in an east and west direction and several spots develop, the larger ones usually preceding the smaller. In fact, the larger spots rarely appear alone, and the disturbed area occupied by a sun-spot group may be as much as $\frac{1}{100}$ of the sun's whole surface. These great spots may be seen through an ordinary smoked glass, or with the unprotected eye when the sun is near the horizon. The Chinese claim to have records of observations of sun-spots made centuries before their discovery by Galileo in 1610.

The umbra of a sun-spot is dark only in comparison with the glowing photosphere which surrounds it; a calcium light projected on it would appear black. It radiates more heat per unit area than the glowing photosphere which surrounds it. In the neighborhood of spots the photosphere is usually more luminous than it averages, and there are nearly always in the vicinity very bright elevated portions called *faculæ* (Fig. 153). These faculæ are especially conspicuous when near the sun's limb, for in this region the photosphere is greatly dimmed by the extensive absorbing material through which its rays must pass, while on the other hand the faculæ project out through this absorbing material and shine with but slightly diminished luster.

An umbra is not uniformly dark, but under good conditions shows many details and often many small, very dark pits. When a spot is about to disappear the photosphere encroaches upon it, forms bridges of glowing material, and ends with the heaping up of photospheric clouds into faculæ.

334. The Rotation of the Sun. — The rotation of at least that part of the sun in which the spots occur can be found from their apparent transits across its disk. It did not take

observers long to find that the sun turns on its axis from west to east in about 25 days ; but observers using different spots arrived at results disagreeing by a day or two. About 50 years ago Carrington made a series of observations and measurements of positions of sun-spots covering a period of 8 years. From the great mass of data secured he found that the time it takes a spot to go around the sun depends upon its latitude, being longer the farther it is from the equator. This "equatorial acceleration" is similar to that occurring on Jupiter and Saturn, though the relative drift is less. The average period of the spots observed by Carrington was 25 da. 9 hr. 53 m. The observations of many astronomers show that spots near the equator revolve in about 25 days, those in latitude 30° in about 26.5 days, and those in latitude 45° in about 27 days. Spots are not seen in latitudes higher than 45°.

The rotation of the sun has been determined from observations of the faculæ, which certainly occupy a higher level than the spots which lie in the photosphere. The photographs of Bélopolsky and Stratonoff show that there is an equatorial acceleration of the faculæ, but that their periods of revolution seem to be a little less than those of the spots in corresponding solar latitudes.

The recent remarkable developments of spectroscopic methods have furnished still a third method of measuring the rotation of the sun. As a consequence of its motion, one limb at the sun's equator approaches us at the rate of about 1.3 miles per second, while the other recedes with the same velocity. Above the photosphere are cooler gases which absorb some of the sun's light and produce dark lines in its spectrum. From determinations of the radial velocities at the two limbs by means of the displacements of these lines, the rate of rotation can be found. Although precise quantitative results can not easily be obtained, yet they show an equatorial acceleration like that of the other methods, and

also the remarkable fact that the periods of revolution at corresponding latitudes are somewhat greater than those derived from the observation of spots.

The reason that the sun rotates in its peculiar manner is not at present known. There is nothing connected with its contraction or radiation which will explain its peculiar behavior. Under the hypothesis that the sun is a mixture of fluids in equilibrium, the work of Wilsing, Samson, and Wilczynski shows that, if it does not rotate as a solid, at least cylindrical portions of it do. That is, every particle at a given distance from the axis of rotation will have the same period. It was found also that the friction between the different layers will not wear down the differences of motion appreciably in millions of years. This theory assumes that in some more primitive state the outer zones had a faster revolution than the inner, from which it follows that there is still an equatorial acceleration. But the material of the sun must be mixed by violent convective currents, and it is not clear that these vertical currents might not rather speedily bring about uniformity of rotation. Moreover, according to this theory, the absorbing layer, which lies above the level of the spots, should rotate in a shorter period than the periods of the faculæ and spots in corresponding latitudes. But the spectroscope seems to show that the periods of the reversing layer are greater than those of the spots in corresponding latitudes.

Notwithstanding these difficulties, no other theory at present is so satisfactory as that the sun's peculiar rotation is the heritage of more extreme conditions which prevailed in the remote past.

The three methods give sensibly the same position for the solar equator. Its inclination to the plane of the ecliptic is, according to Carrington, 7° 15′, and the longitude of its ascending node is 74° 20′. The sun's axis is directed toward a point almost midway between Polaris and Vega ; it will be

remembered that the sun is moving nearly in the direction of Vega. About the first of June and December the earth is in the plane of the sun's equator, and spots appear to travel across the sun's disk in straight lines; from June to December the apparent paths curve downward, and from December to June upward.

335. The Distribution and Periodicity of Sun-spots. — The spots on the sun are distributed in a remarkable manner, and this distribution is related to their numbers, which vary periodically, as Schwabe first announced in 1852. When the spots are most numerous, and largest, they nearly all appear in two narrow zones whose solar latitudes are about $\pm 16°$. At these times there are nearly always from 5 to 15 groups, and from 15 to 100 individual spots (Fig. 153). After about 11.11 years, on the average, the spots again appear in large numbers in the same zones. But sometimes the interval between successive maxima is as short as seven years, and then the latter maximum is very pronounced, as if a fixed amount of sun-spot activity had been crowded into a shorter interval. Sometimes the interval between successive maxima is as great as 16.5 years, and then the latter maximum is less marked than ordinarily. There is a less strongly marked cycle whose period is $5 \times 11.1 = 55.5$ years, and one is suspected having a period of $4 \times 55.5 = 222$ years.

After a sun-spot maximum has passed, the spots appear year after year for about five years, on the average, in successively lower latitudes, and they are continually less numerous. At about the sixth year a few are still visible in latitudes $\pm 6°$, and a new cycle starts in about latitudes $\pm 35°$. After this the spots in the low latitudes disappear, the spots in the higher latitudes increase in numbers, and appear in lower and lower latitudes until the maximum activity is reached. The areas covered by spots in years of maximum activity are from 15 to 45 times those covered in years of minimum activity.

Since accurate records of the numbers and dimensions of all the sun-spots have been kept, the sun's southern hemisphere has been somewhat more active than the northern. For example, from 1874 to 1902 inclusive there were 20 different years in which the total area covered by spots in the southern hemisphere was greater than that in the north-

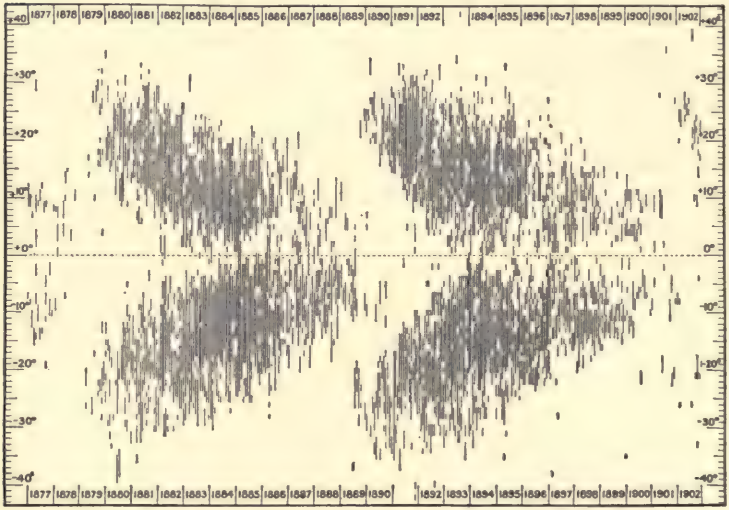


FIG. 155. — Maunder's Figure showing the Distribution of Sun-spots in Latitude and their Relative Dimensions from 1877 to 1902. The positions of the lines show the dates and latitudes of the spots, while their lengths are proportional to the areas covered.

ern, while the opposite was true in only nine years. For this entire period 57 per cent of the total spot area was in the southern hemisphere and 43 per cent in the northern. That is, the activity in the southern hemisphere was about one-third greater than that in the northern. Whether this difference is permanent or what it means can not at present be determined.

336. The Proper Motions of Sun-spots and their Internal Motions. — Individual spots drift both in latitude and longitude, and often have complicated and violent internal motions. As a rule, those spots whose latitudes are less than 20° drift slowly toward the equator, and those which are in higher latitudes away from it. There are frequent exceptions to these statements, and the motion of any particular spot may be irregular.

Spots generally have motions in longitude somewhat different from the average of those which appear in their latitude. In a large group the spot which is ahead usually has a proper motion forward, while the one which is behind lags continually farther in the rear. If a large spot divides, its two components recede from each other, sometimes at the rate of a thousand miles an hour.

Sun-spots sometimes have spiral motions, but the phenomenon can scarcely be said to be characteristic, for less than 3 per cent of them show it. Moreover, the whirling may be confined to a portion of the spot; it may suddenly develop, stop, and even start in the opposite direction. Sun-spots seem to have no analogy with terrestrial cyclones.

As has been stated, the sun must be agitated near its surface by vertical convection currents. In the neighborhood of spots these vertical motions are often very violent. The spectroscope shows that sometimes masses of incandescent gas rise from spots or descend into them with velocities as great as 300 miles per second.

337. The Reversing Layer. — In 1802 Wollaston studied sunlight by passing it through a narrow slit instead of a pinhole as Newton had done. He found that the solar spectrum is crossed by 7 dark lines. In a few years the work was taken up by Fraunhofer, who soon found that the spectrum is covered by an immense number of dark lines. He mapped 324 of them in 1815, and they have since been known as "Fraunhofer lines." A greatly improved map of

these lines was made by Kirchhoff in 1861–1862, and still another by Ångström in 1868. In 1886 Langley mapped the spectrum with the aid of his bolometer far into the infra red region, and in 1886, 1889, and 1893 Rowland published exten-

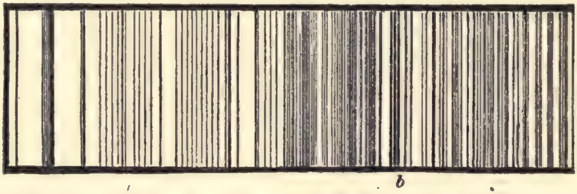


Fig. 156. — A Portion of the Solar Spectrum as shown by a Spectroscope of Moderate Power.

sive and very accurate maps from measurements of the spectrum obtained with his powerful grating spectroscope.

The spectrum of the sun is continuous except for the very numerous dark lines which cross it. Therefore, in accord-

Fig. 157. — The *b*-Group of Lines as shown by a Powerful Grating Spectroscope.

ance with the principles of spectrum analysis, the photosphere of the sun is liquid or a gas under great pressure, and a cooler gas intervenes between it and us. The positions of the lines show that they are due to many heavy metals, and since they must be in a vaporous condition

to produce this absorption, it follows that they are in the solar atmosphere instead of in our own. This absorbing material is the *reversing layer*.

If the reversing layer could be viewed not projected on the brilliant photosphere, it would give a bright line spectrum, the bright lines appearing exactly at the places ordinarily occupied by the (relatively) dark lines. At the total eclipse of the sun in 1870, Young placed the slit of his spectroscope tangent to the limb of the sun ; just as the moon cut off the last of the photosphere the spectrum suddenly flashed out in bright lines where the dark ones had previously

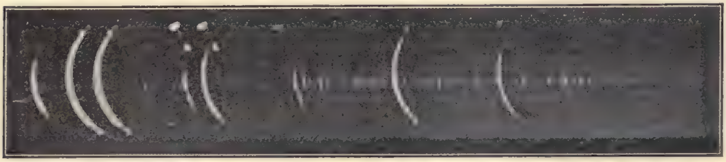


FIG. 158.—The Flash Spectrum. *Photographed by Frost at the second contact of the eclipse of May 28, 1900.*

appeared. Since 1895 the bright lines of the reversing layer have been frequently photographed, showing the identity of their positions with the Fraunhofer lines. From the duration of their appearance as bright lines and the known rate at which the moon apparently passes across the disk of the sun, it is found that the reversing layer is some 500 or 600 miles deep. Probably the dusky veil is below and mixed with the reversing layer, though their absorption differs greatly in an important respect. The continuous absorption of the dusky veil is many fold greater near the sun's limb than near its center, while the selective absorption of the reversing layer is very nearly uniform over the whole surface.

The spectroscope shows that the reversing layer contains the most refractory known materials in a vaporous state, yet

at present it is assumed that the photosphere is both considerably hotter and in a partially condensed condition. Another peculiarity is that the Fraunhofer lines show that the reversing layer is in a fairly quiescent state, while the photosphere below it is in a condition of violent agitation, and the chromosphere above is the seat of enormous eruptions. If the reversing layer is an atmosphere, it follows from the great gravitative power of the sun for matter at its surface that it should be millions of times denser in its lower parts than in the higher regions. But the absorption lines indicate that the difference in density is comparatively slight.

338. Chemical Constitution of the Reversing Layer. — Shortly before his death, Rowland was engaged in making a new, though "preliminary," map of the solar spectrum. In this map, which he left unfinished, the positions of nearly 20,000 lines were given. Something like one-third of them are due to the absorption by our atmosphere, and the remainder to the reversing layer. By comparing their positions with the positions of those which laboratory experiments show the various elements give, it is possible to infer the chemical constitution of the material producing the absorption. In this manner 39 terrestrial elements have been shown to exist in the sun. The elements which have been found and their atomic weight are given in the table on the following page.

The presence of iron is established by more than 2500 line coincidences, calcium by 75, sodium by 13, while lead and potassium have but one line coincidence each. It will be noticed that nearly all the elements in the list are metals, the exceptions being hydrogen, carbon, and oxygen. On the other hand, a number of metals, as gold and mercury, are missing. Likewise such non-metals as nitrogen, chlorine, sulphur, and boron do not appear, although they are found in abundance on the earth.

Element	Atomic Weight	Element	Atomic Weight
Hydrogen	1	Zinc	65
Helium	4	Zirconium	65
Glucinum	9	Gallium	70
Carbon	12	Germanium	72
Oxygen	16	Strontium	87
Sodium	23	Yttrium	89
Magnesium	24	Niobium	94
Aluminium	27	Molybdenum	96
Silicon	32	Rhodium	103
Potassium	39	Palladium	106
Calcium	40	Silver	108
Scandium	44	Cadmium	112
Titanium	48	Tin	117
Vanadium	51	Barium	137
Chromium	52	Lanthanum	139
Manganese	55	Cerium	140
Iron	56	Neodymium	140
Nickel	58	Erbium	166
Cobalt	59	Lead	207
Copper	63		

While the presence of the spectral lines of an element proves its existence, their absence does not show that it does not exist. In the first place, such heavy metals as gold and mercury would have a strong tendency to sink below the level of the reversing layer. Then, again, the characteristic spectra of many elements, particularly non-metals, are suppressed by the presence of certain other elements, particularly metals. Sometimes a very small percentage of the suppressing agent is sufficient to obliterate the spectrum of another substance. Some elements have spectra which change radically under different conditions of temperature, pressure, and electrical excitation. One of these is oxygen, whose presence in the sun was not firmly established until 1897, although it was diligently sought for by many observers. Finally, as

Lockyer has suggested, many of the so-called elements may be in reality compounds which are broken up under the extreme conditions prevailing in the sun, and in this manner their characteristic spectra destroyed.

The reversing layer is undoubtedly constantly receiving material from below and above, so it is safe to say that its composition is not qualitatively different from that of the remainder of the sun. It is interesting that so many terrestrial elements have been found in the sun, for it shows that the earth and sun probably have had a common origin. The fact that about 40 terrestrial elements have not yet been found may be accounted for in several ways besides assuming that they do not exist in the sun. There are 12,000 Fraunhofer lines which have been measured but not identified, and also a vast number of faint ones which have not been measured. There also remains an enormous amount of laboratory work to be done on the spectra of the elements, particularly under extreme conditions of temperature and pressure.

339. The Chromosphere. — Above the reversing layer lies the chromosphere, a gaseous envelope 5000 to 10,000 miles in depth. At the time of a total eclipse it can be seen as a brilliant scarlet ring surrounding the sun, and its surface seems to be seething with tongues of leaping flames.

The spectrum of the chromosphere is made up of bright lines, some of which are permanent while others come and go. The permanent lines are mostly due to hydrogen, helium, and calcium. The intermittent lines are mostly bright "reversals" of the dark Fraunhofer lines (see Art. 341). They are due to many elements which, while highly heated, have been thrown up through the reversing layer. The color of the chromosphere is largely due to a red hydrogen line. There is always a bright yellow line near those given by sodium, which was ascribed to an element, unknown on the earth until 1895, called *helium* (from *helios* = sun), because it was found only in the sun. In March, 1895, on

examining the spectrum of the mineral clevite, Ramsay found this helium line. It was then a problem of chemistry to separate this hitherto unknown element and to find its properties. Helium was found to have, next to hydrogen, the lowest atomic weight of any known element. It is very inactive, entering into no known chemical combinations with other elements. It has the lowest refractive index of any known substance, and is an excellent conductor of electricity. Its rate of diffusion is 15 times its theoretical value, while its solubility in water is almost zero. It is the only gas that can be obtained in considerable quantities which has not been liquefied. When the element radium disintegrates, helium is one of the products.

It is a remarkable fact that calcium, which in the gaseous state is 40 times as heavy as hydrogen under similar conditions, is so widely diffused throughout the tenuous chromosphere. It is at least coextensive with hydrogen and helium. It seems that for some reason gravity is almost entirely balanced by opposite tendencies in these regions. This conclusion is supported by the fact that the enormous chromosphere has, like the reversing layer, nearly the same density at all levels.

A further remarkable fact is that helium, which is universally present in the chromosphere, gives no dark Fraunhofer lines. This seems to be a direct contradiction of Kirchhoff's law which holds true in other known cases.

340. Prominences. — In sharp contrast with the quiescent condition of the reversing layer, the chromosphere is in a state of violent commotion. Vast eruptions, called *prominences*, rise up from it to altitudes of from 50,000 to 300,000 miles, with velocities sometimes as great as 500 or 600 miles per second. If the materials of which the prominences are composed did not encounter some resistance, their enormous velocities would carry them away from the sun never to return (see Art. 111).

Prominences were formerly visible only at the times of total eclipses of the sun, for the illumination of the earth's atmosphere extinguishes them at other times. But if the light from the limb of the sun is passed through a spectroscope, the prominences can be seen at any time. The spectroscope spreads out and correspondingly enfeebles the continuous light of the atmospheric illumination. The light of the prominences is made up of bright lines whose intensity, therefore, is not diminished by the spectroscope. Consequently the atmospheric illumination may be reduced so that the prominences are visible.

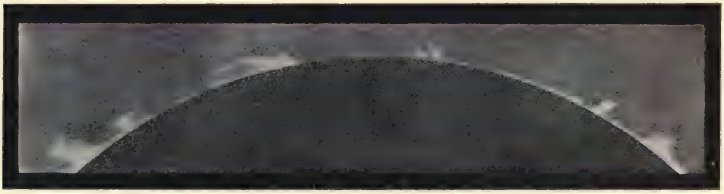


FIG. 159. — Solar Prominences. *Photographed by Fox at the Yerkes Observatory.*

The velocities in prominences are determined both from apparent motions perpendicular to the line of sight, and also from line displacements in accordance with the Doppler-Fizeau principle. In any particular case the actual motion is the resultant of the two components, which are generally of the same order of magnitude. It must be admitted that the velocities found are suspiciously large for the actual motion of material, but no other satisfactory explanation of the phenomena has been suggested.

Not all the prominences are eruptive. Besides those which burst out suddenly, rising to great heights and quickly subsiding, there are others called quiescent prominences which spread out, like the tops of banyan trees, with here and there a stem reaching to the denser regions below. Curiously they sometimes seem to develop at great heights,

as though material already in those regions had for some reason suddenly become visible.

341. The Spectroheliograph. — The photosphere radiates a continuous spectrum, while above it is the reversing layer which produces the dark absorption lines. Some of these lines, as the *K*-line due to calcium, are broad because of the great extent of the absorbing layer. Now calcium is abundant in the prominences, and moreover it shines with an intensity greater than that of the reversing layer. The result is that the reversing layer makes a broad dark line, say the

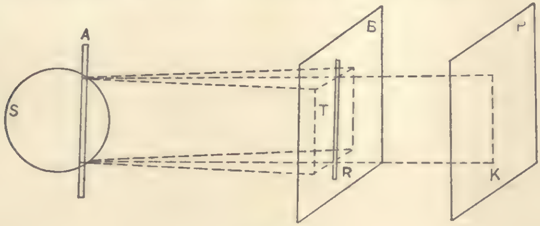


FIG. 160.

K-line, and above it is more luminous calcium in a rarer state which produces a narrow bright line in the midst of the dark one. The line is said to be "doubly reversed."

The spectroheliograph is an instrument invented and perfected by Hale in 1891 for the purpose of photographing the sun with the light from a single element. The ideas upon which it depends were almost simultaneously developed and applied by Deslandres. In this instrument, or combination of instruments, the sunlight is passed through a spectroscope and is spread out into a spectrum. The *K*-line, which is most frequently used, is doubly reversed in the regions of faculæ and prominences. All the spectrum is cut off by an opaque screen except the bright part of the *K*-line, which passes through a second narrow slit. That is, the only light which passes through both slits is the calcium light from that

portion of the sun's image which falls on the first slit of the spectroscope.

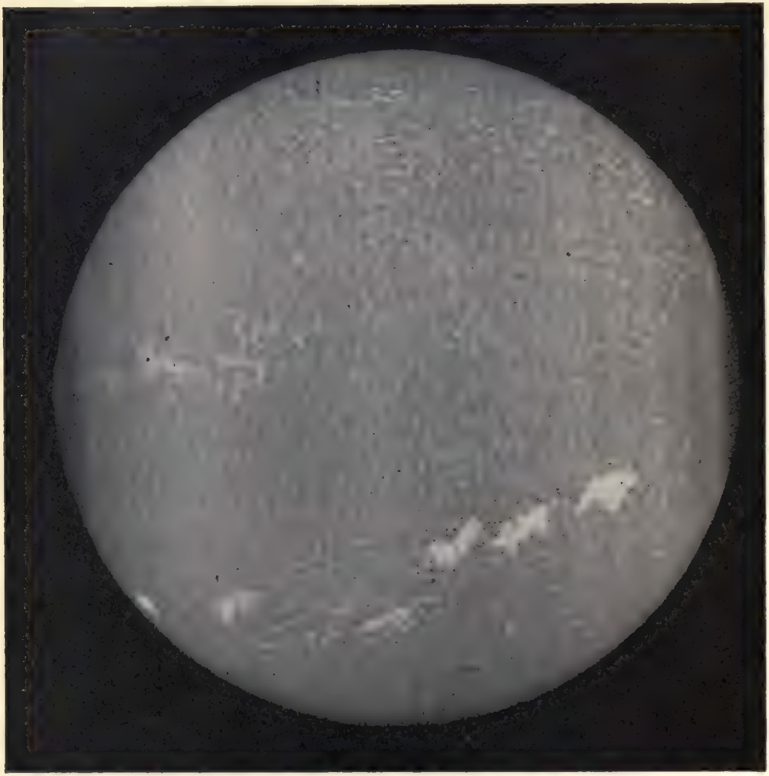


Fig. 161. — Spectroheliogram of the Sun by Hale and Ellerman. The doubly reversed calcium line was used.

In Fig. 160, S is the image of the sun at the focal plane of the telescope, A is the slit of the spectroscope (the prisms are not shown), T is the spectrum which falls on the screen B, R is a slit in the screen B which is adjusted so that it admits the bright center of the doubly reversed K-line, and

P is a photographic plate on which the K-line falls. The apparatus is made so that the slit A may be moved across the image of the sun S, and the slit R simultaneously moved so that the K-line falls on successively different parts of the photographic plate P. In this manner a photograph of the hot calcium vapors which lie above the reversing layer may be obtained. Some other lines have also been used in this way.

The width of a line depends upon the density of the gas which emits it. Suppose a thick layer of calcium gas which is rare at the top and denser at the bottom gives a bright K-line. The central part will be due to light coming from all depths, particularly from the higher layers where the absorption is unimportant. On the other hand, the marginal parts of the line will be due to light coming from the lower levels where the gas is denser. Following out these principles, and using a very narrow slit, Hale first obtained photographs of different levels of the solar atmosphere.

342. The Corona. — During total eclipses the sun is seen to be surrounded by a halo of pearly light, called the *corona*, extending out 200,000 or 300,000 miles, while some of the streamers reach out at least 5,000,000 miles. So far it has not been possible to find any observational evidence of the corona except at the times of total eclipses of the sun. One of the reasons that eclipses are of great scientific interest is that they afford an opportunity of studying this remarkable solar appendage. The brief duration of total eclipses and their infrequency have made progress in the researches on the corona rather slow.

The corona is not arranged in concentric layers like an atmosphere, but is made up of complicated systems of streamers, in general stretching out radially from the sun, but often simply and doubly curved, and somewhat resembling auroras. Many observers have declared that its finely detailed structure resembles the Orion nebula.

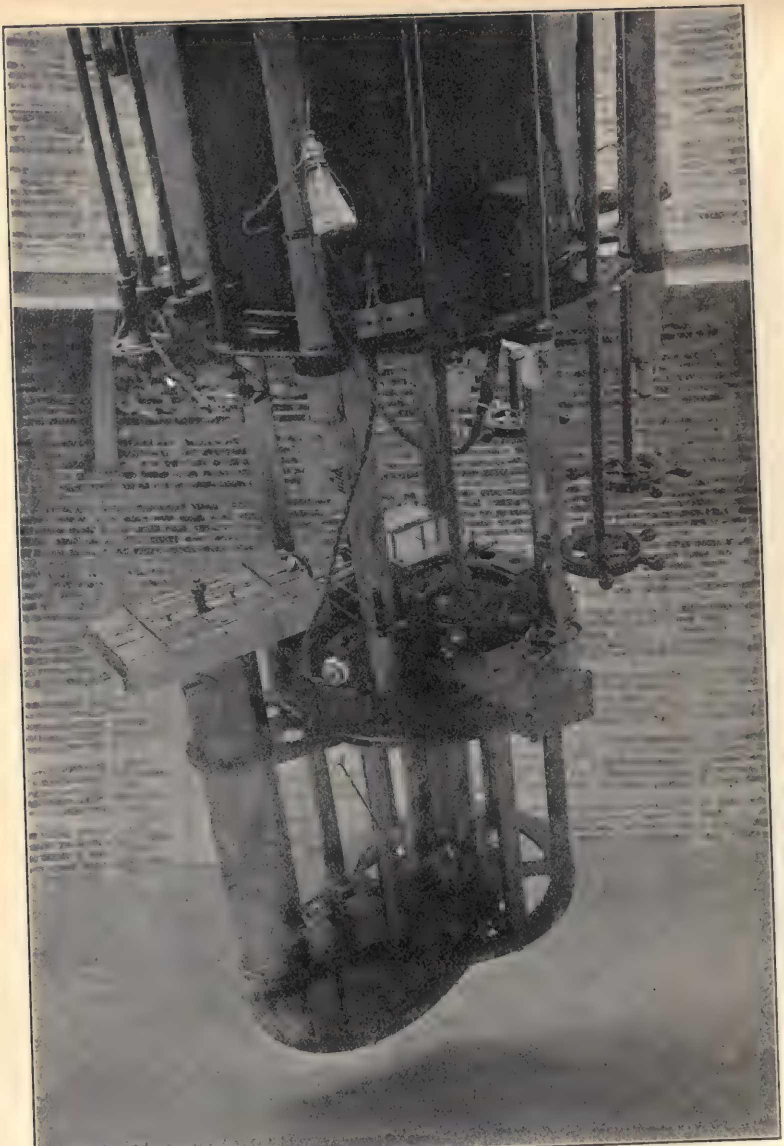


FIG. 162. — The Rumford Spectroheliograph attached to the 40-Inch Telescope of the Yerkes Observatory. The light is turned through 180° by the train of prisms, and the plate holder is nearly in the plane of the slit.

Fig. 163. — Photographs of Different Levels of a Group of Spots by Fox The highest level is shown in the upper picture.

The coronal streamers often, perhaps generally, have their bases in the regions of active prominences, but exceptions have been noted. That they are in some way connected with activity on the sun, is shown by the fact that the form of the corona changes in a cycle of about 11 years, the same as that of sun-spot activity. At sun-spot maxima the coronal streamers radiate from all latitudes nearly equally.

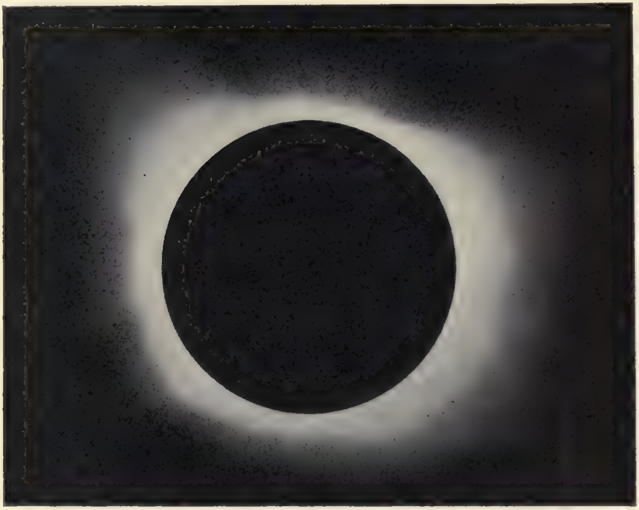


Fig. 164. — Photograph of the Sun's Corona at the Eclipse of Jan. 21, 1898, by the Lick Observatory Expedition.

As the maxima pass, the coronal streamers gradually withdraw from the poles of the sun and extend out to greater distances in the sun-spot zones. At the spot minima the corona consists of short rays in the polar regions, curved away from the solar axis, and long streamers extending out in the equatorial plane.

The spectroscope shows that the corona emits three kinds of light. First, there is a small quantity which is known to

be reflected sunlight, for it gives, though faintly, the Fraunhofer absorption lines, and it is polarized. Second, there is white light whose source, according to Kirchhoff's laws, must be incandescent solid or liquid particles. Lastly, there is a bright line spectrum whose source, according to Kirchhoff's laws, is an incandescent gas. The most conspicuous line is in the green and is emitted by an element, called coronium,

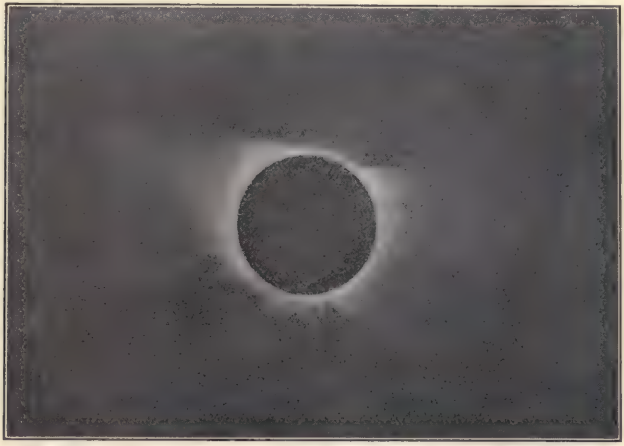


FIG. 165.—Photograph of the Corona, at the Eclipse of May 28, 1900, by the Yerkes Observatory Expedition. This was at a time of minimum sun-spot activity.

which is not yet known on the earth. There seems to be at least one other substance present, but no known elements.

According to present ideas the corona consists of dust particles, liquid globules, and small masses of gas which are widely scattered. From the amount of light and heat radiated, and from the temperature which masses so near the sun must have, Arrhenius computes that there is one dust particle, on the average, in every 14 cubic yards of the corona. The excessive rarity of the corona is shown by the

fact that comets have plunged through hundreds of thousands of miles of it without being sensibly retarded. The dust particles and liquid globules give the reflected light, the liquid the continuous spectrum, and the gases the bright line spectrum. The form of the corona shows that its condition of equilibrium is not at all similar to that of an atmosphere like the one surrounding the earth. Its increase of density toward the sun is inexplicably slow, though doubtless light pressure and electric forces are opposed to gravity. Its radial structure and periodic variation in general form are without satisfactory explanation.

343. The Eleven-year Cycle. — It has been explained that sun-spots vary in frequency and distribution on the sun's surface in a period averaging a little more than eleven years. There are a number of other phenomena which undergo changes in the same period.

The faculæ are most numerous in the sun-spot zones, although they occur all over the sun. Both their number and the positions of the zones where they are most numerous vary periodically in the sun-spot period. This is quite to be expected, for the sun-spots and faculæ are both photospheric phenomena.

The eruptive prominences are frequent in the sun-spot belts and vary in position with them. The evidence so far also shows periodic variations in numbers. The quiescent prominences, on the other hand, cluster in the polar regions.

The coronal types clearly vary in the eleven-year cycle, as was explained in the preceding article. Doubtless the total solar radiation varies to some extent in the same period, though this has not been verified observationally, but the time is now ripe for the investigation. The spectra of sun-spots vary with the period of the spots, but the Fraunhofer lines are singularly invariable.

The great vibrations which so powerfully agitate the sun extend to the earth and doubtless to the whole solar system.

It has been long known that both the horizontal and vertical components of the earth's magnetism vary in the sun-spot period, and that magnetic disturbances ("storms") are most frequent at the times when sun-spots are most numerous. Likewise auroras occur most frequently at the epochs of

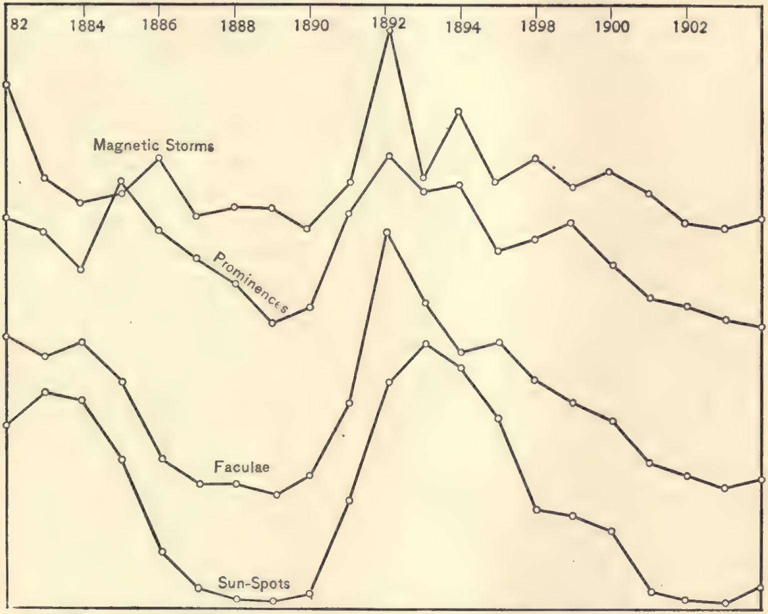


FIG. 166. — Curves of Magnetic Storms, Prominences, Faculæ, and Sun-spots, from 1882 to 1904.

great sun-spot activity. In fact, magnetic storms and auroras never occur except when there is great activity in the sun in the form of sun-spots or prominences; but there are frequent disturbances on the sun without accompanying terrestrial phenomena.

The first suggestion was that the sun induces changes in the earth's magnetic state by sending out electromagnetic

waves. Lord Kelvin raised the objection that if the sun sends out these waves in every direction, it would give out as much energy in eight hours of an ordinary electric storm as it radiates in light and heat in four months.

A recent exhaustive discussion of the data has led Maunder to the conclusion that the source of the periodic magnetic storms is in the sun, that the magnetic disturbances are confined to restricted areas on the sun, and that their influences are propagated out from the sun in cones which rotate with the sun; that when these cones of magnetic disturbances strike the earth, magnetic storms are induced, and that these magnetic storms have intimate, though unknown, relations with sun-spots. The most important contribution of this investigation was that there is much observational evidence to show that the sun is not to be regarded as surrounded by a polarized magnetic sphere, but that there are definite and intense stream-lines of magnetic influence, probably connected with the coronal rays, reaching out principally from the spot zones in directions which are not necessarily radial. It is a little too early to formulate a precise theory as to whether these streams are electrified particles driven off by magnetic forces and light pressure, or whether they involve the minute corpuscles of which atoms are composed, or whether they are phenomena of matter and energy of a character and in a state not yet recognized by science.

QUESTIONS

1. Enumerate the different known layers of the sun.
2. What evidence is there that the sun is surrounded by a dusky veil in addition to the reversing layer?
3. Show by a diagram that the fact that the absorption at the sun's limb is much greater than at its center proves that the dusky veil is thin.
4. Explain by a diagram how Frost's observations that spots lose less light by absorption near the sun's limb than the ordinary photosphere does, prove that the spots are in elevated regions.
5. Describe the rotation of the sun.

6. Describe the periodicity of sun-spots.
7. Enumerate some of the unsettled questions respecting sun-spots.
8. Describe the physical condition and the chemical constitution of the reversing layer.
9. Describe the chromosphere and prominences.
10. What are the uses of the spectroheliograph?
11. Why can not the corona be observed through the spectroscope at any time as prominences are observed?
12. Describe the supposed constitution of the corona.

CHAPTER XV

EVOLUTION OF THE SOLAR SYSTEM

344. Evolution. — A very slow change, especially if it be from the simple and unorganized to the complex and organized, is said to be an evolution. Curiously, the idea of evolution has been repugnant to some minds, although rapid and radical change in most things is a matter of universal experience. Perhaps the reason is that an evolution is, on the whole, a progression in one direction, while many of the changes of ordinary experience, such as the succession of the seasons, are periodic.

As our knowledge increases we find that everything is in a state of change. Individuals change, institutions change, languages change, and even the "eternal hills" are washed away in a moment of geological time. Now we are about to consider a series of changes in which the sun and planets in their present conditions and relations constitute but a single phase of a great evolution.

345. The Data of Evolution. — It is obvious that a knowledge of the system at present is necessary in order that its evolution may be worked out. It is certain that not everything is known which is pertinent to the inquiry, nor, indeed, will *everything* about it ever be known; but this will not prevent a discussion of at least the broad outlines of its development. There are plenty of examples where the general features of a series of changes can be predicted without knowing every small factor which enters into the problem. For example, the temperature of a place depends not only upon its latitude and the seasons, but also upon a

vast multitude of minor influences. It is impossible to foretell precisely what the temperature of a place will be at any given instant ; but from the knowledge of the seasons, and the differences in night and day, the general outlines of temperature variations can be worked out. Similarly, it is not to be expected that all the details of the evolution of the solar system can be given, but the general outlines are not hopelessly and permanently beyond us.

The greatest source of danger to a theory which is to be applied during immense ages is that there may be influences which are imperceptible in intervals of time covered by experience, but which operate continually in one direction. Thus, to take again the illustration of the weather, if it were true that the sun will radiate continually less and less heat, and if we were ignorant of this fact, our predictions of temperature conditions in the remote future would not be verified. It follows that the conclusions, especially in the details, which may be drawn from a theory of evolution, are more uncertain the more remote they are from the present.

In addition to the knowledge of the present condition of the solar system, we must know also the laws of its change in order to work out its evolution. It is something like the simpler problem of predicting where the planets will be a year from now. To do it we must know both where they are now and also the laws of their motions.

Hence, the data of the evolution of the solar system are both its present condition and also the laws according to which its phenomena proceed.

346. The Value of a Theory of Evolution. — Facts are important in proportion to the number of their known relations to other facts. Since a theory of evolution is concerned largely with relations, an attempt at a development of one forces our attention toward the correlation of facts. Moreover, the relations are examined in a critical spirit. Therefore an attempt at the construction of a theory of

evolution is of value because it leads to a better understanding of the material upon which it is being based.

A scientific theory invariably demands data in addition to that upon which it was founded. In this way it stimulates and directs investigation. In spite of the fact that we sometimes hear that the scientist should keep himself clear of preconceived notions, it is true that a great majority of discoveries have been made by those who were looking for the very things found. In most other cases the discoveries were made in attempting to find the opposite thing, or at least a different one of the same type, under the promptings of a false or of an imperfect theory. Hence a second value of a theory of evolution is that it leads to the discovery of new facts.

A broad scientific theory involves many secondary theories depending upon special groups of phenomena. For example, in the solar system there are the theories of the motions of the planets, of the capture of comets, of Saturn's rings, of the zodiacal light, of the equatorial acceleration of the sun, etc. In the construction of a general theory of evolution of the system these secondary theories must be related to the whole, and in this manner they are subjected to a searching examination. This criticism of secondary theories, whether it be constructive or destructive, constitutes another important value of a wide generalization.

It is by means of scientific theories that events may be controlled for our benefit, or that we may adjust ourselves to them. Although the direct contributions of astronomy to the welfare of mankind are important and many, yet it is not so largely directly utilitarian as most of the other sciences. But the indirect benefits of astronomy in enriching our intellectual experiences are not surpassed by those of any other science.

When we think of it, we find that very many of our activities are directed to satisfying our mental wants. Thus, we

do not travel to get more to eat or wear, but to acquire broader views of the world by gaining a knowledge of unfamiliar things. The important thing in traveling is not that we go to any particular place, but that we get the intellectual experiences. Astronomers can not travel through the vast regions of space which they explore, but the long arms of their analysis reach out and gather up the facts and bring them to their consciousness with a vividness scarcely surpassed in any experience. In this way, astronomy virtually extends our experience over an incomparably wide field. But the thing of chief interest in this connection is that when a satisfactory theory of evolution has been developed, it extends our experience similarly in time.

Finally, a theory which gives a unity to a great variety of observational data is of rare æsthetic value. It corresponds to the catalogue of imperfectly related facts upon which it is based as a finished and beautiful house does to the unsightly heaps of stone, brick, and wood from which it may be built. In some reflections along this line, near the end of his work on descriptive astronomy, Laplace said, "Contemplated as one grand whole, astronomy is the most beautiful monument of the human mind, the noblest record of its intelligence."

HISTORICAL

347. Wright and Kant. — A theory of the evolution of the solar system, and the much broader one of the evolution of the whole sidereal universe, on an approximately scientific basis, was first published in 1750 by Thomas Wright of Durham, England. He supposed that the Milky Way is composed of a vast number of gravitating systems like our own, spread out in a great double ring which rotates around an axis perpendicular to its plane. He treated the solar system only incidentally as exemplifying some of his conjectures respecting the sidereal system.

In 1751 the work of Wright fell into the hands of the young philosopher Kant, who at once turned his brilliant mind to the contemplation of the problems of cosmogony. In 1755 he published a work on this subject in which he accepted the general ideas of Wright, especially respecting the structure of the galaxy, but his principal contribution was in the development of a theory of evolution of the solar system. He frankly considered the question of the growth of the present solar system from a more primitive condition, and to guard against the prejudices of his time he showed in the preface that a belief in such an evolution need not in the least alter one's belief in a Supreme Being.

Kant advanced the hypothesis that in the beginning all of the material which is now in the various members of the solar system was in a state of ununited elements, and that it was scattered uniformly throughout the space now occupied by these bodies. He considered that this is the simplest possible hypothesis for the origin of the system. He ascribed the destruction of the homogeneity of the original mass to the diversity of the elements and to their different powers of attraction. He supposed that the heavier molecules would attract to themselves the lighter ones in their vicinity, and that these small aggregations would continually grow by the accretion of smaller masses. Motions would be developed, and because of the initial uniform distribution of matter, the resultant attractions would be toward the center of the whole system. He called attention to the fact that attraction would be opposed by gaseous expansion, and in some obscure way he supposed that these repulsive forces would generate lateral motions in the small nuclei. At first the nuclei would be moving in every possible direction, but he assumed that successive collisions would eliminate all except a few moving in the same direction, and in nearly circular orbits.

Kant considered in successive chapters the densities and

ratios of the masses of the planets, the eccentricities of the planetary orbits and the origin of comets, the origin of satellites and the rotations of the planets, the origin of the rings of Saturn, the zodiacal light, and a theory of the constitution and physical condition of the sun.

The beauty and generality of Kant's theory are enticing, but it involves some obvious difficulties. First, the repulsive forces would not be competent to set up a revolution of the whole system, for as long as it is not subject to exterior influences the whole amount of rotation is constant. Besides this, even if the revolution were granted, it is by no means evident that there would be one dominating central mass, and that all motions except those nearly in one plane and in the same direction would be destroyed.

Kant's treatment of the many special topics shows a wonderful keenness of intellect and power of generalization, but his arguments were often faulty because his knowledge of physical laws was very imperfect. His work on the evolution of the solar system is historically of the highest interest; but as his theory fails completely in certain essential respects, it will not be profitable to give more of the details in this place.

348. Laplace's Ring Hypothesis. — In 1796 Laplace published a most excellent and charmingly written popular exposition of astronomy. In it he explained with pride and exultation how one difficulty after another had been overcome by astronomers until practically every question had been settled. It is improbable that astronomers will ever again have so few problems demanding solution as they did at that time. In the last chapter of his work, Laplace advanced his ideas respecting the development of the solar system, neglecting entirely the more general question of the evolution of the sidereal universe. In the several editions which appeared before his death in 1827, numerous additions to his original remarks were made, but there is no evidence that he had ever heard of the work of Kant.

Laplace called attention to the fact that all of the motions of revolution and of rotation in the solar system then known were almost in the same plane and in the same direction. He calculated that this condition would be the result of chance only once in more than 500,000,000 of cases, showing, therefore, that it is due in all probability to some initial state from which the system has developed. He likewise called attention to the remarkable fact that the orbits of the planets are nearly circular, while those of comets are sensibly parabolic.

Buffon had advanced the theory that the planets are composed of material which has been driven off from the sun by the impact of a comet which has fallen into it. Laplace said that this was the only theory of the origin of the planets with which he was familiar; and he pointed out the fatal objection to it that matter driven off in this manner would, unless it receded to an infinite distance from the sun, return to the point from which it started and reunite with the original mass.

The theory of Laplace was advanced "with that distrust which everything ought to inspire that is not a result of observation or of calculation." In outline the theory was that originally the solar atmosphere (in later editions a nebulous envelope), in an intensely heated condition, extended out beyond the orbit of the farthest planet; the whole mass rotated as a solid in the direction in which the planets now move; the dimensions of the solar atmosphere were maintained largely by gaseous expansion of the excessively heated vapors, and slightly by the centrifugal acceleration due to the rotation; as the mass lost heat by radiation, it contracted under the mutual gravitation of its parts; simultaneously with its contraction its rate of rotation necessarily increased, for the total quantity of rotation (the moment of momentum) must have remained constant; after a time the centrifugal acceleration at the equator equaled the attraction, and

a ring was left off, the remainder continuing to contract; a ring was abandoned at the distance of each planet; a ring could scarcely have had perfect uniformity, and, separating at some point it united at some other, because of the mutual attractions of its parts, and formed a planet; finally, the satellites were formed from rings, left off by the contracting planets, Saturn's ring being the only example still remaining.

The theory of Laplace was less pretentious than that of Kant in that it did not attempt to explain the whole universe, and for that reason possibly less liable to general suspicion respecting its soundness. It carried with it the immense prestige of the name of Laplace, and was soon quite generally accepted. It has exercised an incalculable influence upon the science and philosophy of the nineteenth century. No one knows to what an extent it has entered into everything from the interpretation of astronomical and geological phenomena to the discussion and explanation of theological systems. Because of this wide influence, the additions which have been made to it and the objections which have been raised against it will be treated in considerable detail.

349. The Bearing of the Contraction Theory of the Sun's Heat. — Laplace assumed that the original solar nebula was enormously distended and heated, and that it has been contracting as a consequence of its loss of heat. After the development of the theory of the conservation of energy and the work of Helmholtz on the contraction theory of the sun's heat, this assumption of an original high temperature was no longer necessary. This theory of the maintenance of the energy poured out in the solar radiation makes it very probable that the sun has been continually contracting. Consequently it agrees very well with the Laplacian hypothesis, though it is obvious that it is by no means necessary to assume that the sun has contracted from the form postulated by Laplace.

350. The Meteoritic Hypothesis. — Lockyer has advanced the hypothesis that all matter was primarily in the meteoric state instead of the gaseous. He supposes that the nebulas are vast swarms of meteors which are visible because the collisions make small portions of their whole masses incandescent.

G. H. Darwin has investigated the mechanical condition of a great swarm of meteors. He found that if the dimensions and mass are like those of the solar system, the conditions are not sensibly different from what they would be if they were gaseous. It amounts simply to enlarging the molecules, and the theory need not be considered separately from

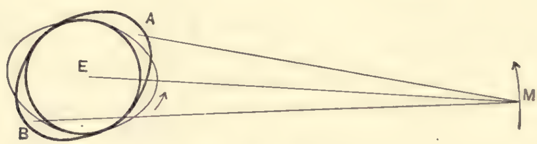


FIG. 167.

that of Laplace. Darwin showed that when the mass was widely extended it would necessarily rotate as a solid.

351. Tidal Evolution. — Before considering the objections to the Laplacian hypothesis, it will be necessary to review briefly the possible effects of the tides on the evolution of the system, as developed in a series of remarkable papers published by Darwin from 1878 to 1882. A large part of the work is of general application, but the detailed numerical illustrations referred to the earth and moon.

Darwin supposed that the earth is a viscous mass subject to the tides of the moon and sun. The viscosity was supposed to be so great that in such small masses as can be subjected to laboratory experiments it would amount to sensibly perfect rigidity. From the fact that there is a lack of perfect rigidity there will certainly be body tides (the

oceanic and atmospheric tides are neglected in this discussion), and from the lack of perfect fluidity the tides will lag (Art. 180). The condition will be as represented in Fig. 167, where the tides are enormously magnified for the purpose of illustration. The light curve shows where the tides would be if the earth were a perfect fluid, and the heavy curve shows the position that they occupy after being carried forward by the earth's rotation.

The problems are to find (1) what effect the moon's attraction on the tides *A* and *B* will have upon the earth's rotation, and (2) what effects the attractions of the tides *A* and *B* will have upon the motion of the moon.

(1) It is easily seen that when the month is longer than the day (the tides being situated as represented in Fig. 167), the length of the day is increased. When the sun's tides are included, the day is lengthened still more. Darwin showed that the plane of the earth's equator and that of the moon's orbit are both changed, the character of the changes depending upon quite a number of factors.

(2) The attraction of the tide *A* for the moon may be resolved into two components, one toward *E* and one perpendicular to $\overline{EM}$ in the direction of the moon's motion. In the case of the tide *B* the component perpendicular to $\overline{EM}$ is opposite to the direction of the moon's motion. But the attraction of the tide *A* is greater than that of *B*, and the angle *AME* is a little greater than the angle *BME*. Therefore the whole resultant is an acceleration of the moon. The effect of an acceleration is to increase the size of the moon's orbit and to lengthen the month.

352. Outline of Darwin's History of the Earth-Moon System. — Carrying the computation backward in time, Darwin arrived at what he considered a probable initial state of the earth and moon, and then he sketched their evolution to the present time. For convenience, the outline will be worded as though no question remained respecting its exact truth.

At least 54,000,000 years ago the earth and moon formed one body having a diameter a little over 8000 miles. This mass rotated on its axis in a period of about 5 of our present hours, and the plane of its equator was inclined 11° or 12° to the plane of the ecliptic. The mass was rotating so fast that it was in unstable equilibrium, and the moon broke off under the stimulus of the sun's tides. At this time the earth and moon revolved around their common center of gravity and on their axes in the same period (about 5 hours). When the earth contracted it rotated more rapidly, the day was shorter than the month, and tidal evolution began. The tides lengthened both the day and the month, *but the month more rapidly than the day*. This increase of periods has continued until the present conditions have been reached.

Similarly, the earth raised tides on the moon and increased the length of the lunar day until it equaled the length of the month. The eccentricity of the moon's orbit increased for a time, and has decreased ever since. The inclination of the plane of the earth's equator to the plane of the ecliptic increased from 11° or 12° to 23.5°, and the plane of the moon's orbit has changed from coincidence with the plane of the earth's equator until it is now inclined to the plane of the ecliptic by only 5° 9′.

In the future the day will increase in length *faster* than the month, the day and the month will become equal at 50 or 60 of our present days, and the moon's orbit will be circular, except as it is disturbed by the attraction of the sun. If the earth and moon were the only two bodies in existence, this condition would be permanent. But the sun's tides will interfere and bring the moon back toward the earth, in such a way that the day will always be a little longer than the month, and the evolution will eventually end by the earth and moon again uniting.

The working of body tides generates heat, and Darwin com-

puted that enough heat has already been produced in the earth by the tides to raise its temperature, if all applied at once, 3000° Fahrenheit.

353. Evidences of Tidal Evolution. — The work of Darwin is based on the assumptions (which seem most certain) that the earth is not perfectly rigid and that there is internal friction when it is distorted. But the tidal reactions are excessively complicated, they are quite different with different degrees of viscosity, rates of rotation and revolution, and in some cases slight changes in the assumptions would make important changes in the conclusions. As a rule the tides with long periods work against those with short periods, whose influences in general predominate. The difficulties of the problem and the uncertainties of the data make it important that the theory be verified, if possible, by the actual phenomena.

The most striking fact, and the one which probably first directed inquiry to this subject, is that the same side of the moon is always toward the earth. This agrees perfectly with the theory, which shows that, under the hypotheses, this condition is the result of the earth's attraction for the tides which it has generated on the moon. Other similar facts are that Japetus constantly turns one side toward Saturn (Art. 279), and that Mercury and Venus probably always present the same face to the sun (Arts. 260 and 262). Of course, there is no way of knowing how fast the original rotations were and how much work tidal friction has done in these cases.

Mars rotates more slowly than its inner satellite revolves (Arts. 264 and 265), instead of faster as it would if it contracted, as the Laplacian theory supposes, from the dimensions of the satellite's orbit. This fact is incapable of explanation directly under the Laplacian hypothesis, but Darwin explains it by supposing that the sun's tides have lengthened the planet's day from the period of the satellite's

revolution to its present period. He regards this as verifying his theory of tidal evolution and as relieving the Laplacian hypothesis of a very serious objection. However, Nolan has pointed out that Phobos is so near Mars that, notwithstanding its small mass, its tides are comparable to those of the sun; and since it revolves faster than the planet rotates the tides which it generates tend to decrease the size of its orbit. He calculated that before the sun's tides would lengthen the day by one minute, the satellite would be precipitated upon the planet by the action of its own tides.

The inner particles of Saturn's ring system revolve much more rapidly than the planet rotates. Since this planet is six times as far from the sun as Mars is, solar tidal influences will be much less effective, for they vary inversely as the sixth power of the distance. The density, dimensions, and rate of rotation are all modifying factors; but taking what seem to be conservative estimates of those which are not well known, it is found that to explain this difficulty as Darwin did the similar one in the case of Mars, more than 3000 times as long an interval would be required. One certainly would not infer either from the Laplacian hypothesis or from the present condition of these bodies that Saturn is 3000 times as old as Mars.

If the earth once rotated on its axis in 5 or 6 hours, it must have been greatly bulged at the equator, as Jupiter is now. The retardation of the tides would be greatest in the equatorial regions, and it might be expected that there would be still some evidence of the equatorial zone lagging behind the higher latitudes, but none is found. Moreover, as the rotation decreased the earth would become more nearly spherical. In changing its shape the most mobile parts would be affected first before the strain became great. Therefore, we should expect, under the theory, that both poles would now be covered deeply with water, and that the equatorial zone would be entirely land. There is no geological evidence

of any radical change of shape during a time which geologists estimate as 100,000,000 years or more.

The satellites of Jupiter and Saturn undoubtedly raise tides upon their respective primaries just as the moon does upon the earth. In a similar way the planets raise tides upon the sun. Moreover, in all these cases the tide-raising bodies revolve nearly in the equatorial planes of the bodies around which they circulate, and the periods of rotation and revolution are such that the tides tend to cause equatorial retardations. But notwithstanding the fact that these forces have been at work probably many tens of millions of years, we find in every one of these three cases that the tidally distorted body has an equatorial acceleration instead of a retardation. Since no instance is known where the opposite is true, these must be considered very significant facts.

Curiously no other known fact is more favorable to the theory of tidal evolution than the character of the moon's rotation, while no other fact is more unfavorable than the absence of evidence of any change in the shape of the earth. Perhaps the explanation of the apparent contradiction lies in the fact that the character and magnitude of the tidal influences depend upon the rigidity of the disturbed body, and upon the period of its rotation compared to the period of revolution of the tide-raising body. The viscosity of the moon and its rate of rotation may always have been related to the month so that the tides always tended to bring about the present state of affairs; while the rigidity and day of the earth may very well have been related to the month so that the earth's rotation has not been sensibly changed.

We may conclude that the theory of tidal evolution depends upon sound principles, but that, because of the uncertain factors involved and the contradictions in the observational evidence, its influence on the evolution of the system can not now be correctly estimated.

354. Facts which Support the Laplacian Hypothesis. — The

following phenomena are in agreement with Laplace's theory (and possibly with other theories also), and have been the basis for its wide acceptance.

1. The planets all revolve nearly in the same plane and in the same direction.

2. Their orbits are all nearly circular.

3. The sun rotates in the direction in which the planets revolve.

4. The planes of the equators of the planets and of the orbits of their satellites are nearly coincident with the planes of their orbits (Uranus and Neptune present exceptions).

5. The satellites revolve in the direction that their respective primaries rotate (the ninth satellite of Saturn and probably the seventh satellite of Jupiter are exceptions, while the positions of the planes of the equators of Uranus and Neptune are not known).

6. According to the contraction theory of the sun's heat, this body was once vastly larger than at present.

The agreement of these facts with what the Laplacian theory would lead us to expect is obvious. There are many other facts which do not seem to be inconsistent with the theory, but which have not been shown necessarily to follow from it.

355. Facts which are Inconsistent with the Laplacian Hypothesis. — For many years astronomers have found it difficult to reconcile the Laplacian hypothesis with all the phenomena presented by the solar system. The most critical examination yet published of the crucial facts and principles is contained in the papers by Chamberlin and the author which appeared in 1900.[1] It was shown in these

[1] Chamberlin, "An Attempt to test the Nebular Hypothesis by the Relations of Masses and Momenta," *Journal of Geology*, February–March, 1900; Moulton, "An Attempt to test the Nebular Hypothesis by an Appeal to the Laws of Dynamics," *Astrophysical Journal*, March, 1900.

discussions that it is necessary to abandon the ring theory because it is inconsistent with certain data given by observation when taken in connection with the principles of dynamics. Some of these things which are contradictory to the theory will now be enumerated and briefly discussed. The principal ones are: —

1. The considerable mutual inclinations of the planes of the planetary orbits, and the inclination of the plane of the sun's equator to the general plane of the system, are not to be expected on the basis of the ring theory.

2. The eccentricities of the planetary orbits are not to be expected on the basis of the ring theory.

3. The orbits of the planetoids contradict the ring theory.

4. The rapid revolution of Phobos and of the particles of the inner ring of Saturn can not be satisfactorily explained.

5. The presence of light elements in the earth is not to be expected.

6. A series of rings could not have been left off.

7. A ring could not have condensed into a planet.

8. The moment of momentum of the present system is less than $\frac{1}{200}$ of that of the supposed initial nebula.

9. The retrograde revolutions of the ninth satellite of Saturn and (probably) of the seventh satellite of Jupiter flatly contradict the theory.

These statements require some explanation.

356. The Inclinations of the Planes of the Planetary Orbits. — According to the Laplacian hypothesis, the original solar nebula is supposed to have maintained its dimensions almost wholly because of the gaseous expansion of its parts. Darwin said in the introduction to his investigations on the mechanical conditions of a swarm of meteorites, "But the very essence of the nebular (Laplacian) hypothesis is the conception of fluid pressure, since without it the idea of a figure of equilibrium becomes inapplicable." He showed in this paper that the meteoric swarm (which includes the

nebula as a special case) would soon, whatever the initial irregularities, rotate as a solid. It follows from this fact that the planetary orbits should all lie in the same plane, or at least that those of the inner planets and the plane of the sun's equator should deviate but very little, if any, from one another. As a matter of fact, the differences in the planes of the orbits of the planets near the sun are not only considerable, but they are much greater than the mutual inclinations of the planes of the orbits of the major planets, and the plane of the sun's equator is inclined several degrees to the general plane of the system.

357. The Eccentricities of the Planetary Orbits. — According to the conditions postulated in the Laplacian theory, the orbits of the planets should be very nearly circular, and the more nearly circular the nearer they are to the sun. But it is found that while Neptune's orbit is very nearly round, Mercury's orbit is more than twice as eccentric as that of any other planet. The general conditions are indicated by the fact that the orbits of the terrestrial planets average more than twice as eccentric as the orbits of the great planets.

358. The Orbits of the Planetoids. — It follows from the ring theory that the orbits of all the members of the system should be distinct from each other. It is found by observation that the orbits of at least 530 planetoids are looped through each other in a most complicated fashion ; some are practically in the plane of the ecliptic, while others are very highly inclined to it ; and some are almost perfectly circular, while others are very elongated. Undoubtedly the perturbations by Jupiter have disturbed the initial arrangement, but it is scarcely possible that they could have developed the present conditions from orbits which were originally concentric circles, or from matter which once revolved around the sun in a circular ring.

Similarly the orbit of the planetoid Eros reaches from near the orbit of the earth out beyond that of Mars. Not

less remarkable from the standpoint of the Laplacian theory is its high inclination (10°) to the orbits of the earth and Mars.

359. The Rapid Revolution of Phobos and the Inner Ring of Saturn. — According to the ring theory, the periods of rotation of the planets should be shorter than the periods of revolution of all of their respective satellites, for as a planet shrunk away from the last ring it would continually rotate more rapidly. But Phobos revolves in a shorter time than is required for a rotation of Mars, and similarly the particles constituting the inner part of Saturn's rings perform a revolution in less than the planet's period of rotation. If the first is explained by tidal evolution, the action of Phobos introduces a more embarrassing difficulty, and it must be assumed that Saturn is 3000 times as old as Mars in order to explain the second in the same way (Art. 353).

360. The Escape of the Light Gases from a Ring. — If a ring were left off, it would be so widely extended that the mutual gravitation of its parts would be very feeble, and according to the kinetic theory of gases (Arts. 110 and 111), all the lighter elements would escape. But the lightest known element, hydrogen, is abundant on the earth, though it is now in chemical combination with other elements.

361. The Leaving off of Rings. — It is easy to overlook the fact that the postulated nebula must have been excessively rare. According to the hypotheses made, it must have been denser at its center than near its periphery. But if we suppose that it was homogeneous and that it reached out to Neptune's orbit, we find that its density was only $\frac{1}{250,000,000}$ that of air at the sea level. Neptune's ring could not have been so dense as this, which is many times rarer than the best vacuum yet produced in our laboratories. Now a ring of such rarity would have had no cohesion and would not have separated except particle by particle. When the process was once started it seems that it should have been

continuous instead of intermittent as the theory supposes. The theory postulates that when a ring was left behind the nebula was made stable for a long period of contraction. Roche has attempted to show that rings of considerable dimensions would be abandoned at certain intervals, but his work on this point is far from conclusive. Every other writer on the subject has keenly felt this difficulty. Thus, Faye, in his modification of the Laplacian theory, supposed that the whole nebula broke up into rings simultaneously.

362. The Condensing of a Ring into a Planet. — We may assume for the sake of argument that rings are abandoned, and inquire whether they will unite into planets or not. The matter of the ring would be very widely spread out and the mutual gravitation of its parts would be very feeble. The appropriate investigation shows that the tidal forces coming from the interior mass would more strongly tend to scatter the material than its gravitation would to gather it together into a planet. Consequently a ring could not even start to condense into a planet. It would be something like a comet which becomes utterly dissipated by tidal forces.

To give every possible advantage to the ring theory, we may assume that all the matter has been gathered into a planet except a ring of very small particles, and then ask ourselves whether this minute remainder will be brought to the planet. Investigation shows a strong probability that only that part of the ring which is within 60° of the planet could be brought on to it in any time however long. That is, if we assume that the process of formation of a planet out of a ring is almost finished, we find that it can not complete itself. This shows the strong improbability that the assumed stage could ever have been reached by condensation from a more uniform ring.

363. The Moment of Momentum of the System. — Whatever evolution the system may have undergone in consequence of the mutual interactions of its parts, its mass and

its whole quantity of rotation (moment of momentum) have remained constant. If no energy had been lost by radiation, we might add to this that the sum of the kinetic and potential energies has also remained constant.

The Laplacian hypothesis supposes that the solar nebula once extended out beyond the orbit of Neptune and that it was in a condition of hydrodynamical equilibrium. Its form depended upon its rate of rotation, and its density upon its rotation and the laws of gaseous expansion. If we knew its form and its law of density, its moment of momentum could easily be found. Neglecting the rotation, the law of density has been computed by Ritter, Hill, and Darwin. The effect of the rotation is to make the body oblate and to place a larger part of its mass far from its axis. Consequently, if we compute the moment of momentum under the assumption that the body is a sphere whose law of density is that found by Ritter, Hill, and Darwin, we shall get a result which is too small. Therefore, since the moment of momentum never changes, if the system has evolved from such an original nebula, its present moment of momentum must be greater than this quantity. But the present moment of momentum is easily computed, and *it is found that instead of being larger than the quantity previously obtained it is less than $\frac{1}{200}$ as great.* This is a crucial test, and the numerical discrepancies are so great that it is certain that the fundamental postulate of the Laplacian theory respecting the original condition of the system is wrong.

Chamberlin presents the difficulty in a different way. Consider the supposed system at the time Jupiter's ring was about to be left off. The whole mass is supposed to have been rotating as a solid. When Jupiter's ring was abandoned, $\frac{1}{1000}$ of the whole mass was separated. Now the moment of momentum of this ring can not have been appreciably changed in its subsequent evolution. The resistance of meteoric matter has decreased it, while tidal reactions,

which have been exceedingly slight, have increased it. We shall not be sensibly in error if we suppose Jupiter's ring had the same moment of momentum that the planet has now. But computation shows that Jupiter has 95 per cent of the moment of momentum of that part of the whole system which is within the orbit of Saturn. That is, the Laplacian theory indirectly affirms that a nebula rotating as a solid and in a state of hydrodynamical equilibrium can abandon a ring containing only $\frac{1}{10}$ of one per cent of its mass, but 95 per cent of its moment of momentum. The thing is quite incredible.

364. The Retrograde Revolution of the Ninth Satellite of Saturn. — The ninth satellite of Saturn revolves in the retrograde direction. This apparently is an impossibility if the Laplacian hypothesis is true, but it has been suggested that it may be explained by tidal evolution. The suggestion is that when Saturn extended out to the orbit of the ninth satellite, it rotated in the retrograde direction with the period of this body. The tides raised in the mass by the sun would tend to bring it to the condition where it always has the same side toward the tide-producing body, just as the moon and Japetus always have one side toward the earth and Saturn respectively. That is, the tides generated by the sun are supposed to have stopped its retrograde rotation and to have given it a forward rotation whose period was equal to its period of revolution, or 29.5 years. As the body contracted from these dimensions it would rotate more rapidly, and the remaining nine satellites are supposed to have been developed from rings which were successively abandoned after its rotation had become direct.

But let us examine the question more carefully. When the rotation period of the nebulous mass equaled that of its revolution, it filled some space as that indicated by the dotted curve in Fig. 168. Up to this time the tides generated by the sun had increased its moment of momentum by changing

it from a negative quantity to a certain positive quantity. After this time the tides generated by the sun decreased its moment of momentum, for they always retarded the rotation. Therefore, if the theory is true, the greatest moment of momentum in the whole history of the Saturnian system should be found when the day and year of its nebula were equal.

It is easy to go to decisive numerical results. The larger the dotted circle was, the larger this maximum moment of momentum. To make the case extreme, we may suppose that it was as large as the orbit of the ninth satellite. Because of the excessively slow rotation (once in 29.5 years)

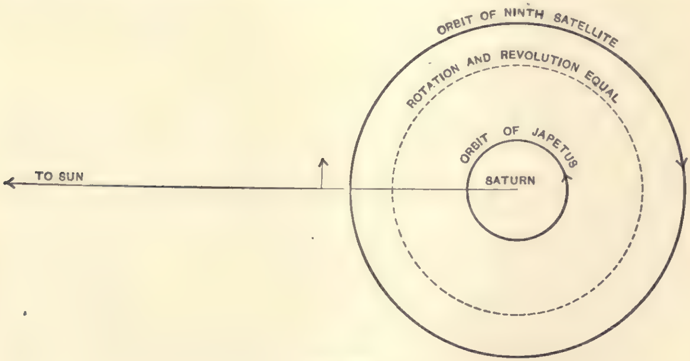


FIG. 168.

the body will have been sensibly spherical. Our result will be too large if we take the mass as homogeneous, for it was certainly denser toward the center. With these assumptions we compute what should be larger than the maximum moment of momentum.

When the mass had shrunk down to the dimensions of the orbit of Japetus, it is supposed to have been rotating in the period of Japetus. If we assume that it was spherical, we shall obtain a result which is too small, for it will have become somewhat flattened. The assumption of homogeneity

will enter about as before, and we shall find, if the Laplacian theory and the suggested explanation are true, that the moment of momentum was then considerably smaller than before. But the computation shows that it was more than seven times as great. Consequently the explanation fails, and the retrograde revolution of this satellite squarely opposes the Laplacian theory.

If the seventh satellite of Jupiter revolves in the retrograde direction, as observations so far seem to indicate, the disagreement is still more striking, for the sixth satellite revolves in the forward direction at almost the same distance from the planet.

QUESTIONS

1. Discuss the value of a theory of evolution.

2. In what ways may a false theory be valuable, and in what harmful?

3. Can you give examples of any phenomena which are not related to any theory?

4. Why should we follow Kant's idea that the evolution of the system has been from the simple and uniform to the complex and heterogeneous?

5. What was the fatal error in Kant's theory? How did Laplace avoid this error? Is it correct to say that Kant's theory is useless because it contains a fundamental error?

6. What phenomena are in agreement with the Laplacian theory?

7. What phenomena are neutral with respect to the theory?

8. What phenomena contradict the theory?

9. In the light of Darwin's work on the mechanical condition of a swarm of meteorites, what dynamical differences would there be in the evolution starting from the meteoritic hypothesis?

10. Draw a diagram showing how a force applied to the moon in the direction of its motion will enlarge its orbit.

11. Draw a diagram showing the position of the tides when the day is longer than the month, and show what their effects are upon the lengths of the day and the month.

12. What facts go to confirm Darwin's theory of tidal evolution? What oppose it?

13. What difficulty with the Laplacian theory can be explained, at least partially, by tidal evolution?

14. What objections to the Laplacian theory apply equally to the theory that lumps were formed in the equator of the solar nebula and, being left off, contracted into the several planets?

15. Suppose a nebulous ring were abandoned and that on one side of the sun it had the distance of Neptune, and that on the other side it was 5000 miles nearer the sun. From Kepler's third law find how many years would be required for the inner portion to overtake the outer in longitude, supposing that the different parts did not disturb each other's motion.

THE SPIRAL NEBULA HYPOTHESIS[1]

365. Hypothesis respecting the Antecedents of our Present System. — The solar system exists and is in the midst of an evolution; the problem is to trace out this evolution. The historical theories have been seen to be untenable, and the question arises whether at the present time an hypothesis can be formulated whose implications are in agreement with observed phenomena. An attempt is now being made to work out a theory along somewhat new lines, and a sketch of its main features will be given.[2]

Instead of supposing that the solar system started from a vast gaseous mass in equilibrium under the law of gravitation and the laws of gaseous expansion, the spiral hypothesis postulates that the matter of which the sun and planets are composed was, at a previous stage of its evolution, in the form of a great spiral swarm of discrete particles whose positions

[1] This is called the "Planetesimal Hypothesis" by Professor Chamberlin for reasons which will be explained. It will be called the Spiral Hypothesis here in order to keep it in sharp contrast with the Laplacian ring theory.

[2] The developments sketched here are the sequence of the papers by Professor Chamberlin and the author previously quoted. The first printed account of the new theory is given in Chamberlin's paper, "Fundamental Problems of Geology," Year Book No. 3 of the Carnegie Institution of Washington. It has also been expounded by the author in the Astrophysical Journal, October, 1905. The work of developing the theory has been much assisted by grants from the Carnegie Institution.

and motions were dependent upon their mutual gravitation and their velocities. Gaseous expansion preserved the dimensions of the Laplacian nebula, while in this the orbital motions were the dominant factor. Because of the fact that every particle is supposed to have moved nearly independently like a planet, Chamberlin calls the theory the *Planetesimal Hypothesis*.

Before discussing the possible origin of a spiral swarm of particles, and the details and merits of the planetesimal theory, attention should be called to the fact that there is not an example of a Laplacian ring nebula among the thousands of nebulas which are known. On the other hand, spirals are very numerous, particularly among the smaller and fainter nebulas. The photographs which Keeler made at the Lick Observatory shortly before his death led him to the conclusion that the spiral is the normal type. He said:

"1. Many thousands of unrecorded nebulas exist in the sky. A conservative estimate places the number within the reach of the Crossley reflector at about 120,000. The number of nebulas in our catalogues is but a small fraction of this.

"2. These nebulas exhibit all gradations of apparent size from the great nebula in Andromeda down to an object which is hardly distinguishable from a faint star disk.

"3. Most of these nebulas have a spiral structure. . . . While I must leave to others an estimate of the importance of these conclusions, it seems to me that they have a very direct bearing on many, if not all, questions concerning the cosmogony. If, for example, the spiral is the form normally assumed by a contracting nebulous mass, the idea at once suggests itself that the solar system has been evolved from a spiral nebula, while the photographs show that the spiral is not, as a rule, characterized by the simplicity attributed to the contracting mass in the nebular (Laplacian) hypothesis. This is a question which has already been taken up by Chamberlin and Moulton of the University of Chicago."

The spiral nebulas have dark lanes down between their arms, and it is evident that, if the distribution of matter in them is even approximately as it appears to be, their forms

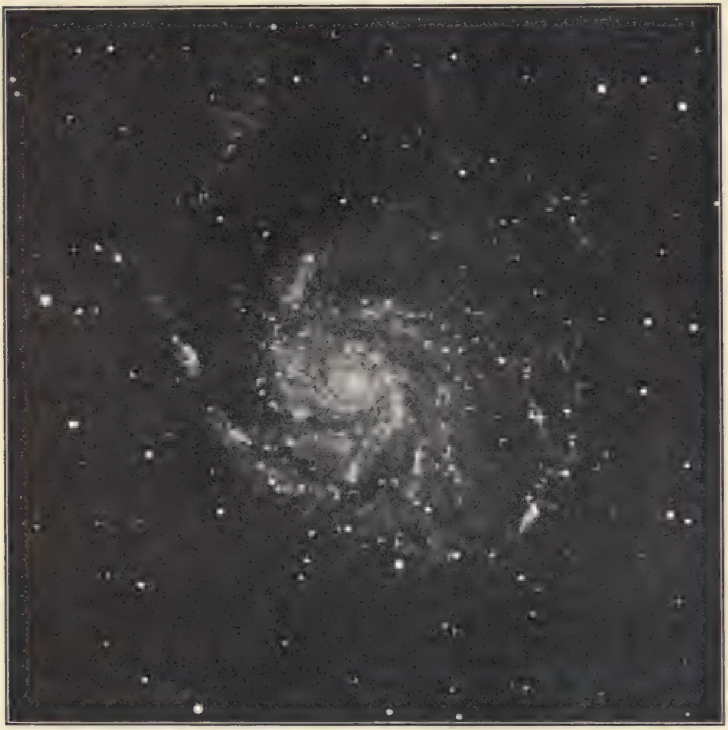


FIG. 169.—Spiral Nebula M. 101 in Ursa Major. *Photographed by Ritchey at the Yerkes Observatory.*

are preserved almost entirely by the motions of their separate parts instead of by gaseous pressure.

366. A Possible Origin of Spiral Nebulas.—The theory of the evolution of the system from a spiral nebula is largely independent of any hypothesis about the origin of the spiral.

However, a possible, and even probable, mode of generation of these remarkable forms has been suggested by Chamberlin; and for the sake of having a definite theory to work on, it will be assumed, at least provisionally, that the solar spiral nebula was developed in this way.

The stars are moving with respect to one another, often with very great velocities, and apparently in every direction. It follows that in the course of a time which may be extremely long indeed they will pass very near other stars, or possibly collide with them. If a collision occurs, the chances are very great that it will be oblique rather than central, and a spiral may be formed; but the chances of simply a near approach are enormously greater, and only this case will be treated here.

When two large bodies come near each other they are subject to great tidal strains, which, according to the researches of Roche, entirely break them up if their distance apart is less than $2.44 \cdots$ times their radii. Suppose, however, that this limit is not reached. Then they do not disintegrate under tidal strains alone, but when these forces are added to the eruptive tendencies of highly heated gaseous bodies, it is almost certain that masses of matter burst out and recede to great distances. It is like the eruptive prominences on the sun, only on a vastly greater scale. These eruptions occur in the directions of the greatest disturbing forces. It follows from the character of the tide-raising forces (Art. 177) that they are straight toward and from the tide-raising body. If this matter were undisturbed it would fall straight back on the body from which it burst forth, but the tide-raising body changes its orbit into an ellipse, as will be shown.

367. The Disturbing Acceleration. — As preparatory to considering the disturbing effects of one sun on the material ejected from another, the character of the disturbing acceleration will now be investigated.

Let the two suns be S and S', and refer the motion of S' to S, from which the matter is supposed to have been ejected. Suppose S', moves around S in a parabolic or hyperbolic orbit in the direction indicated by the arrow. Consider a small mass of matter at P and find the disturbing acceleration which S' exerts upon it. Let $\overline{SA}$ represent the acceleration of S' upon S in direction and amount. Let $\overline{PB}$ represent the acceleration of S' upon P in direction and amount. Since S' is nearer to P than it is to S, $\overline{PB}$ is longer than $\overline{SA}$. From the fact that the attraction varies inversely as the square of the distance, it follows that

$$\overline{SA} : \overline{PB} = \frac{1}{\overline{SS'}^2} : \frac{1}{\overline{PS'}^2}.$$

Now resolve $\overline{PB}$ into two components, one of which shall be equal and parallel to SA. That is, when $\overline{PC}$ is drawn equal and parallel to $\overline{SA}$ the acceleration $\overline{PB}$ is equivalent to the two accelerations $\overline{PC}$ and $\overline{PD}$. But the relative positions of S and P are not changed by the equal parallel acceleration $\overline{SA}$ and $\overline{PC}$. Hence the disturbing acceleration is the remaining component PD. The corresponding figure is drawn when the disturbed particle is at P'.

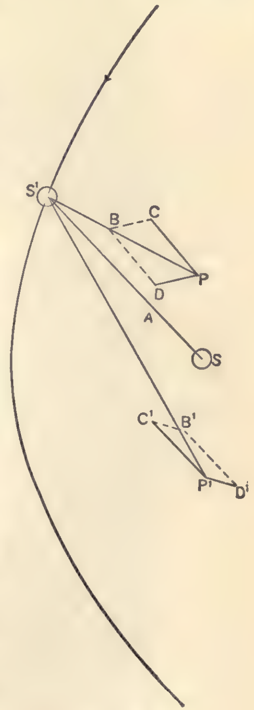


FIG. 170.

The general result is that the disturbing component of acceleration is always directed *toward* the line joining S and S'. When the disturbed particle is in the region of P or P', the disturbance is in the direction of motion of S'.

368. The Development of Elliptic Orbits of Particles ejected from the Sun in Straight Lines. — Suppose a small mass is ejected from S toward S'. It will for a considerable time move out from S in a nearly straight line. In the meantime S' will move forward in its orbit, and the conditions will be as represented in Fig. 170. It follows from the character of the disturbing force that the small mass will be turned in the direction of revolution of S'. The amount of turning depends upon the mass of S', the eccentricity of its orbit, the nearness of its approach to S, its distance from the perihelion at the time the matter is ejected, and the velocity with which it is ejected. It is a difficult and laborious process to compute the curve which will be described in any particular case, but the computations so far made indicate that the ejected masses will be left moving in elliptical orbits after S' has receded so far that its disturbing influence is no longer of importance. Precisely similar remarks apply to the matter which is ejected in the direction opposite to S'.

369. The Formation of a Spiral Nebula. — The material is presumably ejected at frequent intervals during the whole time that S' is in the neighborhood of S. The separate parts will have traveled along the paths indicated by dotted lines in Fig. 171. The particles which were ejected first when S' was yet at a great distance moved in the small curves marked 1 and $1'$. As S' approached its perihelion the matter was ejected with greater velocities and described the curves marked 2 and $2'$. The particles describing the curves 3 and $3'$ were ejected when S' was nearest S. The remainder

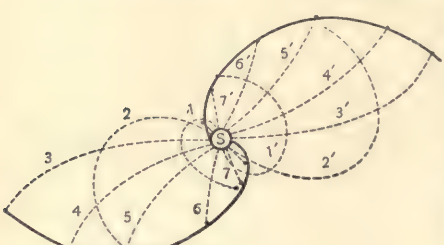


Fig. 171.

left S after S' had passed its perihelion and when its distance from S was increasing. These curves are smaller, something like those described by the particles first ejected. When we see a spiral nebula we do not see the paths which the separate masses have described, but the positions which

FIG. 172.—Spiral Nebula in Canes Venatic (M. 51), showing the Two Arms. *Photographed at the Lick Observatory.*

they occupy at the time. In the present case if a smooth curve is drawn through the regions where the matter is densest, it will form a sort of double spiral as represented by the full lines. There will be nuclei here and there along the arms of the spiral where large masses have been ejected, and the whole space will be more or less filled with finely divided and nebulous material. It must be remembered that

the matter does not move along the arms of the spiral, but in orbits which cross them at large angles. The particles in the smaller orbits will move faster than the outer ones, and the spiral will become more and more coiled with age until its spiral character can no longer be discerned. In the photographs of spiral nebulas the *two arms* can nearly always be easily made out, and it is significant that no other number is certainly found. But it is almost certain that the spiral nebulas which have been photographed are much greater than the one from which our system may have developed.

370. The Development of the Solar System from a Spiral Nebula. — The consequences of the hypothesis that the solar system has developed from a spiral nebula of the type just considered will now be worked out, and the results will be compared with the known condition of the system.

For simplicity in the exposition, the statements will be made positively as though there were no further question about the evolution of the system, and as if this were simply an account of its development. All those phenomena which were enumerated as supporting or opposing the Laplacian hypothesis will be reviewed in connection with the spiral theory.

371. The Origin of Planets. — The various planets have grown out of the original nuclei by the gradual accretion of the smaller particles whose orbits crossed, or passed near, their orbits. The larger nuclei, especially those that passed through regions rich in the finer material, gave rise to the larger planets. Marked irregularities in the masses are to be expected rather than the opposite. The planetoids have formed from a mass of material with no large dominating nucleus.

372. The Origin of Satellites. — When the planetary nuclei left the sun they were accompanied by smaller secondary nuclei. Those secondary nuclei which had large velocities with respect to their primary nucleus escaped from its gravi-

tative control and became independent bodies. When their relative velocities were very small, they fell upon the primary nucleus. When their velocities were moderate, they became satellites.

373. The Planes of the Planetary Orbits. — The planes of the orbits of the planets are very nearly the same as the plane of the orbit of S' when it passed by S. It is clear that small differences in the planes are to be expected because of the various conditions which control the direction of projection, and that the greatest difference may well be in the nearest planet whose material was for a shorter time under the influence of S'. Moreover, the process of sweeping up the scattered material, which in general would be distributed symmetrically with respect to the plane of the orbit of S', would tend to reduce the planes of the orbits of the various bodies to coincidence. In general, the more a nucleus grew by sweeping up the scattered material, the more nearly the plane of its orbit coincided with the general plane of the system.

Turning to the observational data we actually find the orbit of Mercury more highly inclined to the average plane of the system than that of any other planet. The orbits of all the great planets are nearly in the same plane. On the other hand, the planes of the orbits of the planetoids are often highly inclined. For example, the orbit of Eros is inclined about 10° to the orbits of the earth and Mars, an unexplainable condition under the Laplacian theory.

374. Rotation and Equatorial Acceleration of the Sun. — The present rotation of the sun is the resultant of its rotation before the appearance of S', and of the disturbances produced by this body. Its original axis of rotation is unknown, but it is very improbable that it was exactly perpendicular to the plane of motion of S'. S' disturbed the rotation of S in two ways. First it raised an enormous tide in S which it pulled around S in the direction of its revolution. In this

way it contributed a large moment of momentum around an axis perpendicular to the plane of its orbit, and this moment of momentum has been constant ever since. In the second place, a considerable quantity of the material which was ejected and had its straight line orbit changed to an elliptical orbit would still have its perihelion distance less than the radius of S. Consequently, it would be precipitated again on the sun, and inevitably in such a way as to increase the moment of momentum of the sun around the same axis. This cause may have been more efficient in determining the character of the sun's rotation than the other. Hence, since the sun's present rotation is the resultant of its original rotation and these disturbing factors, we should expect to find its equator near, but not exactly in, the average plane of the planetary orbits.

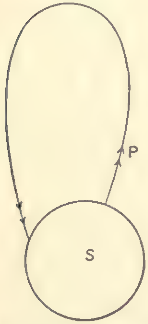


FIG. 173.—Ejected Matter falling back on the Sun gives it Moment of Momentum in the Direction of Revolution of S'.

Both of the factors which have just been noted were more important in the equatorial zone than in any other. Consequently there was an original equatorial acceleration which has not yet been worn out by friction on the lower parts. The spots are in those zones where the relative motions of the different layers are the greatest, and they are probably due in some way to these relative motions.

375. The Small Eccentricities of the Planetary Orbits. — The eccentricities of the orbits of the nuclei were on the average undoubtedly greater than those of the orbits of the present planets. This decrease in eccentricity has been brought about by the accretion of the scattered material. This subject offers some difficulties and it is not possible in an elementary work to give an exhaustive treatment of it. However, the general order of ideas will be explained, and the argument will be made conclusive in the most important case.

It follows from the mode of generation of the spiral that its separate bodies were moving in a great variety of orbits. Consider the orbits of a nucleus M, and of a small body m intersecting it at P_1 and P_2. If a collision occurs at P_1 the body M will move in the direction P_1Q_1, and if at P_2 in the direction P_2Q_2. When the collision is at P_1, the orbit of M will intersect the radius more nearly at right angles than it did before. But this alone is not sufficient to make it more circular, for the velocity in a circular orbit passing through P_1 is greater than that in the ellipses at that point. When the collision is at P_2, the orbit intersects the radius at a more acute angle than it did before. But this does not necessarily mean that the orbit is more eccentric, for the velocity in the ellipses at this point is greater than it is in the circle through it. This follows from the equation given in celestial mechanics for the square of the velocity,

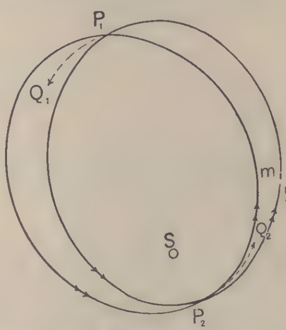


Fig. 174.—Reduction of the Eccentricity of the Orbit of a Nucleus by Collisions.

$$V^2 = \frac{2}{r} - \frac{1}{a}, \tag{1}$$

where the mass of the sun is taken as unity, and where a is the major semi-axis of the orbit.

Let us consider the case of a nucleus M moving in a certain orbit whose semi-axis is a_0 and whose eccentricity is e_0. Suppose there is a small mass m moving in an ellipse whose semi-axis is also a_0 and whose eccentricity is also e_0. We shall make no restrictions upon the way in which the two ellipses intersect each other. The problem is to find the effect of the union of m and M by collision upon the eccen-

tricity of the orbit of M. This is the most important case, for every nucleus will move in a region which is rich in material describing similar orbits, and the chances of encounters are greatest when the orbits are most similar.

The kinetic energy of the two bodies before their union is

$$\tfrac{1}{2}\,(MV_0^2 + mv^2),$$

where V_0 and v are the velocities of M and m at the instant preceding collision. Their kinetic energy after their union is

$$\tfrac{1}{2}\,(M + m)\,V^2,$$

where V is the velocity of the united mass. Now after their union the kinetic energy will be less than it was before, for a certain part of it will have been transformed into heat by the impact of the two parts. Hence

$$MV_0^2 + mv^2 > (M + m)V^2.$$

Hence, at the instant just following collision we may write as a consequence of (1),

$$M\left(\frac{2}{r} - \frac{1}{a_0}\right) + m\left(\frac{2}{r} - \frac{1}{a_0}\right) > \left(M + m\right)\left(\frac{2}{r} - \frac{1}{a}\right),$$

where a is the semi-axis of the new orbit. It follows from these equations that

$$\text{(2)} \qquad \frac{M + m}{a_0} < \frac{M + m}{a},$$

or

$$a < a_0.$$

The r is the same in each term, for the distance of both bodies from the sun at the instant preceding collision was the same as their common distance the instant after collision.

It is shown also in celestial mechanics that the sum of the

products of the masses and the areal velocities is a constant, and this is true whether there are collisions or not. (These statements are subject to slight modifications if the rotation of M is changed.) It is another statement of the fact that the total moment of momentum of the system is a constant. The formula which expresses the fact that it was the same before collision as after is

$$(3)\quad M\sqrt{a_0(1-e_0^2)} + m\sqrt{a_0(1-e_0^2)} = (M+m)\sqrt{a(1-e^2)}.$$

Since $a < a_0$, it follows that

$$\sqrt{1-e_0^2} < \sqrt{1-e^2},$$

whence

$$e < e_0.$$

This result is true however the motions of the bodies may have been disturbed by their mutual perturbations before their collision. Hence we have the general result: *If any two bodies whose orbits have the same major axes and eccentricities are subject only to their mutual perturbations and unite in any way by a collision, the orbit of the combined mass will be less eccentric than the original orbits were.* When the conditions of equality of axes and eccentricities are anywhere nearly fulfilled, the same results follow in a very great majority of cases.

There are cases in which a collision will increase the eccentricity of the nucleus, but unless the matter is distributed in an extraordinarily special manner, they are relatively very few. It follows that in general the more stray matter a nucleus has swept up the more nearly circular its orbit will be. Now among the planets the smallest one of all, Mercury, has an orbit more than twice as eccentric as that of any other; and the next smallest planet, Mars, has the next most eccentric orbit. The orbits of the earth-like planets average more than twice as eccentric as the orbits of the great planets. In contrast with the small eccentricities

of the orbits of the planets, we find that the planetoids move in orbits whose average eccentricity is three times as great as the average eccentricity of the orbits of all the planets. About one planetoid out of four has an orbit more eccentric than that of Mercury, a fact quite inexplicable under the Laplacian theory. It is not remarkable in the light of the present theory that the orbits of these small bodies are looped through one another in a complicated fashion.

376. The Rotations of the Planets. — The character of the rotation of a planet depends upon the direction and rate of rotation of its original nucleus, and upon the effects of the collisions of the scattered material with it. There is no apparent reason for assigning any particular direction of rotation to an original nucleus, or of supposing that its angular velocity was large. Chamberlin has advanced and developed the idea that the rotations have been made forward by the impacts, and this question will now be considered.

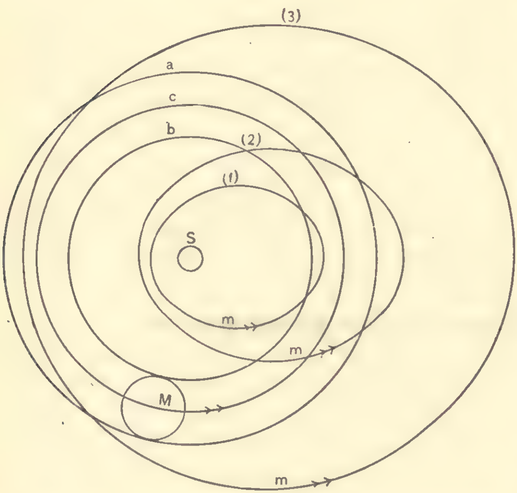


FIG. 175. Collisions of Scattered Material give the Planets Forward Rotations.

For simplicity the nucleus may be assumed to be revolving around the sun in a circle, though if this assumption were

not made, the discussion would not be essentially different. Suppose the nucleus M revolves around the sun so as to fill the space between the curves a and b, and so that its center travels along the circle c. For this discussion the orbits of the particles will be divided into three classes. The first class will consist of those whose aphelion points do not lie outside of c; the second class will consist of those whose perihelion points lie within c and whose aphelion points lie outside of c; the third class will consist of those which lie entirely outside of c. They are designated as (1), (2), and (3) respectively in Fig. 175.

It follows from equation (1) of the last article that the small bodies describing orbits of the first class will always be moving more slowly than M when the collision takes place. Consequently M will overtake m, the collision will be on the side of M toward the sun, and the tendency will be to give M a *forward* rotation.

The small bodies describing the orbits of the second class will be moving faster or slower than M, according as their semi-axes are greater or less than the distance from S to c. In either case the collision may tend to make the rotation either direct or retrograde, and in the long run the two tendencies will nearly balance. Besides this, the effects on the rotation will be small because the impacts will generally be nearly in the line of centers instead of tangential.

The bodies describing the orbits of the third class will always move faster than the nucleus at the time of collision. Consequently, since they overtake it outside of the center of gravity they tend to give it a *direct* rotation. To summarize, the two classes of bodies which are most efficient in changing the rotation of the nucleus by impact tend to make it *direct*, while the effects of a much larger intermediate class are unimportant because they mutually destroy one another.

On the basis of this discussion it follows that we should expect to find the planets rotating in the same direction as

they revolve. Any radical departures from this rule would be expected, if at all, in the outermost planets, where the orbits of the third type have been very few. These effects on the rotation for a given amount of material have been greater the larger the nucleus, both because of the greater differences in velocity at the instants of impact, and also because the effectiveness of a given force in changing the rotation is greater the farther it acts from the axis of rotation. The general agreement of these results with the observed phenomena is too evident to require comment. On the other hand, the direct rotations of the planets have always given trouble under the Laplacian hypothesis.

377. The Direction of Revolution of the Satellites. — When the planetary nuclei left the sun they were attended by the smaller secondary nuclei. While in any particular case the secondary nuclei may have had some common direction of revolution around their primary nucleus, there is no apparent reason for supposing that there was any general direction of revolution. Consequently, we shall examine the various cases which could have arisen.

For the purposes of the discussion we shall divide the orbits of the secondary nuclei into three classes depending upon the positions of the planes of their orbits and their directions of revolution. (*a*) The first class will consist of those whose orbits were highly inclined to the planes of motion of their respective primary nuclei; (*b*) the second class, of those which revolved nearly in the planes of motions of the primary nuclei and in the forward direction; (*c*) and the third class, of those which revolved nearly in the planes of motion of the primary nuclei and in the retrograde direction.

(*a*) Let us consider the effect of the scattered material upon the secondary nuclei of the first class. Whenever one of these nuclei passes through the plane of motion of its primary it encounters the scattered material, which acts

upon it like a resisting medium and decreases its velocity. The result of the resistance to its motion is a decrease in the size of its orbit. In addition to this, the mass of the primary nucleus continually increases by the accretion of meteoric matter, and this also decreases the size of the orbit of the secondary nucleus. These two causes always operating in the same direction would in time precipitate the secondary nucleus upon its primary. Of course, the farther it was originally from its primary, the better was its chance of maintaining its separate existence.

(*b*) Consider a satellite nucleus having a direct revolution in the plane of motion of its primary. The small particles

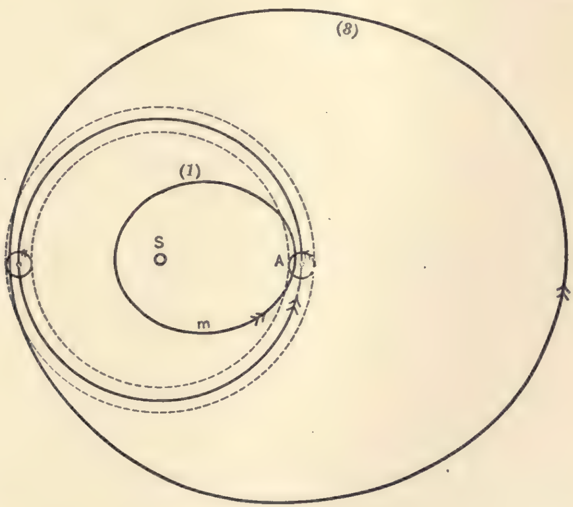


FIG. 176.

which it may encounter will be divided into three classes: (1) those whose orbits are entirely within the orbit of the planetary nucleus M; (2) those whose orbits cross the orbit

of M; and (3) those whose orbits are entirely outside of the orbit of M. In Fig. 176 the full circle represents the orbit of the planetary nucleus M, the dotted circles the limits of the orbit of the satellite nucleus as it revolves around M, and (1) and (3) the orbits of particles of the first and third class respectively. The orbits of particles which lie between these two are not shown.

The particles of class (1) will encounter the satellite nucleus at (or near) A. The problem is to find which will be moving the faster at this point. The satellite nucleus is carried forward by the motion of M, while it moves backward in its revolution around M. The latter is a much slower motion than the former. The velocity forward is the difference of the two velocities. Since the major semi-axis of m is less than that of M, it follows from equation (1), Art. 375, that it moves more slowly than M does. If its major axis is considerably less than that of M, it will move more slowly than the difference in the motion of M and the satellite nucleus. In this case the satellite nucleus will overtake m. But it follows from the direction of motion of the satellite nucleus that in this case its motion around M will be accelerated by its collision with m. It is found by a mathematical discussion that this always results if the eccentricity of the orbit of m is greater than

$$\frac{r}{R} + 2\sqrt{\frac{MR}{r}},$$

where R is the radius of the orbit of the planetary nucleus around the sun, r the radius of the satellite nucleus around M, and M the mass of the planetary nucleus expressed in terms of the sun's mass. In the case of the earth and moon the limit comes out 0.035, but in the case of the larger planets and closer satellites it is very much larger. How-

ever, when the collisions were most numerous, the planetary nuclei, M, had not yet grown to their present size. Since the orbits of the scattered material must have been as a rule considerably eccentric, it follows that the collisions on the whole accelerated the satellite nuclei.

The formula and results are the same for the particles moving in the orbits of the type (3). The results are varied in the case of particles moving in the orbits (2), which are intermediate between the two classes considered.

The effect of the accelerations by the scattered material is to enlarge the orbit of the satellite nucleus, and to prevent its being drawn down upon the growing planetary nucleus. For this reason the satellite nuclei of this type are likely to retain their separate existence and to develop into satellites.

(*c*) The satellite nuclei having retrograde revolutions are acted upon in the opposite way from those having direct revolutions. Therefore their orbits will continually shrink both because of collisions with the scattered material, and also because of the increasing attraction of M as its mass grows. These satellites will for these reasons in general be precipitated upon their respective primaries. Conditions will be favorable for their survival only if the scattered material is distributed in a very special way, or if they are started at a very great distance from their primaries.

It follows from these discussions that satellites may revolve around their primaries in any direction, but that the chances of surviving as independent bodies are against those satellite nuclei which do not revolve in the forward direction. The same planet may have satellites moving in both directions.

378. The Eccentricities of the Satellite Orbits. — In general, the orbits of the satellite nuclei were originally quite eccentric. It will now be shown that the orbits of those having direct revolution are rendered less eccentric by the impact of the planetesimal material.

Let S be the sun and C the curve in which the planetary

nucleus revolves around it (Fig. 177). Let P_1 be the position of the planetary nucleus when the satellite orbit has its farthest apse outside of C, and P_2 its position when this condition is reversed. If the orbit were filled with a body rotating as a solid around P_1, the infall of meteors would tend to increase the rotation in the same direction as was shown in Art. 376. Similarly, the effect on the satellite nucleus is to accelerate its motion, and this effect is the greatest when the nucleus is farthest from P_1.

The effect of an acceleration of a body when it is at the farthest point of its orbit is to increase the distance of the nearest point and to make the orbit more circular. Thus, in Fig. 178, if the body were originally moving in the closed orbit, and were accelerated when at s, the distance of s' from P would be increased and *the orbit would be less eccentric.* The same remarks apply when P has moved half-way around its orbit and conditions are as represented at P_2 (Fig. 177). That is, the impacts of scattered material upon satellite nuclei having a forward revolution increase their velocities, but most when they are most remote from their respective primaries; *the results of the accelerations are that these satellite orbits are enlarged and made less eccentric.* It is to be noted that the results are the opposite in the case of satellite nuclei having retrograde revolutions.

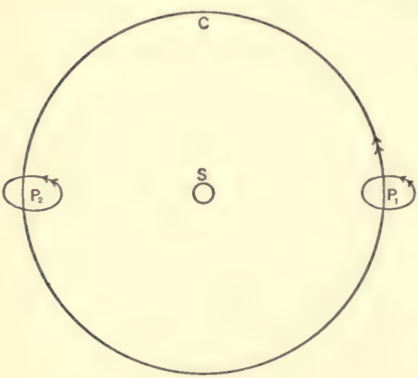


FIG. 177.

In the actual system the satellite orbits are found on the whole to be very nearly round. By far the most eccentric

orbit of those satellites whose motion is direct is that of Hyperion, which is very greatly perturbed by the large neighboring satellite Titan. The eccentricity of the orbit of Hyperion is 0.12. But the ninth satellite of Saturn, which revolves in the retrograde direction, has an orbit whose eccentricity is 0.22. The high inclinations of the orbit of the satellites of Uranus to the orbit of this planet would not have been anticipated on the basis of this theory; still they are not definitely contradictory to it as they are to the Laplacian theory.

379. The Inner Satellite of Mars and Saturn's Rings. — The inner satellite of Mars was originally a small nucleus circulating very near the nucleus of Mars. Its period was longer than it is at present, and it steadily decreased with the increase of the mass of the planet. The planet never extended as a continuous body out even to the orbit of this satellite. Phobos is outside of Roche's limit, and there is no difficulty in its short period of revolution.

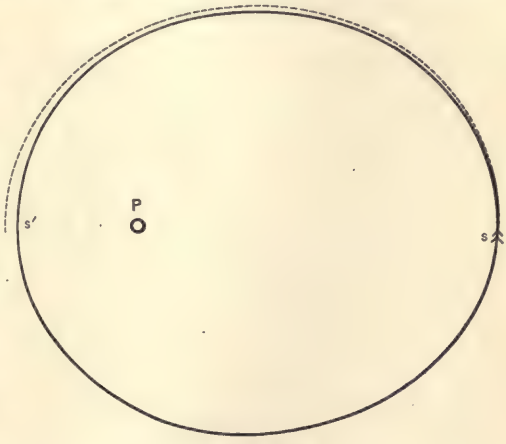


FIG. 178.

The rings of Saturn have developed out of material originally revolving close to the nucleus of the planet. They are inside of Roche's limit, and the disrupting tendencies of the tidal strains have more than balanced the collecting tendencies of the mutual gravitation of their parts. Collisions

have pulverized the material and have destroyed the divergences of motion until all the particles move nearly in the same plane.

380. The Moment of Momentum of the System. — The moment of momentum of the system is nearly all possessed by the planets, Jupiter having over 95 per cent of that which is within Saturn's orbit. This condition of affairs is precisely that which the postulated origin of the spiral nebula would lead to, and it is entirely in agreement with the theory which has been outlined. The total moment of momentum of the system is the measure of the disturbances produced by the sun S'.

381. The Zodiacal Light. — The zodiacal light is probably the light reflected from the vast number of particles moving in orbits so nearly parallel to that of the earth that they have been swept up very slowly. Many of them are the remains of the original material scattered by S', though some of them are the remains of disintegrated comets. The large meteorites which fall upon the earth, often containing large quantities of occluded gases, are probably part of the material ejected from the sun, though some of them give very strong evidence of having been once parts of a large solid body. They may have come from planets which existed before the advent of S', and which were broken up and destroyed by this body.

382. The Evolution of the Planets. — The evolutions of the small and large planetary nuclei have been quite different. They were all very hot at the time of their ejection. The small nuclei did not have sufficient gravitative control to retain their lighter gases. In a comparatively short time they had no appreciable atmospheres, and they speedily cooled until they became solid. The meteoric matter which fell in upon them was also in a solid state. The relative velocity was in general so small that no great amount of heat was generated by the impact, and what was produced speed-

ily radiated away. After the masses began to assume earth-like dimensions the interior pressure became very great and they diminished in volume. This shrinking produced *interior* heat just as it does in the case of the sun. Computation shows that if the earth shrank so that its density increased from the average density of known meteorites (3.5) to its present density (5.5), enough heat would be generated to increase its temperature (assuming a specific heat of 0.2, which is about that of rock) more than 10,000° Fahrenheit. This heat would be conducted to the surface and lost very slowly, and it is much more than sufficient to account for all the known igneous action in the case of the earth. But the earth as a whole has been solid throughout its history.

The earth acquired its atmosphere chiefly after it became about as large as Mercury. The atmospheric gases came from the interior, squeezed, as it were, out of the heated and compressed material. Bodies much smaller than Mercury have never retained any real atmospheres. This applies to most of the satellites and to all of the planetoids.

On the other hand, the large planetary nuclei were so massive that they never lost their light gaseous envelopes. Because of this their original heat was largely retained, and they have not yet contracted to any great extent. They are less dense than the smaller planets both because they retained nearly all of the original light elements, and also because the conditions have been unfavorable to their cooling and contracting.

383. The Age of the Solar System. — No certain answer can be given to the question of the length of time that has been required for this evolution of our system to take place. It is certainly very great. The greatest difficulty is in accounting for the apparently undiminished vigor of the sun. If it was in its maturity at the time of the visitation of S', apparently it ought to be far in its decline now. The

only explanation available, and the one which had to be evoked also in the Laplacian theory, is that probably the contraction theory of the sun's heat accounts for only a small part of the energy which becomes available in this form (see Arts. 318 and 319).

384. The Future of the System. — That part of the evolution of the system in which the scattered materials were gathered into the various nuclei seems to be sensibly completed. The future will probably be very much like the present until the sun's store of energy becomes reduced to such an extent that its radiation appreciably diminishes. Then an age of increasing frigidity will come on, and, so far as we can see, will last indefinitely unless some influence exterior to the system becomes operative.

One event seems probable. Unless the motions of the stars are ordered in some special way so that these bodies never approach one another closely, a condition which now is not even suspected, our sun at some remote epoch will pass near some other sun. If this second sun is in its youth a new spiral nebula will be formed, probably involving at least a part of the matter now making up the remote planets of our system. In this manner the conditions for a new evolution may be developed. If the two suns are both dark, they still may be more or less broken up and heated by the work of the body tides.

Formerly the sidereal evolution was supposed to be almost entirely a process of aggregation, and astronomers pictured to themselves how sun will add to sun by collision until the material universe will end in one or a few immense suns which will eventually become cold and lifeless. But the chances of near approach without collision are enormously greater than those of actual collision. A near approach produces a breaking up of suns and the scattering of material instead of a greater aggregation of it. Now we recognize the chances for dispersion as well as the less favorable ones

for concentration. In addition, we are just now finding in connection with radio-active substances that some atoms at least contain within themselves forces which lead to their dissolution.

385. Summary. — The first word should be one of warning that the theory which has been sketched briefly should not be accepted as final. There are many points where quantitative results must be obtained and compared with our actual system. There may be many modifications of it possible and necessary. For example, the genesis of spiral nebulas may be different from that postulated above.

The hypothesis of an original spiral nebula is suggested by recent photographs of nebulas as well as by the system itself. The conditions which are supposed to have given rise to the spiral nebula seem most reasonable in view of the motions of the stars. The development of a spiral nebula by the near approach of two suns seems to be a necessary consequence, though this point needs further elaboration. The development of some such a system as ours from a small spiral nebula of the type considered seems to be inevitable. So far as the details have been worked out nothing directly contradictory to the theory, or even seriously questioning it, has been found, while it explains admirably all the main features of the system. It can be safely said that, at present, this hypothesis satisfies all the requirements of a successful theory much better than any previous one.

QUESTIONS

1. Explain the fundamental difference in the dynamics of the Laplacian nebula and of the spiral nebula.

2. Explain carefully why the greatest outbursts of material from the disturbed sun will be directly *toward* and *from* the disturbing sun.

3. Would both suns be turned into spiral nebulas by a near approach?

4. Construct diagrams showing the disturbing acceleration for four different positions of P (Art. 367).

5. What facts of the system are more satisfactorily explained under the Laplacian hypothesis than they are under the spiral nebula hypothesis? (Make a complete examination.)

6. What facts are more satisfactorily explained under the spiral nebula hypothesis than they are under the Laplacian hypothesis?

7. Draw a figure to represent a case where a collision will increase the eccentricity of the orbit of a nucleus.

8. What explanation can be suggested for the equatorial accelerations of Jupiter and Saturn?

9. Why should we expect that the large planets would rotate more rapidly than the small planets?

10. Why should we expect the planetoid orbits to be on the average more eccentric than the planetary orbits?

CHAPTER XVI

THE STARS AND NEBULAS

386. Problems of the Stars and Nebulas. — The problems relating to the stars are their numbers and magnitudes, their apparent distribution, their proper motions, their grouping into systems, their distances and actual motions, their spectra and constitution, and their evolution. The problems of the nebulas are their dimensions, character, evolution, and their relations to the stars.

387. The Number of Stars of Various Magnitudes. — The magnitude of a star was defined in the chapter on Constellations. A first-magnitude star gives us as much radiant energy as the average of the 20 brightest stars in the sky. The light received from a first-magnitude star is $2.51 \cdots$ times that received from a second-magnitude star, and so on. The ratio of a first-magnitude star to a sixth is 100 to 1, and a sixth to an eleventh is 100 to 1, and so on for every five magnitudes. The ratio of a first-magnitude star to a sixteenth-magnitude star, which is nearly as faint as can be seen with our best instruments, is 1,000,000 to 1.

The magnitudes of the stars depend both upon their light-giving power and also upon their distances from us. Evidently the amount of light received from a star is directly proportional to the amount it radiates, and varies inversely as the square of its distance from us. There is another factor which is probably not entirely negligible, and that is

that the interstellar regions may not be absolutely transparent. If the ether does not transmit light waves perfectly, or if there is scattered meteoric or other material which absorbs light, the stars are fainter than they would otherwise be, and the loss of light is greater the farther they are away.

One method of measuring the magnitude of a star is to cut off its light by a wedge of neutral tinted glass until its apparent brightness equals that of an artificial star. The eye decides when the two stars have the same brightness. But suppose they are of different colors and that the eye is unequally sensitive to the two colors. Then the one which has the color to which the eye is the more sensitive will be actually fainter than the other when the eye decides they are equally bright. Since the eye is most sensitive to yellow light, the yellow stars will come out relatively too bright as compared with the blue stars. But the magnitudes of stars are also determined by comparing the dimensions of their images when they are photographed. Since ordinary photographic plates are very much more sensitive to light in the blue end of the spectrum than they are to light in the other end, the blue stars will come out relatively too bright by this method. Hence it is apparent that this problem is to some extent involved in the color of the stars.

E. C. Pickering is carrying out a survey of the whole heavens at Harvard and at a sub-station in Peru, for the purpose of determining as accurately as possible the magnitudes of all the brighter stars. Müller and Kempf, at Potsdam, are carrying out a similar undertaking for the northern sky, though with instruments of a different type.

It has been found that, down to stars of the ninth magnitude, the number of stars of any magnitude is from 3 to 3.75 times as great as the number of stars in the next magnitude brighter (Art. 38). Since a star of any magnitude sends us $2.5\cdots$ times as much light as one of a magnitude fainter, it follows that the stars of each succeeding magnitude, down to

the ninth, send us more light than do those of the next brighter magnitude. Newcomb has computed that the stars of the first ten magnitudes send us 75 times as much light as we get from the stars of the first magnitude. Now if this ratio of increase, or any ratio greater than or equal to unity, were indefinitely kept up, the whole amount of light received from the stars would be an infinite number of times that received from the first-magnitude stars. That is, if the stars were scattered at random, the whole sky would be as bright as the average star, or something like the sun. The sky is quite dark, all the stars together giving us only about $\frac{1}{80}$ as much light as the full moon, and consequently there must be a very great falling off in the ratios of the numbers of stars below the ninth magnitude. Precise, or even approximate, numbers can not be given beyond the tenth magnitude, but it is certain that the ratio decreases very rapidly below the twelfth, and it is probable that in the first 16 or 17 magnitudes there are only about 100,000,000 stars. If the ratio which prevails down to the ninth magnitude were maintained, there would be more than ten times this number.

388. Apparent Distribution of the Stars. — The brighter stars are quite irregularly distributed over the sky; but a careful examination of them shows that they are considerably more numerous near the Milky Way than elsewhere. The condensation is even more marked when those faint stars are included whose combined light produces this hazy band of light entirely around the sky.

Precise numbers are not known for stars below the ninth magnitude, and the list is not complete thus far for the stars around the south pole. But the Galaxy runs diagonally across the sky, and a very satisfactory idea of the distribution of the stars with respect to it can be obtained, even if those in a particular declination are not included. Seeliger has made the count from the catalogues of Argelander and Schönfeld of all stars down to the ninth magnitude north of

declination $-24°$. He has divided the sky into 9 zones, each one 20° wide. Number I has its center in the center of the Milky Way. Then $+$II is 20° wide lying along the north side of zone I, and $-$II is of the same width and lies along the south side of zone I. Similarly $+$III and $-$III are zones 20° wide lying adjacent to $+$II and $-$II respectively. Zones $+$V and $-$V are around the north and south poles of the Galaxy respectively. Celoria has counted about 200,000 stars between the equator and $\pm 6°$ declination. Many of his stars are much fainter than those used by Seeliger, but they have the disadvantage of having been taken from a very limited region of the sky. However, they show a very marked condensation toward the Galaxy. The results are given in the following table: —

Zone	No. of Square Degrees in Count		No. of Stars counted by		Average No. of Stars per Square Degree	
	Seeliger	Celoria	Seeliger	Celoria	Seeliger	Celoria
I	4520	285	32,267	41,820	7.4	147
+II	4590	255	24,492	29,469	5.3	116
−II	3972	285	23,580	31,706	5.9	111
+III	5127	285	19,488	22,551	3.8	79
−III	2954	330	11,790	25,618	4.0	78
+IV	3147	404	10,185	27,352	3.2	68
−IV	1791	314	6,375	22,264	3.6	71
+V	1399	0	4,277	0	3.1	—
−V	468	0	1,644	0	3.5	—

Two things follow from this table. One is that the stars are much more numerous in the region of the Milky Way than they are elsewhere. The other is that the change is on the average gradual from the Milky Way to its poles. When fainter stars are used the disparity in numbers in the two

regions is still more marked. The counts made by the Herschels in many parts of the sky showed that the stars revealed by their telescopes are nearly 20 times more

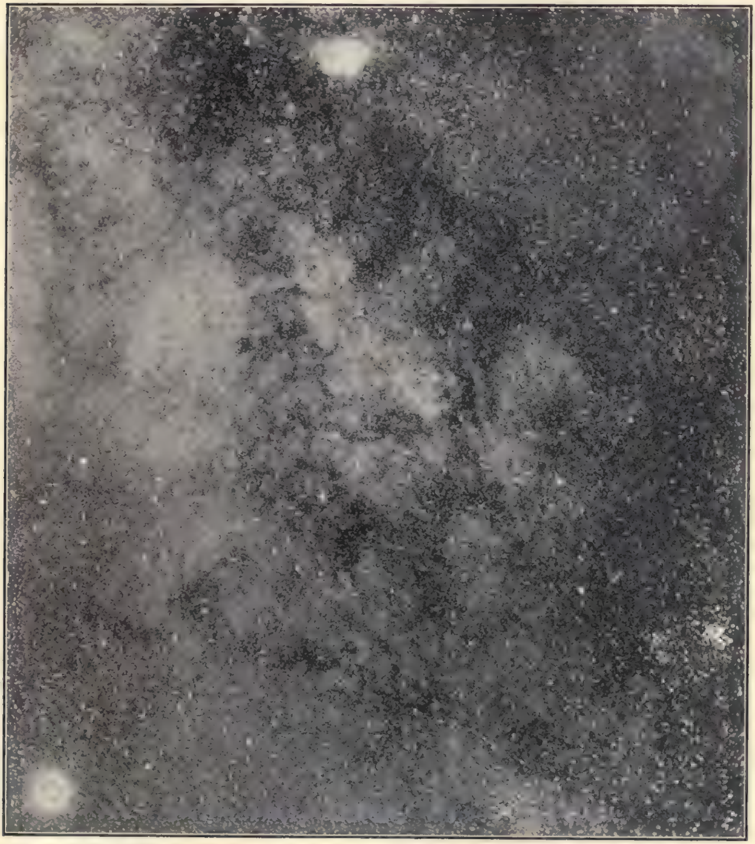


Fig. 179. — Milky Way in Sagittarius. *Photographed by Barnard.*

numerous in the Milky Way than they are 90° from it. At the poles of the Milky Way, Celoria saw with his small tele-

scope nearly as many stars as the Herschels did with their large ones, showing that in these regions the ratio in numbers for the faint stars very rapidly decreases. But in the Galaxy itself Celoria found only one-fourteenth as many as the Herschels had observed.

389. Proper Motions of the Stars. — The drift of a star with respect to a fixed system of reference lines is called its *proper motion.* It is expressed in angular measure and depends upon the component of velocity of the star perpendicular to the line joining it with us, and upon its distance from us. Other things being equal, the more distant the star the less its proper motion.

Most of the bright stars have proper motions which are sensible in the course of a few years with our modern instruments of precision; yet they are so small that the stars are really quite "fixed." The largest proper motion known is that of an eighth-magnitude star in the southern sky which drifts $8.7''$ annually. At this rate it will take more than 2000 years for it to move as far as the apparent distance between the pointers of the Big Dipper. Perhaps an even better idea of the slowness of this motion is given by the fact that the proper motion of this star amounts in 25 years only to the apparent distance between the components of Epsilon Lyræ. The great majority of the stars, even of those brighter than the ninth magnitude, move so slowly that their proper motions are given in seconds of arc per century.

The study of the proper motions of the stars has shown one very interesting fact. Kapteyn has examined the stars observed by Bradley, and he finds that those whose proper motions per century are $5''$ or more are very uniformly scattered over the sky, while those whose proper motions are less than this amount cluster in the region of the Milky Way. Newcomb has examined the more extensive lists observed by Boss and Auwers, and he has found that those stars whose proper motions are as great as $10''$ per century show no tendency

whatever toward a condensation in the region of the Milky Way. While the brighter stars have on the average larger proper motions than the faint ones, there are stars of all magnitudes having proper motions greater than 5″ per century. Hence if all those stars which have no sensible proper motions were destroyed, the sky would be strewn nearly uniformly with stars of all magnitudes. These are

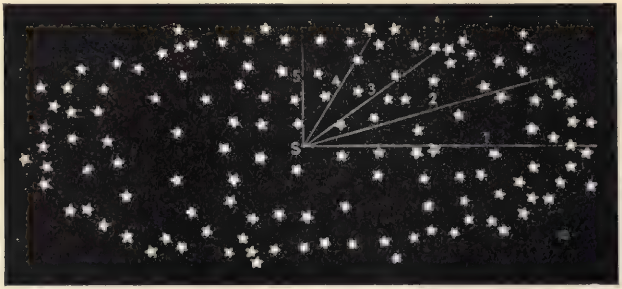


FIG. 180. — Herschel's Conception of the Milky Way.

very important facts, for if the proper motions are in a way a measure of the nearness of the stars, as seems reasonable, then it follows that those stars which really belong to the Milky Way are very far from us.

390. The Structure of the Milky Way. — Two general theories respecting the structure of the Milky Way have been advanced, and each is supported by certain facts. William Herschel advanced the so-called "grindstone" theory, which asserts that the stars which make up the Galaxy are spread out somewhat uniformly in the form of a vast disk, whose diameter is many times its thickness. The solar system is somewhere in its interior, and near enough its center so that it appears nearly equally thickly studded with stars all the way round. This explains why the stars should apparently be most numerous in the plane of the Milky Way, and why

the numbers should diminish gradually as its poles are approached.

Thus Fig. 180 represents a cross section of a disk with the solar system at S. The lines 1, 2, 3, 4, 5, show five directions, and the corresponding distances before the borders of the system are passed. If the stars are distributed uniformly, the apparent numbers in the different directions will be proportional to the lengths of these lines. The general agreement with the facts given above is quite evident.

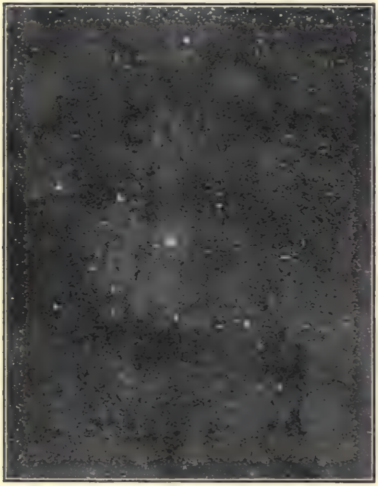


FIG. 181. — Milky Way in Ophiuchus, showing Rifts. *Photographed by Barnard.*

The things which oppose Herschel's theory are that the Milky Way is extremely irregular. In the first place, its width varies greatly. This means that in some directions from the sun the disk must be pinched down even out to its remotest borders, while it may be very wide in the opposite direction. Besides, it is divided for a long distance, which means, if this theory is true, that the division must be fairly complete from perimeter to center. More troublesome still are the numerous dark rifts in it which are almost devoid of stars. They can be seen on many photographs, and often in the vicinity of dense star clouds. This means, if the theory is true, that there are tunnels converging toward the solar system, which extend through the entire disk of stars, a supposition which is extremely improbable.

The other theory is that the Milky Way is a vast belt of stars, all about equally distant from us, encircling the sky. Of course, there are many stars which do not belong to this belt. This explains the general appearances, and the irregularities and dark holes do not cause serious difficulty, for they are partial rifts in the belt of stars; but the fact that the average number of stars per square degree decreases regularly from the Galaxy to its poles is quite contradictory to this hypothesis respecting their distribution.

The trouble with both theories seems to be that they are too simple. It appears more probable that there are a number, if not many, belts of stars approximately in the same plane, and besides a large number of other stars more uniformly distributed. Thus Fig. 182 shows a cross-section of the

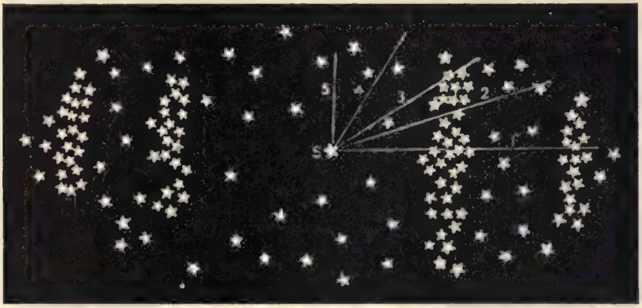


Fig. 182. — Cross-section of the Milky Way according to the Belt Theory.

galactic region according to this conception of the distribution of the stars. *S* is the sun, and the lines 1, 2, 3, 4, 5, reach out to the boundary of the stars in different directions. It is evident that the stars are most numerous in the direction of the Milky Way, and that they gradually diminish in numbers toward its poles. There are, besides, opportunities for rifts anywhere in the Milky Way.

According to Gould, the sun probably belongs to a belt of about 400 stars, which encircle the sky in a plane inclined

about 20° to the plane of the Milky Way. It intersects the Milky Way at its most northerly and most southerly points, and passes through Orion, Canis Major, Carina, the Southern Cross, and Scorpio south of the celestial equator, and through Lyra, Cygnus, Cassiopeia, and Taurus north of it. There are many bright stars in this region which, compared with the distances to the stars of the Milky Way, are relatively near one another.

391. The Parallaxes of the Stars. — Since the earth revolves around the sun the stars are apparently in slightly different directions from it at different times of the year. The difference in direction of a star as seen from two points on the earth's orbit which are separated by the mean distance to the sun is the *parallax* of the star. In other words, the parallax of a star is the angle subtended by the major semi-axis of the earth's orbit as seen from the star.

If the earth revolves around the sun, the stars will have parallactic displacements. Hence their parallactic motions prove the heliocentric theory, and all of the early attempts to find a star with a measurable parallax were made for the purpose of settling this question. But now the heliocentric theory is firmly established, and parallax measurements are made for the purpose of finding the distances to the stars and the structure of the sidereal universe.

The direct method of finding the parallax of a star is to find the way its right ascension and declination vary throughout the year as a consequence of the earth's revolution. Between 1835 and 1840, Bessel, Struve, and Henderson followed this method, and each found almost simultaneously a star whose parallax was sensible. The stars are all so very remote that their parallaxes so far as known are less than 1″, and the difficulties in detecting such small variations after all the changes the observer and instruments undergo in six months are very great. This method has given positive results in only a very few cases.

Another method which avoids this difficulty is to find the parallax of one star with respect to another. Thus, in Fig. 183, suppose S is the sun, and that A and B are two points on the earth's orbit at the distance $\overline{AS}$ from each other. Suppose S' is the star whose parallax is desired, and that there is another star nearly in the same direction but so remote that the lines from A and B to it are sensibly parallel.

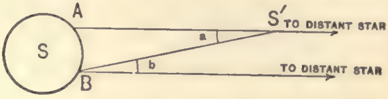


FIG. 183.

Suppose that, as seen from A, the star S' and the distant star are in the same line. When seen from B their apparent angular distance apart will be the angle b, which equals the angle a. Since a is the parallax of S', the observed angle b is also the parallax. It is evident that it is not necessary that S' and the distant star should be in a line with either A or B. The difference in their apparent distance apart as seen from the two points is the parallax of S' with respect to the distant star. Now it is not possible to determine in advance whether a star is so distant that it may be used in this way. The plan is to compare S' with a number of faint stars whose proper motions are small. The presumption is that some of them at least will be so distant as to be suitable for the determination of the parallax of S'.

A slight modification of this method is to photograph the stars at different times of the year and to measure their apparent distances apart on the photographic plates. Kapteyn inaugurated this method by measuring plates taken by Donner at Helsingfors. A telescope of great focal length is advantageous in this work because of the large scale of the photograph secured. The results obtained by Schlesinger with the 40-inch refractor of the Yerkes Observatory show that the possibilities of the method have not yet been fully realized.

So far 43 stars have been found whose parallaxes, as

measured, are $0.05''$ or over. The nearest one known is Alpha Centauri, whose parallax is $0.75''$. Since there are 206,265 seconds in a radian it follows that this star is $\frac{206265}{0.75} = 275{,}020$ times as far from us as we are from the sun. A star whose parallax is $0.05''$ is 15 times as far away. These statements mean that only a *very few* of the stars are within 400,000,000,000,000 miles of the sun. Beyond a doubt the great majority of them are many times this distance from us. Or, expressed in light years, the great majority of the stars are so far away that more than 66 years are required for their light to come across the vast space which separates them from us.

392. The Motion of the Sun. — Since the stars as a rule have proper motions, it follows that they move with respect to the sun as a reference point. But if the observer were in the vicinity of a star having a proper motion, our sun would appear to him to have an equal proper motion in the opposite direction. That is, motion so far as we know is purely relative, for we have no fixed points in space. But we may assume that on the average the stars move as much in one direction as another. This is equivalent to assuming that the center of the whole system of stars is at rest, and this assumption defines fixed space for us. Now we may attempt to find how the sun moves with respect to the system of stars, and this will be called *the motion of the sun.* Attempts have been made, chiefly by Michelson and Morley, to find the motion of our system with respect to the ether, but they have so far failed.

A little over a century ago Herschel found from a study of the proper motions of the stars that, on the whole, those stars which are in the direction of the constellation Hercules apparently are becoming more and more separated from one another, while those in the opposite part of the sky apparently are approaching one another. He interpreted this as meaning that the sun is moving toward the stars in Hercules, and it is

easy to see that this hypothesis explains the phenomena. The explanation is like that of the fact that the rails of a railway track appear closer and closer together the farther they are from the observer and apparently separate at any point as the point is approached. Herschel's results were derived from a limited number of stars and were subject to corresponding uncertainties. Moreover, only stars with large proper motions, and therefore presumably near the sun, were used.

More recently several other investigators, among whom may be mentioned Boss, Stumpe, Porter, L. Struve, and Newcomb, have determined the direction of the sun's motion. Their results agree approximately with one another and with those of Herschel, though there has been a tendency to place the apex of the sun's way a few degrees to the northeast of the point found by Herschel, and within 5° or 6° of the star Vega. The opposite point in the sky, or *antapex*, is in the general direction of Sirius. When stars with large proper motions are used, the apex of the solar motion comes out farther south than when those with smaller proper motions are used. That is, the motion of the sun with respect to the one group, which is presumably nearer to us, is different from its motion with respect to the other stars, which are farther away. This means that the one group is moving as a whole with respect to the other, and confirms the opinion that the sun is a member of a large group of related stars.

Another method of determining the solar motion is by spectroscopically measuring the radial velocities of the stars. Those in the direction in which the sun is moving will, on the average, relatively approach the sun, while those in the opposite direction will, on the average, recede. Campbell has recently made a determination of the apex of the sun's way from the radial motions of 280 stars. He found a point a little west and about 10° south of that given by the proper

motions of the stars. He necessarily used rather bright, and therefore presumably near, stars. This method gives not only the direction of the sun's motion, but also its velocity. The measurements of Campbell show that the velocity of the sun with respect to the system of 280 stars which he observed is about 13 miles (20 kilometers) per second. At this rate the sun moves about 4 astronomical units per year. It is very likely that the motion of the sun with respect to a greater number of stars is somewhat, though not greatly, different. There is nothing to lead us to suspect that we shall be far wrong if we assume that this is the rate of motion of the sun with respect to the center of gravity of all of the stars which are visible.

The stars are so remote that the sun will not move over a distance equal to that to the nearest one, Alpha Centauri, in less than 68,000 years. The relative rate of approach of the sun and Vega is about 10 miles per second, and the parallax of Vega is 0.11″. Consequently it will take the sun 558,000 years to pass by Vega. But Vega has a component of motion perpendicular to the line of sight of nearly 0.5″ per year. This is about 4 times its parallax, which is the angle subtended by the earth's radius as seen from the star. Consequently this component of Vega's motion is about 4 astronomical units a year, or a little more than its radial component with respect to the sun. It follows that, although we are moving nearly in the direction of Vega at present, we shall not pass very near this star. In fact, if Vega continues to radiate the same amount of light as it does now, the motion of our system with respect to it will never increase its apparent brightness so much as one magnitude, and it will be 275,000 years before it reaches its greatest brightness. This illustrates how little the stars will in general change in apparent brightness because of the motion of the sun.

393. Distances of the Stars determined from the Motion of

the Sun. — The stars are so very far away compared to the diameter of the orbit described by the earth that the parallaxes of only 43 of them so far have been found measurable. If the earth's orbit were as large as that of Neptune, the problem would be easier on account of the large base line which could be used. But the sun's motion can be made to afford an indefinitely large base line.

Suppose first that all the stars of the observable sidereal universe are relatively at rest except the sun. Its motion among them will give them an apparent displacement, or proper motion, in the direction opposite to that in which the sun is moving. The farther a star is away the smaller this proper motion will be. If it is so remote that no displacement can be observed in 1 year, then 10 years, 100 years, or any other interval, may be used. But when the proper motion, which under the hypotheses is entirely due to the motion of the sun, has been found, the distance to the star can be computed. Since the sun travels about 4 astronomical units per year, it follows that the parallax of a star is one-fourth of its annual proper motion.

The false hypothesis that all the stars except the sun are relatively at rest has greatly simplified the problem. As a matter of fact, the stars are moving with respect to one another, and the proper motion of a star is due both to its own motion and also to the motion of the sun with respect to the system. Since the actual motion of any particular star is unknown, it is necessary to take the average motions of many, and then the results will be consistent, for the motion of the sun is defined with respect to the many. This statistical study of the stars has been taken up by Kapteyn and pursued with rare skill and diligence. He takes a large list of stars, say all of a given range of magnitudes. He resolves their proper motions into two components, one in the direction of the sun's motion, and the other at right angles to it. The former components depend upon the motion of the sun,

while the latter are independent of it. Since it has been assumed that the average motion in every direction is zero, it follows that the average of the former components of velocity would equal the average of the latter if it were not for the sun's motion. Consequently, the difference in the two averages is the average of that part of the component of motion along the sun's path which is due to the sun's motion. From this result it is possible to compute the average parallax of the stars under consideration because the sun's velocity is known.

The principles which form the basis of this method of determining parallaxes are the only ones which give us much hope of finding out a great deal about the actual distribution of the stars. It is interesting to note the different kinds of work which contribute to the result. First, the whole problem of finding the apparent positions of the stars at different times is involved, for the results depend upon the proper motions of the stars. Then the spectroscope is employed in furnishing the velocity of the solar system, for without it the actual numerical results could not be found.

394. Distances of the Stars. — The ancients supposed that the stars are fairly near the earth. Tycho Brahe tried to detect their parallaxes with the crude instruments he had in his day, which was before the invention of the telescope. As he could observe no annual displacements, he took the results as meaning that the earth is stationary rather than that the stars are so remote that their displacements are insensible. Since the errors in his observations were some minutes of arc, we see how near he supposed the stars must be. Bradley took up the question of finding parallaxes in 1725 and discovered aberration. While he did not prove the motion of the earth by the parallactic motions of the stars, he did do it by their aberrational displacements. He supposed that the stars are much nearer together than they have been found to be. In fact, the imagination has never equaled the truth in this matter.

It is necessary to select a suitable unit of distance in describing the distribution of the stars in space. Let us suppose a star is at the distance unity when it is 200,000 times as far from the earth as the earth is from the sun. Its parallax is then 1″, and light comes from it to us in a little more than three years. In round numbers 20,000,000,000,000 miles equals the unit of sidereal distance, equals three light years. The unit of sidereal space will be taken as the sphere whose radius is the unit of sidereal distance. Since no star has been found whose parallax is so great as 1″, it follows that the unit sphere whose center is the sun contains no other known sun. The earth compares in volume to this enormous space about as a minute particle only $\frac{1}{20}$ of an inch in diameter does to the whole earth. The parallax of a star can not be directly measured with certainty if it is distant more than 10 sidereal units, and the results are very uncertain when this limit is approached. It is not likely that we know the parallaxes of all the stars that are within this distance from our system, but down to 0.20″ they are probably nearly all known. Stars whose parallaxes are greater than this amount are within the sphere whose radius is 5. There are 20 of these stars known, and the volume of the sphere is 125. That is, in the vicinity of the sun there is 1 star to about 6 units of space. This is approximately the star density in our part of the sidereal system.

We shall see presently that there are two main types of stars. Type I, of which Sirius and Vega are examples, are white or bluish white. They are intensely hot and have hydrogen atmospheres. Type II are the yellowish stars which include the sun, Capella, Arcturus, etc. Their atmospheres contain many metals. From his statistical studies of proper motions, Kapteyn has derived formulas for computing the average distances of the stars of various magnitudes of the two types. The following table shows the results obtained from his formulas down to the fifteenth magnitude, though

they must be regarded as very uncertain beyond the ninth magnitude. It must be remembered that these are mean results derived from the study of the proper motions of the stars. They may be much in error for the first few magnitudes because there are not enough bright stars to make

Magnitude	Distance	
	Type I	Type II
1	33.7 = 101 light years	14.4 = 43 light years
2	43.2 = 130 light years	18.5 = 56 light years
3	55.4 = 166 light years	23.7 = 71 light years
4	71.0 = 213 light years	30.4 = 91 light years
5	91.0 = 273 light years	38.9 = 117 light years
6	117.0 = 351 light years	50.0 = 150 light years
7	150.0 = 450 light years	64.0 = 192 light years
8	192.0 = 576 light years	82.1 = 246 light years
9	246.0 = 738 light years	105.0 = 315 light years
10	316.0 = 948 light years	135.0 = 405 light years
11	405.0 = 1215 light years	173.0 = 519 light years
12	519.0 = 1557 light years	222.0 = 666 light years
13	666.0 = 1998 light years	285.0 = 885 light years
14	854.0 = 2562 light years	365.0 = 1095 light years
15	1090.0 = 3270 light years	468.0 = 1404 light years

the statistical method safe. They may also be much in error for the fainter stars which were not used in deriving the formulas.

Our sun is surpassed in brilliance by many other suns. The measurements of Wollaston, Bond, and Zöllner show that its magnitude on the stellar basis is about -26.4. From this we can compute its magnitude as seen from any distance. It is found that if it were removed to the average distance of a first-magnitude star of Type I it would be only

a little brighter than an eighth-magnitude star. That is, at the same distance it would be only about $\frac{1}{600}$ as bright as an average star of the Type I. If it were removed to the average distance of a first-magnitude star of the Type II, it would be of about the sixth magnitude. Or, at the same distance it would be only about $\frac{1}{100}$ as bright as an average star of the Type II.

395. Motions of the Stars. — The motions of the stars with respect to one another are shown by their relative proper motions, and by their different radial velocities. Since the absolute motion of a star can not be found unless its parallax is known, it is necessary to treat many stars by the statistical method. Our general knowledge of this subject is largely due to Kapteyn, who found that the average speed with which the stars move is about 1.86 times the sun's velocity. Since the sun's motion with respect to the average of the stars is about 12 miles per second, their average speed with respect to the remainder of the stellar system is about 22.3 miles per second, or about 7.4 times the earth's distance from the sun per year. Since the unit of sidereal distance is 200,000 times the distance from the earth to the sun, it follows that, on the average, the stars move through a unit's distance in 27,000 years.

Kapteyn found, as has been stated, that there is one star to about 6 units of sidereal space. Consequently their distance apart averages about 3.5 units, and on the average they move over this distance in about 100,000 years. It is evident from these numbers that, if the motions of the stars are quite devoid of system, they will after very long intervals pass relatively near one another. It is found that there is about one chance in 18,000,000 that some sun will pass within a billion miles of any particular sun once in 100,000 years.

There are stars, however, moving with much greater velocities than the average. The star known as 1830 Groombridge has a proper motion of 7.04″ per year, although

it is so remote that its parallax is only $0.15''$. Therefore it travels yearly $7.04 \div 0.15 = 47$ times the distance from the earth to the sun in a direction at right angles to the line joining it with us. This component of velocity is 138 miles per second. Its actual velocity is probably considerably greater. In 1897 Kapteyn discovered from measurements of plates taken by Gill and Innes at the Cape Observatory a star having a still larger proper motion of $8.7''$ per year. Its actual velocity will not be known until its distance has been measured, but it is certainly very great. There are now quite a number of stars known whose velocities with respect to the average of the system exceed 100 miles per second.

The large velocities of some of the stars have an interesting bearing on some of the more general speculations in cosmogony. Many who have reflected on the development of the sidereal universe have attempted to show that it could have evolved from a chaotic state in which the matter of which it is composed was scattered somewhat uniformly through the vast domains which it now occupies, and in which the various parts were relatively at rest. According to such ideas every relative motion which now exists has been developed by gravitational forces in the system. But it can easily be shown by the principles of celestial mechanics that the mutual gravitation of the stars is not competent to generate such enormous velocities. If we make the very liberal estimate that there are 10,000 millions of suns of the mass of ours spread out in a space which it takes light 10,000 years to cross, and if we suppose that a star has come from an infinite distance toward this sidereal system under the impulsion of its gravitation, we shall find that after having been subject to its entire gravitating force in one direction for an infinite time it will arrive at the borders of the system with a velocity less than 100 miles per second. This shows the weakness of such speculations. But aside from

them, there is no reason for assuming that the stars were originally relatively at rest, and hence we are under no obligations to account for their motions any more than we are for their existence.

But the interesting fact of the very high velocities remains. So far as we can see, such stars as 1830 Groombridge will pass along in nearly straight lines and forever leave the visible sidereal universe in a very few millions of years. Likewise they must have entered the part of the universe with which we are acquainted not many millions of years ago. We are not yet in a position to appreciate the significance of such startling facts as these.

The large average velocities of the stars show that their motions are nearly rectilinear and practically independent of their mutual attractions except at the times when two or more of them pass near one another. In this respect they are something like the molecules in a gas which move in sensibly straight lines except when they collide.

The comparison of the motions of the stars of our sidereal system with the motions of the molecules of a gas is in one respect apt to be misleading. The motions of the stars are not altogether at random; there are many examples where groups of stars have sensibly the same motions. The Pleiades and a large number of fainter stars in the same region move in the same direction with the same speed. It is not to be supposed that this is the result of their mutual gravitation. It indicates rather a common origin, and their similar spectra point strongly in the same direction. Some more widely scattered groups such as the stars in the Big Dipper have parallel equal motions. Such cases are remarkable, for these stars are about one-tenth as far apart as they are from us. An observer near one would see the others as stars not much brighter than Sirius seems to us. This explains how they can move in nearly parallel lines without speedily rushing together because of their mutual gravitation. As has been

remarked, the sun is probably one of a group of something like 100 stars, and we may find that the great majority of stars are members of a limited number of star families.

396. Groups of Stars — the Pleiades. — Here and there throughout the sky are places where the brighter stars seem to be clustered. These families of stars are of such magnificent proportions as to stagger the imagination. Among the best known are the Pleiades, the Hyades, Coma Berenices, Præsepe in Cancer, and Orion. While they differ greatly among themselves, a general idea of their character can be obtained from a description of the Pleiades.

The seven brightest stars in the Pleiades are of about the fourth magnitude and they cover nearly 3 square degrees. As was stated in Art. 56, their proper motions show that they are so remote that their light comes to us in something like 267 years. At this distance the sun would shine as an insignificant ninth-magnitude star. But it must not be imagined that these stars are really close together. Those which can be seen average something like half a degree apart. That is, they are something like $\frac{1}{100}$ as far from one another as the group is from us. The two or three years which would be required for the light to pass from one to another are comparable to the time required for light to come from the nearest stars to the earth.

The telescope reveals a large number of stars in this region which one would not suspect from observations with the unaided eye. (See Fig. 21.) The measurements of Elkin show that 45 of these fainter stars have the general motion of the group, and are consequently very probably a part of it, while 8 were found having different motions. Besides this there are nearly 2000 fainter stars which were not examined. However, the fainter stars are less numerous than they are in most parts of the sky, and even in neighboring regions.

About 50 years ago Tempel noticed a faint nebulosity

near one of the brighter stars of the Pleiades, and in 1890 Barnard noticed another one. In fact, he became convinced that the whole region of the Pleiades is strewn with very faint nebulous wisps. Photographs have made the most wonderful revelations. The main stars of the group are found surrounded with nebulous banks whose enormous dimensions are indicated by the scale of the system. There is plainly a condensation around the principal stars, but the nebulous materials do not have the uniform globular forms of atmospheres stretching out billions of miles from the stars. Yet faint wisps extend from star to star in a way which proves their intimate connection with these bodies. In 1893, Barnard made long-exposure photographs of the sky around the Pleiades, and found the faint nebulosities which he had suspected from direct telescopic observations. They cover more than 100 square degrees in a region so remote from us that suns of the brilliance of ours are invisible without considerable telescopic aid; in fact, so remote that stars which are separated from one another as far as our sun is from Alpha

Fig. 184.—**The Pleiades and Surrounding Nebulosities.** ***Photographed by Ritchey.***

Centauri seem to us to be less than a degree apart. Beyond a doubt the stars in all this vast space are intimately related by their origin and their evolution.

397. Globular Star Clusters. — Perhaps more wonderful than the groups of related stars are the dense globular clusters. (See Figs. 5 and 20.) They cover portions of the sky generally less than one-half a degree in diameter, that is, less than the apparent diameter of the moon. There are more than 100 of these systems known, and they often contain several thousand stars. A photograph of the great cluster Omega Centauri in the southern heavens showed over 6500 stars. Deducting 1500, which the average star density of the region showed were seen in the cluster only by projection, 5000 remained as members of this great system. Palmer counted 5482 on a photograph of the great Hercules cluster taken at the Lick Observatory. There are also fine dense clusters in Canes Venatici and Pegasus in the northern sky.

The stars in most of the clusters are very faint, ranging from about the twelfth to the sixteenth magnitude. It is a question of great interest whether these systems are made up of great suns like ours which appear feeble and near together only because of their great distance from us, or whether they are examples of evolution in which the matter was distributed nearly equally among a very large number of small bodies. In answering this question the first thing would be to find their distances. It is not possible to measure their parallaxes by direct processes, and their probable distances can be inferred only from their proper motions. Unfortunately we do not yet have any positive data bearing on the problem. It seems probable, however, from their apparent fixity that they are distant at least 400 light years. At any rate, we may make this assumption in order to obtain some sort of a picture of what a cluster really is. At that distance our sun would appear as an eleventh-magnitude star. From this it is apparent that the stars in clusters are quite respectable suns,

though possibly smaller than our own. But it must be remembered that our estimate of their distance may be altogether too small.

According to our hypothesis as to the distances of the clusters, their diameters are something like the distance from our system to Alpha Centauri. Whatever their distances, stars from one side of a cluster would appear from the opposite side about as bright as the Pleiades do to us. Their immense size means that the individual stars of which they are composed can not, on the average, be very near together. In a cluster of the assumed dimensions containing 5000 stars, the average distance of the stars from one another would be 30,000 times the distance from the earth to the sun. When we remember that gravitation varies inversely as the square of the distance, we are not surprised that these bodies do not fall to the center of the cluster under the influence of their mutual gravitation, or even that in the few years during which they have been carefully studied no relative motions have been observed. There is plenty of room in them for almost indefinite motion without collision, and there is no gravitational necessity for any higher velocities than other stars possess on the average.

When we consider the dimensions of star clusters and the great distances between the separate stars, we are apt to conclude that they have no intimate relation to one another. But this conclusion is certainly false. About 10 years ago Baily announced that many clusters, perhaps 1 out of 5, contain very many variable stars. In a given cluster the range of variability is nearly the same, usually a magnitude or two, the character of the light variation is essentially the same, and the periods of variation are approximately, though not exactly, the same. The variable stars flash out for a short time with from 2 to 6 times their ordinary light at regular intervals of usually a few hours. Their approximately equal periods suggest the idea that they are very

much alike, and that possibly the periods were originally the same, but that they have become slightly different because of some slight inherent differences in the stars, or of their different environments. Although the discovery of these variables is very recent, and the search for them by no means exhaustive, yet more than 500 of them have been found. Their relative frequency in apparently quite similar clusters varies greatly. There are 128 of them in Omega Centauri, 2 have been found out of 1000 examined in the Hercules cluster, while 132 out of 900 in the cluster in Canes Venatici were found to be variables. We have at present no idea whatever of the reason of their variability, but the fact points to fundamental similarities in the members of these great aggregations of stars.

398. Double Stars. — A few double stars have been known almost since the invention of the telescope, but Herschel was the first astronomer to search for them systematically, and to measure their distance and direction from each other. Assuming, perhaps unconsciously, that the solar system is the normal type, he supposed the stars were only in the same direction and that they were not intimately related. His purpose in measuring them was to find the apparent motions they would have with respect to each other if they were unequally remote, due to the earth's motion around the sun. From the magnitude of these apparent displacements it would be possible to compute the distance of the nearer star. Instead of finding what he sought for, to his surprise he found in the course of a few years that some pairs were actually revolving around each other. Instead of a star having always a retinue of planets, in some cases certainly there are twin suns of essentially the same dimensions. They may have planetary attendants or not, so far as we know, because such objects would be beyond the range of our instruments even though they were a thousand times more powerful than any yet constructed.

In the past century many observers have devoted themselves diligently to double-star astronomy. The names that stand out most prominently are William Struve, Dawes, John Herschel, and Burnham. About 12,000 pairs of double stars are now known, and new very close pairs are constantly being discovered. Hussey and Aitken found that about 1 star in 18 brighter than the ninth magnitude is shown by the Lick telescope to be a double. In not every case are two stars which are apparently near each other in the sky actually connected physically. "Near" means in this connection that the two components are not separated from each other by more than 20″, which is about one-tenth of the smallest angle the eye can distinguish without optical aid. Sometimes one component of a pair is much farther from us than the other, when they form a double simply by perspective. But the chances are very much against there being many pairs in the sky accidentally so nearly in the same direction from us as the two components of a double star. Hence it is probable that a very large proportion of the double stars are physically connected. Those which are known to form systems are called *binaries*.

399. The Orbits of Binary Stars. — The stars are so remote from us that the components of a binary system can not be seen as two separate stars unless they are a great distance apart. But when they are far from each other their periods are long, and observations must extend over many years in order to furnish data for the computation of their orbits. The double stars first discovered were those which were not very close together, and while in many cases we are now sure that the two components of a pair are revolving around each other, in only 35 or 40 have the arcs described been sufficient to give the orbits with precision. The orbits of 40 of the best-known binary stars were computed by See about 10 years ago.

The periods of binary stars range from about 5 years to

hundreds and perhaps thousands of years. The planes of their orbits are inclined at all angles to the lines joining them with our system, so that, as a rule, we see their orbits in projection. Perhaps the most interesting thing about their orbits is that generally they are quite eccentric. See found that in the 40 orbits he computed the eccentricity averaged 0.48, or more than 12 times that of the planetary orbits.

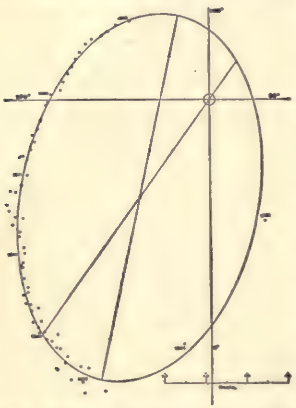


FIG. 185. — Apparent Orbit of Xi Boötis (See).

400. Masses of Binary Stars. — The masses of the planets which have satellites are found from the periods of the satellites, and the computation is a simple matter (Art. 181). The masses of Mercury and Venus have been found from their attractions for other bodies, chiefly comets. The mass of every celestial body is found from its attraction for some other body. It is evident, therefore, that the mass of a single star remote from all other visible bodies can not be found. On the other hand, when the size of the orbit of a binary pair is known and the period of their revolution, their combined mass can be computed just as the mass of a planet and satellite is computed. The apparent size of the orbit is observed, and the actual size can be computed when the distance of the star is known. At present there are only six cases of visual binaries in which all the required data are at hand. They are given in the following table, where the

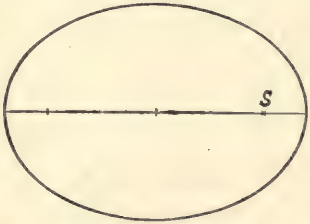


FIG. 186. — Actual Shape of the Orbit of Xi Boötis (See).

columns are in order : (1) the name of the star, (2) the apparent major semi-axis of its orbits, (3) its eccentricity, (4) its parallax, (5) major semi-axis in terms of the earth's distance from the sun, (6) period in years, (7) combined mass of the pair in terms of the sun's mass, and (8) the light radiated in terms of the sun's light.

Star	Apparent Semi-axis	Eccentricity	Parallax	Actual Semi-axis	Period	Mass	Light
α Centauri	17.70″	0.53	0.75″	23.6	81	2.0	1.7
Sirius	8.03	0.62	0.37	21.7	52	3.7	32.0
Procyon	3.00	—	0.30	10.0	40	0.6	8.5
η Cassiopeiæ	8.21	0.51	0.20	41.0	196	1.8	1.0
70 Ophiuchi	4.55	0.50	0.19	24.0	88	1.8	0.7
85 Pegasi	0.78	0.39	0.04	19.5	24	11.3	2.2

There are not enough pairs given in this table to enable us to draw any general conclusions with safety. The average distance of the stars of a pair from each other is 23, or a little greater than the mean distance of Uranus from the sun. It is altogether probable that in most cases they are farther apart than these stars, for those which are at great distances from each other have such long periods that their orbits are not yet known. The average mass of a pair is 3.5 times that of the sun, while the average radiating power is nearly 6 times that of the sun. This seems to indicate that the stars are on the average relatively more brilliant than the sun, and this idea is supported to some extent by the excessive power of radiation of many stars, as, for example, the brighter ones in the Pleiades.

When the orbits of the two stars of a pair with respect to their center of gravity is known, their separate masses can be computed. Of the 6 stars in the table we have the necessary

data with some approximation in the case of Sirius, Procyon, and 85 Pegasi. The masses of Sirius and its companion are respectively 2.5 and 1.2 times that of the sun. The corresponding numbers in the case of Procyon are 0.6 and 0.2. It is remarkable that in both of these systems where the disparity in the masses of the two bodies is not great, the primary is nearly 10,000 times as bright as its companion. Perhaps the companions have more rapidly run through their evolutions as great luminous suns, and are now approaching extinction.

According to the computations of Comstock in the case of 85 Pegasi, the masses are respectively 4.3 and 7.0 times that of the sun. In this system the smaller mass is 100 times as luminous as the larger. Moreover, the spectroscopic method of classifying stars according to their age and state of development leads to the conclusion that this is a much older system than that of Sirius, and of about the same age as that of Procyon. Therefore we should expect to find the smaller mass approaching the dark stage instead of being so exceedingly brilliant. However, the data respecting masses are yet rather uncertain, and the contradiction with what has been found in the study of Sirius and Procyon may be due to errors in the data.

401. Spectroscopic Binary Systems. — The spectroscope has contributed to binary star astronomy most interesting and important results. It has been mentioned in several connections that radial velocities can be measured by the displacement of the spectral lines. The application of this principle to the stars was begun by Sir William Huggins in 1868, and approximate results were obtained for 30 stars. But it is only since 1890 that this has become an important part of observational astronomy.

Suppose the spectrum of a binary system whose plane of motion passes through the earth is photographed. When one star, as *A*, Fig. 187, is receding from us, the other, as *B*,

is approaching. We may suppose, for simplicity, that they are equal and that their center of mass is at rest with respect to us. The lines of the spectrum of A will be shifted toward the red, and those of B toward the violet. If the two stars have the same spectra, all the lines will be apparently double. After a quarter of a revolution when they are at A' and B' respectively neither star will be approaching or receding, and their lines will coincide and appear single. After another quarter of a revolution B will be receding and A approaching, and the lines will be double again.

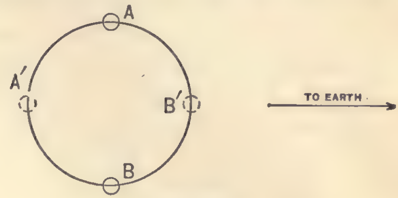


FIG. 187.

The discovery of the first spectroscopic binary of this kind was made by Miss Maury at the Harvard Observatory in 1889. In the apparatus used large prisms were placed in front of the objective, so that the spectra of all the stars in the field were obtained at one time. Miss Maury noticed in certain spectrograms of Mizar (Zeta Ursæ Majoris), which were taken in 1887 and 1889, that some of the lines were double, and in others that they were single. An examination of 70 plates showed that all the lines were doubled periodically every 52 days, and it was inferred from this that the star is a binary. The components are not the ones which can be seen with a small telescope. One of these telescopic components is the binary, and its two parts are so close together that they can not be seen separately with the most powerful instrument. Many other stars of the same class have been discovered.

The method which has just been described fails if the spectra of the two stars are not of approximately the same brightness. Telescopic doubles often differ greatly in actual dimensions and light-giving power, and the same things are

to be expected of the closer pairs. If one star were dark, the lines of the other star alone would be seen, and they would shift one way and then the other from their mean position as the radial component of velocity changed. In order to detect these very slight shifts in the spectral lines it is necessary to use a slit spectroscope at the eye end of the telescope, and a comparison spectrum. That is, means must be adopted to determine the absolute position of the lines at any time. The first binary of this type was discovered by Vogel, at Potsdam, in 1889. He found that the lines of Algol shifted back and forth with a period the same as that of its variability (2 da. 20 hr. 49 m.). This confirmed the theory long held that this star at times becomes dim because it is partially eclipsed, for the shifting of the lines agreed perfectly with the requirements of the explanation.

In 1898, 13 spectroscopic binaries were known, but the discoveries have proceeded so rapidly in recent years that 140 pairs, 6 of which were also visual binaries, were known at the beginning of 1905. In the catalogue issued by Campbell and Curtis it is stated that 72 have been discovered at the Lick Observatory and its branch in South America, 41 at the Yerkes Observatory, 8 at the Harvard Observatory, 7 at the Lowell Observatory, 6 at Pulkova, 4 at Potsdam, 2 at Meudon, and 1 at Cambridge. To this list should be added 6 visual binaries which are also spectroscopic binaries. Seven pairs were discovered independently at two observatories. In 17 cases the spectra of both components were visible.

402. The Orbits of Spectroscopic Binaries. — In the case of a visual binary all the elements of the orbit are known except its absolute size. If the distance were known, the dimensions of the orbit could be easily computed; but as we have seen the stars are so remote that only very seldom can their parallaxes be directly obtained. If the relative velocities of the stars toward us are measured by the spectroscope, it is an

easy matter to find the size of an orbit whose period is known, for its circumference equals the product of the relative velocity and the period. Since the apparent separation of the stars has been observed, their actual distance from each other can be computed. There is a good prospect of finding the parallaxes of many stars in this way in the course of time. This is again a case in which the most diverse processes unite to give important results. In all of these cases both the periods and the dimensions of the orbits will be known, from which the combined masses of the stars can be computed.

Let us now consider the case of a non-telescopic binary in which the lines of both stars are observed. The relative radial velocity of the stars is determined by the amount of the separation of the lines. If the plane of the orbit of the pair passes through the earth, the maximum observed radial velocity is the whole relative velocity. If the plane of the orbit is inclined to the line joining the earth to the star, the observed component is only a fraction of the whole relative velocity. The extreme case is where the plane of the orbit is perpendicular to the line of sight, when no relative radial velocity can be observed. Now the spectroscope does not tell us how the plane of the orbit lies. Consequently all we can say is that the actual relative velocity is at least as great as the observed radial velocity.

Since the periods of this class of stars are always known, it is possible from this lower limit on the relative velocities to compute a lower limit for the size of the orbit. This enables us to compute a lower limit for the sum of the masses. In most of these cases the orbits are probably not greatly inclined to the line joining them with us; hence the results obtained in this way will be on the whole fairly approximate. Since we know that the stars get at least a certain distance apart, we may infer that they must be at least a certain distance from us or they could be seen as telescopic doubles.

In this case the computed limit may not be even approximate, for any distance greater than this limit will satisfy the conditions. These results are very precious because they may be obtained however far the stars are away so long as they send us light sufficient for spectroscopic purposes. This means that with present spectroscopic equipment they must not be fainter than about the eighth magnitude.

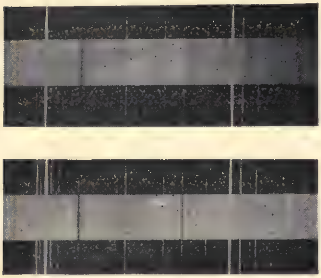


FIG. 188. — Shift in the Lines of Mu Sagittarii. (Frost and Adams.) The central strips are the spectra of the star whose dark lines are shifted differently in the two pictures with respect to the bright comparison lines beside them.

The last class to be considered is that where the spectrum of but one star of a pair can be observed. The variation in its radial velocity gives a lower limit (because of the uncertainty in the inclination) to the dimensions of its orbit around the center of gravity of the two stars. The darker of the two stars is presumably smaller than the brighter one, and if so its orbit around the center of gravity will be larger than that of the brighter component. Consequently the relative orbit of the pair will generally be at least twice as great as the orbit of the brighter component. Since the period is known, a lower limit to the sum of the masses can be found.

The spectroscopic binaries are surprisingly numerous. Campbell and Curtis found that of the stars studied with the Mills spectrograph 1 in 7 was a spectroscopic binary, while Frost and Adams found the ratio 1 in 3 in the class of stars known as the Orion type. These results are very significant, for it must be remembered that all those stars whose orbits are anywhere near to perpendicularity with the line of sight escape detection ; also, nearly all of those whose periods are

more than a few months have not been found because of the limited number of determinations of the velocities of most stars ; those whose masses and distances apart are such that their velocities are low have escaped notice ; and finally some whose periods are about commensurable with a day may not have shown variable velocities in the limited number of observations so far made. Taken in connection with the numerous visual binaries, these results strongly suggest that systems of two or more stars may be the common type rather than that exemplified by the sun and its retinue of planets. When the Laplacian ring theory was developed our system was supposed to be the dominant type.

The periods of spectroscopic binaries so far range between 1.45 days and 3.3 years, and possibly longer in some cases where the determination is not yet complete. There is now no appreciable gap in the periods between spectroscopic and visual binaries, and there is no reason for supposing that they are essentially different classes of stars. The eccentricities are not yet well determined for many orbits, but enough is known to show that they range from about the eccentricity of the earth's orbit to something like 0.9. This is in substantial agreement with the visual orbits. The lower limits of their mean distances lie between 100,000 miles and 100,000,000 miles.

403. Interesting Spectroscopic Binaries. —*Mizar.* The first spectroscopic binary discovered was Mizar. Vogel's later work showed that its period is about 20.5 days, proving in connection with the size of its orbit that the mass of the system is something like 20 times that of the sun. According to the parallax found by Klinkerfues, which it is likely is too large, the pair radiates 38 times as much light as the sun. That is, the radiation seems to be greater in proportion to the mass than in the case of the sun.

Spica. Vogel found that the velocity of Spica varies in a period of 4 days, showing a radial motion of 57 miles per

second for the brighter component. If the dark one is of equal mass, their combined mass is about 2.5 times that of the sun, and their mean distance apart is something like 6,000,000 miles. Since it has been impossible to find any parallax for this star, it must be excessively brilliant in proportion to its mass.

Capella. The parallax of this star has been measured with the utmost care by Elkin. It is at a distance of 35 light years from us. At that distance our sun would be an insignificant object near the limits of observation without optical aid. While Capella and its companion emit 100 times as much radiant energy as our sun, their mass is only 17 times as great. The spectra of both stars can be distinguished, though one is much fainter than the other. From the absolute shift of the two sets of lines it has been found that the two masses are about equal. The brighter one has a spectrum almost exactly like that of the sun, while the spectrum of the other is like that of a supposedly earlier type of star. The stars of this earlier type are believed to be ordinarily hotter and more luminous than those of the solar type. Dyson and Lewis believed they obtained observational evidence of the duplicity of the star with the 28-inch Greenwich equatorial, although it appeared perfectly round to Hussey through the 36-inch Lick telescope.

Polaris. This star has two darker companions discovered spectroscopically by Campbell. One is near the bright star, the two going around their center of gravity in a little less than 4 days, while the third is more distant and revolves in a period of several years.

404. Origin and Evolution of Binary Stars. — The ideas of astronomers respecting the origin and evolution of stars have been unconsciously influenced to a great extent by the Laplacian nebular hypothesis. According to present opinions, the stars have condensed from more or less widely extended nebulas. This seems most reasonable in the light of

our present knowledge. The rotations of the stars are the result of the relative motions of the different parts of the nebulas, for the moment of momentum of a contracting mass remains constant. As the bodies contract they continually rotate more and more rapidly. When they rotate very rapidly they break into two parts, forming a close binary system.

When a binary system has been formed in the manner indicated the tidal reactions of the bodies on each other become important factors in their further evolution. Darwin's discussion of the effects of tidal evolution pertained to the solar system alone, but See has extended the same ideas to the binary stars in attempting to explain the peculiarities of their orbits, particularly the high eccentricities. It has been shown (Art. 351) that the general effects of the tides under the conditions which would prevail would be to lengthen both the common period of revolution and the periods of rotation. It can also be shown that another consequence of tidal interactions under the same conditions is to increase the eccentricities of the orbits if they have any original eccentricity. See believes that the high average eccentricity of the orbits of binary systems is a direct evidence of the operation of tidal forces.

According to the theory which has been outlined, the older a system is the larger its orbit will be. Therefore, on the average the visual binaries should be older systems than the spectroscopic binaries. Now astronomers have a theory (Art. 411) of the relative ages of different stars based on the study of their spectra. According to this classification some spectroscopic binaries are old stars, and some telescopic binaries are still in the earlier stages of their development. There seems to be no definite indication of correspondence in the two theories. Their failure to agree must act as a stimulus for new researches in both directions.

There are serious difficulties in accounting for the wide

separation of many of the binary pairs on the basis of this theory. In the whole evolution the moment of momentum remains constant. That is, the present orbital moment of momentum of any pair and their rotational moments of momentum together must equal the rotational moment of momentum of the system at the time it separated into two bodies.

Consider the system of Sirius and its companion. If they once formed one mass 20,000,000 miles in diameter, its average density was less than $\frac{1}{1000}$ that of water. It follows from the present orbital momentum of the system alone that it must then have rotated on its axis in about 3.5 days. On the other hand, if the mass divided when it had shrunk to these dimensions, the two components must have revolved around each other, according to Kepler's third law, at this mean distance in a period of 56 days. This shows simply that the theory of the origin of binary stars which has been described must not be hastily accepted. It can not be doubted that tidal interactions have important effects upon the evolution of binary systems; but if they have not arisen from the division of rapidly rotating larger masses the rotations of their components may be in any direction, and consequently their tidal evolution may be quite different from that which has commonly been accepted.

FIG. 189. — Trifid Nebula in Sagittarius. *Photographed with the Crossley reflector of the Lick Observatory.*

It is perhaps suggestive that there are many nebulas which seem to have broken in parts, and which appear to be capable of developing into several separate stars (Fig. 189).

405. Variable Stars. — A star whose brightness changes is said to be a variable. The first one known, Omicron Ceti,

was discovered by Fabricius in 1596. The variability of Algol was discovered by Goodricke in 1783. The next year he recorded the variability of Beta Lyræ. But variable stars were not discovered in any considerable numbers until toward the close of the nineteenth century. Now more than 300 of these objects are known in addition to those which have been found so abundantly in some of the star clusters. Some of them vary periodically with periods ranging from less than a day to over two years. Others vary irregularly without any apparent rule or system. Some flash out brilliantly for a short time and then sink back into permanent oblivion. It is practically certain that the brightness of

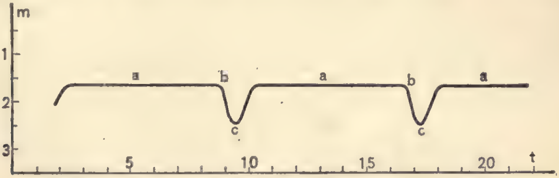


FIG. 190. —Light Curve of Eclipsing Variables.

every star varies slowly, but there is no observational evidence of a slow secular change in any case unless it be that of Castor or Pollux. In 1610 Bayer recorded that Castor was at least as bright as Pollux, while it is now certainly a little fainter. Variable stars are classified according to the character of their light variation.

406. Eclipsing Variables. — If the plane of the orbit of a binary pair passes very nearly through the earth, the stars will more or less completely eclipse each other every time they are in a line with the earth. If one of the two is a dark star and nearly as large as the other, it is clear the light received from the pair will remain constant except when the brighter star is eclipsed. As the dark star begins to eclipse the brighter, the light will diminish rapidly until the time of greatest obscuration, after which it will rapidly

regain its normal value. The variation in light is represented by a curve (Fig. 190), where the intervals of time are marked off along the horizontal line t, and where the amount of light received is proportional to the distance of the curve above this line. The parts marked with a's measure the light when the star shines undimmed by an eclipse, the b's show where the light begins to wane as the eclipse commences, and the c's give the brightness at the time of greatest obscuration. These stars evidently must be spectroscopic binaries if their spectra are bright enough and are displaced enough by radial motions to enable them to be observed. The reason that not all spectroscopic binaries are eclipsing variables is that the planes of their motion do not pass near enough the earth.

The typical eclipsing variable where one component is dark is Algol (Beta Persei). Its light curve is essentially like that given in Fig. 190. There are about 25 variables of this type known, and they are usually called Algol variables. They are all characterized by the shortness of their periods, most of which are less than 5 days. Doubtless the explanation is partly that when they are far apart they do not eclipse unless the plane of their motion passes very exactly through the earth. Russell has shown that these variables are of very low average density, which is a remarkable fact, since one of each pair is dark.

The period of Algol is 2 da. 20 hr. 48 m. 55 sec. It is normally a star of the second magnitude, but at the time of eclipse it loses five-sixths of its light. In 1889 Vogel found that it was a spectroscopic binary, and we consequently know much about its orbit. The bright star is a little more than 1,000,000 miles from the center of gravity of the system, and has a diameter slightly exceeding 1,000,000 miles. From the diminution of light at the minima, Pickering calculated that the diameter of the dark companion is about 760,000 miles. Vogel estimated 830,000 miles. Yendell found that the incli-

nation of the orbit to the line of sight is 7°. If the stars are of equal density, the bright one is about twice as massive as the other, and their combined mass is two-thirds that of our sun. From the distance assigned to it by Chase's measures of its parallax (0.035″), it is found that it radiates 80 times as much light as the sun, and that its surface therefore is 52 times as bright as that of the sun. As classified by its spectrum it is a very young star, while, strangely indeed, its companion is old and dark. Slight irregularities in its light curve led Chandler to suspect the existence of a third dark star disturbing the motion of the first two, while Tisserand supposed the irregularities might be produced by the equatorial bulges of the stars.

There are several variations from the type of star just considered. One is that in which the stars are of unequal size and both bright. Then each eclipses the other, but the amount of light shut off in the two cases is different. The light curve has two minima of different depths below the normal brightness. In one case at least both spectra have been recognized. Often there are irregularities which have not yet been explained. Sometimes the periods increase for a number of years and then decrease again, showing possibly the presence of a third disturbing body. In the case of Y Cygni the stars seem to be of equal size and brightness. The eccentricity of their relative orbit (0.145 according to Dunér) causes the minima to occur at unequal intervals, and consequently distinguishes the two eclipses from each other. The fact that the minima are equal proves the equality of size and brightness of the two stars.

407. Variable Stars of the Beta Lyræ Type. — There is another class of stars closely related to those which have been considered. Their light varies continuously, instead of being constant except at the epochs where it descends for a short time to the minimum. There are, besides, two minima, one generally much more pronounced than the other, while their

maxima are equal. The standard star of this type is Beta Lyræ, which is one of the earliest known variables.

The explanation of these variables is that two stars eclipse each other, and that they revolve in such small orbits compared to their dimensions that the intervals in which neither obscures the other are very short. Beta Lyræ has two spectra which change greatly with the period of the star's variability, though it can not be said that the changes in the spec-

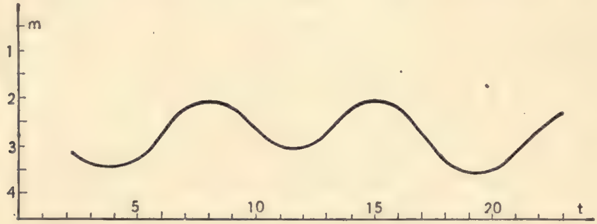


Fig. 191. — Light Curve of Beta Lyræ Variables.

tra agree fully with the eclipse theory. Myers has solved the problem of satisfying the light variations by the eclipse theory. He finds that the mean distance of the stars from each other is 31,000,000 miles, and that their orbit is nearly circular, having an eccentricity of only 0.018. The stars are so large that they are nearly in contact, and their mutual tides have elongated them in the line joining their centers. The large star is two-fifths as bright as the smaller one. Their average density is less than that of the air at the sea level. It is suspected that the period of Beta Lyræ is slowly increasing, as it should under the influence of the tidal disturbances.

Another star of the same type is U Pegasi, which has the remarkably short period of 9 hours. In this case, according to Myers, the stars are probably actually in contact. These stars seem to have separated recently, and are to some extent confirmatory of the theory of the origin of binary systems.

408. Variable Stars of the Omicron Ceti Type. — The stars of this type have long and very irregular periods of variability. The example best known is Omicron Ceti, which has been observed through more than 300 of its cycles. Yet it has not been found possible to formulate any law describing its phases. Its maxima and minima are subject to as great irregularities as its period, and there is no discoverable relation among them. In 1779 Herschel saw the star when it was nearly as bright as Aldebaran, while 4 years later it was not visible even through his telescope. This means that at maximum it was at least 10,000 times as bright as at minimum. Ordinarily its maximum is much below that observed by Herschel and its minimum considerably above. Omicron Ceti was called *Mira*, the wonderful, and 300 years of observations have only added to the mysteries associated with its peculiar behavior.

FIG. 192. — Light Curve of Stars of Omicron Ceti Type.

The general characteristics of the light curves of stars of the Omicron Ceti type is a slow, but gradually accelerated, increase in brightness, and then a much more gradual decline. The change is continuous. The spectroscope shows marked changes in the spectra of these stars, but no evidence of their being spectroscopic binaries. They are generally red and seem to be far advanced in the process of cooling. The cause of variation seems to lie within the stars themselves, yet we can imagine no internal disturbances which would produce the remarkable fluctuations observed in many objects of this class.

409. Irregular Variables. — In addition to the classes of variables enumerated, there are others whose variations have no semblance of periodicity. Some are distinguished by

flashing out with relatively great brilliance after intervals usually counted in years. They are generally red stars. Others unaccountably fade away now and then and sometimes become invisible through good telescopes, even though they had been visible with the unaided eye. These stars are sometimes at least apparently associated with faint nebulosities.

410. Temporary Stars. — Occasionally stars have been observed to blaze forth with great brilliance in parts of the sky (so far as observed always in the Milky Way) where none had been previously seen, and then to sink quickly away into obscurity. They are characterized by a sudden rise to one great maximum which, notwithstanding later possible fluctuations, is never repeated. One of the most remarkable ones of which we have any records shone as bright as Venus in 1572 in the constellation Cassiopeia. This is the star which attracted the attention of Tycho Brahe, and turned him to astronomy. Kepler also was stimulated by his discovery of a temporary star in 1604 in Ophiuchus. At its maximum it was as brilliant as Jupiter. There are few which have attained such magnitudes. It is altogether probable that, until recent times, most of those whose maxima were less marked entirely escaped notice. To the great number of observers now attentively watching the sky and comparing it with accurate star charts, is added the powerful assistance of photographic processes. Mrs. Fleming has discovered more temporary stars on the Harvard plates in 10 years than were found by all observers from Tycho Brahe to Herschel. Of the 11 stars which were discovered in the nineteenth century, 5 were found by Mrs. Fleming. Unfortunately the photographic records are read often only after the star has vanished.

Temporary stars are called *novæ*, or new stars. The first one whose spectrum was examined in any detail was Nova Aurigæ, discovered by Anderson in January, 1892. From

examinations of photographs of this region of the sky which had been taken previously, it was inferred that it was at its maximum brightness (magnitude 4.4) on December 20, 1891. In March it began to decline rapidly, and by April had sunk to the twelfth magnitude. In August it blazed up again to the ninth magnitude, and then gradually sank away. This history is fairly characteristic of the light changes of temporary stars.

The spectrum of Nova Aurigæ was full of surprises. It was double, consisting of a set of bright lines and one of dark lines. The first thought was that the two sets of lines came from two different stars. The shifting in the position of the bright lines showed, on the basis of the Doppler-Fizeau principle, a velocity away from the earth of over 200 miles per second, while the dark lines showed in the same way that their source was approaching the earth at the rate of more than 300 miles per second. There are abundant reasons for doubting the correctness of this interpretation of these line shifts, but no satisfactory explanation is at hand. Equally puzzling was the radical change in the character of the spectrum. As the star became fainter the dark lines, characteristic of a star like the sun, disappeared, and the bright lines became like those of nebular spectra.

A remarkable and more carefully studied temporary star was discovered by Anderson, February 22, 1901, in Perseus. On the 23d it was brighter than the star Capella, while the photographs of Pickering and Stanley Williams showed that on the 19th it was not brighter than the twelfth magnitude. In this short interval its light had increased more than 20,000 times. A day later it had lost one-third of its light. The changes in its luminosity are shown in Fig. 193.

The spectrum changes of Nova Persei were much like those of Nova Aurigæ. The star speedily changed from white to red, and its spectrum ultimately became like that

of the gaseous nebulas. But the most interesting thing in connection with the star was the nebulous matter which was later found around it. Its existence was first shown on photographs by Wolf taken August 22 and 23. Later photographs by Perrine and Ritchey showed that it was gradually becoming visible at greater distances from the star. It looked as though the star had ejected nebulous matter, but it was found on computation that, if this were the correct explanation, the ejected matter must have been leaving the star with about the velocity of light. The theory was suggested by Kapteyn and W. E. Wilson, and expounded in detail by

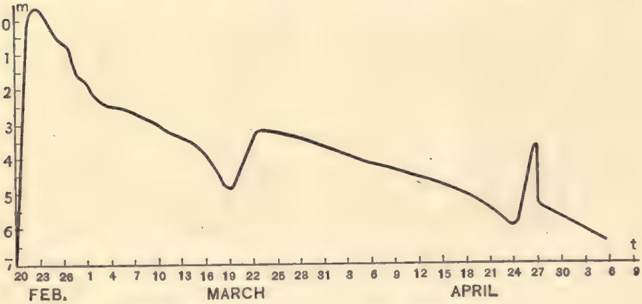


Fig. 193. — Light Curve of Nova Persei.

Seeliger, that the nebulous matter was lying invisible in that part of space before the appearance of the star; the star was an invisible dark one moving with high velocity; it rushed through the nebula and its surface blazed into incandescence because of the friction, just as a meteor is made to glow when it strikes into our atmosphere; the heating was superficial and died away quickly when the star left the nebula, to be partially revived once or twice as it encountered stray nebulous wisps; and more and more of the previously dark nebula became visible as it was lighted up by the brilliant rays of the star. If this is the true explanation of the phe-

nomenon, it was one of the most interesting things ever observed. The star and nebula are so far from us that about four months were required for light traveling at right angles to the line of sight to illuminate the nebula to a distance of 3′, an arc barely appreciable to the unaided eye. How great the distance when an object moving with the velocity of light will not cover the apparent distance between the components of Epsilon Lyræ for months! The actual collision and bursting out of Nova Persei occurred in about the year 1600, and the waves of light from that far-off source reached us only after the lapse of 300 years.

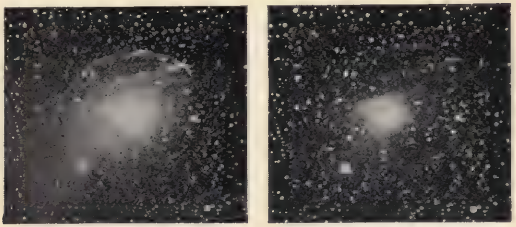


FIG. 194.—Nebulosity surrounding Nova Persei on Sept. 20 and Nov. 13, 1901. *Photographed by Ritchey.*

411. The Spectra of the Stars. — The spectra of the stars differ as greatly as their colors. They were classified first by Secchi in 1863. More powerful instruments have shown many new facts, and have led to a number of new and more extended systems of classification, particularly by Huggins, Vogel, and Miss Maury. The four types described by Secchi still form a general basis for classification, while the modifications are largely additions of sub-classes. The work is very intricate, and we shall have to content ourselves with a mere outline of it.

Type I. Stars of this type are blue or bluish white, as Sirius and Vega. Nearly one-half of all the stars are included

in it. Their spectra are very bright toward the violet end, presumably indicating very high temperatures. They may be conveniently divided into two classes : (*a*) those in which the absorption is largely due to helium, while the metallic lines so abundant in the sun's spectrum are almost entirely wanting ; and (*b*) those in which there is strong hydrogen absorption.

The brighter stars in Orion and the Pleiades are excellent examples of class (*a*). They are often associated with nebulous material, and they are supposed to be in their earliest stages after having condensed from nebulas. But the recently discovered fact that helium is one of the products of disintegration of radium raises a question. These stars are often called the "helium stars." They show a strong tendency to be relatively most numerous in the Milky Way, and all those known are certainly very remote from us. They are also very brilliant. The star Rigel, which is a good example of this class, is inferred from its proper motion to be distant from us more than 330 light years. Its great brightness means therefore that it shines with 8000 times the luster of our sun.

The stars of class (*b*) are supposed to be next in order of evolution. They are often referred to as "Sirian stars" because Sirius is such a splendid example of them. They show little or no helium absorption. This probably means that its spectrum has been suppressed by the other elements, as in the case of the sun, rather than that the element is absent. The absorption in the atmospheres of the Sirian stars is slight, and their great brilliancy may be due partly to this, although they are usually regarded as very hot stars.

Type II. These are the solar stars. They are somewhat yellowish, and their spectra are characterized by numerous fine metallic lines. The hydrogen lines are still present, and in fact the gradations are such that there is no sharp line of demarcation between the hydrogen and solar stars. The

indications are that the spectra show an evolution with age rather than differences in size and mass, for these stars vary in dimensions from those which are much smaller than the sun up to Canopus (a star of early solar type), which seems to be

FIG. 195. — Spectrum of Sirius with the Comparison Spectrum of Titanium on the Sides. Frost and Adams.

certainly 200,000 times as great as our sun. These stars are about as numerous as those of Type I.

Type III. Stars of this type are red, and Antares and Betelgeuse are examples. Only about 500 of them are known. Many of them are variable. Their spectra show heavy absorption bands which are sharp toward the violet,

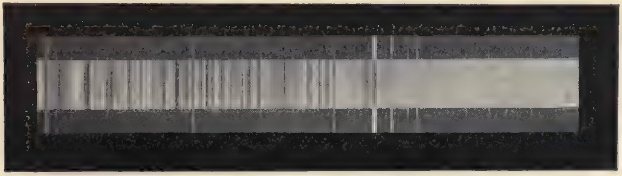


FIG. 196. — Spectrum of Arcturus with Titanium Comparison Spectrum. Frost and Adams.

and which gradually disappear toward the red. Metallic lines are visible, but the cause of the so-called flutings is at present unknown. The fixity of the known red stars argues that they are immensely remote. Hence such bright stars as Antares and Betelgeuse, whose light is so largely absorbed,

must be enormous suns. They are almost certainly many thousands of times greater than our sun. We do not know whether their spectra depend upon the stage they have reached in their evolution or upon their constitution.

Type IV. The 250 stars of this type are all faint and of a deep red color. Like those of Type III, they are often variables. They have heavy dark absorption bands, sharp on the red side and indefinite on the violet, and thus in a way just opposite to those found in Type III. The absorption bands are probably due to carbon, and the stars are called "carbon stars." Their faintness precludes their study except with powerful instruments, such as were employed by Hale and Ellerman at the Yerkes Observatory in their extensive work upon these objects. They have been thought to form the last luminous stage in the evolutions of the stars. The fact that they have no measurable proper motions is indicative of their great distances, and therefore, possibly, of their great masses and luminosity. They show a preference for the Milky Way, though they are not confined to it.

Additional Types. (1) There are stars with fluted spectra which have also bright lines. That is, in the midst of general absorption there are glowing gases brighter than the photospheres of the stars themselves. These stars are always variables, and this fact furnishes a convenient method, employed incidentally at the Harvard College Observatory, of finding variable stars. Omicron Ceti belongs to this spectral type.

(2) Certain helium stars show bright lines in addition to their dark absorption lines. Curiously the dark and bright lines belong to the same elements, and the bright lines are those which occur toward the violet end of the spectra. Alcyone shows some of these bright lines, though the stars of this class lie mostly in the Milky Way.

(3) Wolf and Rayet, at the Paris Observatory, discovered some stars of a peculiar spectral type in 1867. On a

fairly continuous spectrum are superposed many dark lines and bands, some few being due to helium and hydrogen, but most of them to unknown substances, and mingled with them are many bright lines. The metallic lines of the solar spectrum are quite unknown in these stars. Of the more than 100 so far discovered all are situated in the Milky Way, except some which are in the Magellanic clouds in the southern sky, which are essentially like the Milky Way. These stars are never variables.

NEBULAS

412. Spiral Nebulas.—It has been remarked (Art. 364) that the spiral nebulas are more numerous than all other kinds combined. According to Keeler's estimate there are at least 120,000 within the reach of the reflecting telescope which he used. In all cases where observations have been competent to settle the question, they have been found to consist of a central nucleus and two opposite arms. (See Figs. 6 and 171.) They are distinguished by being white while all other nebulas have a greenish tinge. Their spectra are either continuous, or continuous except for a few dark absorption lines. It is true that they are so faint that they have not yet been

Fig. 197. — Spiral Nebula in Ursa Major, M. 81.
Photographed at the Lick Observatory.

FIG. 198.—Great Nebula in Andromeda. *Photographed by Ritchey with the 2-foot reflector of the Yerkes Observatory.*

studied extensively, but dark lines were found by Scheiner in the spectrum of the great Andromeda nebula. Their spectra indicate that they are perhaps largely in a solid or liquid condition. On the other hand, their transparency indicates their tenuity. Hence they seem to be perhaps vast swarms of incandescent solid or liquid material surrounded by gaseous material. The dynamical conditions require that the motions in them shall be at a large angle to the arms of the spiral, rather than along them. An hypothesis as to their possible origin was suggested in Art. 365. The chief difficulty is to explain their continued luminosity. Lockyer attempted to account for the light of all nebulas by the heat generated in the collisions of the meteorites of which he supposed they were largely composed.

The Andromeda nebula is the greatest spiral which we know. It is tilted to the line of sight so that it is about 1.5° long and 0.5° wide. Its distance is unknown, but it is improbable that its parallax is greater than $0.01''$, and it may be very much smaller. If this is its parallax, its diameter must be more than 500,000 times the distance from the earth to the sun, and light would require 8 years to travel across it. Unless it is, on the average, of an unimaginable tenuity, it may exercise appreciable attraction on bodies as remote from it as the sun. Suppose, for example, that its apparent dimensions were only as great as those of the sun, and assume for simplicity that it is spherical like the sun. Then if they were in the same direction as seen from the observer at O (Fig. 199), they would fill the same part of the sky. Since they are by hypothesis similar, their volumes are as the cubes of their distances from O. With a parallax of $0.01''$ this ratio is $(20{,}000{,}000)^3$ to 1^3. If their densities were equal, this would be the ratio of their masses. Since attractions vary inversely as the squares of the distances, the nebula and the sun, if equally dense, would attract O in the ratio 20,000,000 to 1. Even if the nebula is $\frac{1}{20{,}000{,}000}$ as dense

as the sun, it attracts the earth as much as the sun does. Since no nebula has yet given any evidence of disturbing the motions of even those stars nearest to them, their extreme rarity may be inferred.

The spiral nebulas are most numerous outside of the Milky Way, and are in this way distinguished from clusters and diffuse nebulas. They seem to be quite unrelated to other types of nebulas both in character and origin, for no connecting links

FIG. 199.

The volumes of nebulas of given apparent diameter are as the cubes of their distances, and, if their densities are the same, their attractions are directly as their distances.

have so far been discovered. Knowledge of their internal motions is as yet entirely lacking. It is worthy of note that they are frequently, if not generally, closely accompanied by other spirals.

413. Planetary Nebulas. — Planetary nebulas are small, round, or elliptical, nearly uniform faint objects presenting a disk much like that of a planet. It has been suggested that they are oblate bodies made thus by rapid rotations. Their spectra contain the bright lines in the green of a substance called "nebulium" because it is not found except in nebulas. Planetary nebulas often contain brighter nuclei, as though they were condensing down into stars at their centers. Still smaller and somewhat brighter nebulas, called "stellar nebulas," are known. They perhaps represent the last stage of planetary nebulas before they become stars.

414. Ring Nebulas. — Only a few ring nebulas are known, of which the one in Lyra is the best example. (See Fig. 19.) It has a very faint star (fifteenth magnitude) near its center, which has been suspected of being variable. The

FIG. 200. — Nebula in Cygnus. *Photographed by Ritchey at the Yerkes Observatory.*

origin and development of these objects are quite beyond conjecture.

415. Irregular Nebulas. — There are many nebulas of irregular form. The Orion nebula is the finest of these and the most beautiful object in the heavens. (See Fig. 22.) It is very remote and enormously large. Its dimensions are something like those of the Andromeda nebula, and per-

haps even larger. Its spectrum contains the bright lines of nebulium and hydrogen. No relative motions of its parts have been telescopically observed, but Campbell has found spectroscopic evidence of different radial velocities in different parts. If nebulas actually condense into stars, we have here the material for hundreds of ordinary suns now in a very early stage of its development. The whole region for degrees around the Orion nebula is more or less covered with faint nebulous matter.

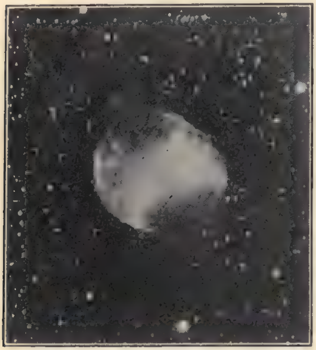


Fig. 201. — Dumb-bell Nebula. *Photographed at the Lick Observatory.*

There are nebulas which appear to have been broken into pieces. Of this type is the Trifid Nebula in Sagittarius. (See Fig. 189.) If they have been broken up, the forces which have done it are unimaginably great; if not, it is strange that gravitation does not draw the parts together.

There are irregular nebulas like the one in Cygnus (Fig. 200). (See also Fig. 7.) Very curiously the stars are much more numerous on one side of it than they are on the other. This seems to be a young nebula. In fact, it suggests the idea that the development of matter from something more primitive, perhaps the ether, may be taking place all the time. At the other extreme of the evolution, matter may be disintegrating by radioactive and possibly other processes, and going back into a more elemental state. However, this is speculation.

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